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# Upper Secondary Mathematics Teachers' epistemological beliefs concerning the nature of mathematics

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*This study aimed to explore the structure and level of Grade 10-12 mathematics teachers' epistemological beliefs regarding the nature of mathematics. Data was drawn from the international comparative study New Open Research: Beliefs about Teaching Mathematics (NorBa) investigating mathematics teachers' beliefs in more than 15 countries. A total of 147 mathematics teachers completed the NorBa's questionnaire in Cyprus. Exploratory factor analysis was employed to explore the underlying structure of mathematics teachers' epistemological beliefs and MANOVA was employed to examine for differences in epistemological beliefs in relation to background characteristics. The results are discussed and compared with those of other countries in an attempt to explore for cultural differences.*

*Keywords: mathematics, teachers, epistemological, beliefs*

## Introduction

It is generally acknowledged that beliefs are part of teachers' professional competence. In line with this view, almost all theoretical classifications of teachers' professional competence (e.g. Ernest 1989; Thompson 1992) include, apart from knowledge, teachers' beliefs and attitudes. Beliefs are thought to be crucial to the perception of situations since they influence teachers' choices of actions (Felbrich, Müller & Blömeke, 2008; Felbrich, Kaiser & Schmotz, 2012). Hence, teachers' beliefs play an important role in teaching as they serve as a bridge between teachers' knowledge and their actual teaching (Wilkins, 2008; Felbrich et al., 2012).

Teachers' epistemological beliefs about the nature of mathematics present an issue that has been prominent in empirical research (Felbrich et al., 2008; Felbrich, et al., 2012; Törner & Pehkonen, 1996). In their recent overview of the research literature concerning mathematical epistemological beliefs, Depaepe, De Corte and Verchaffel (2016) state the importance of teachers' epistemological beliefs since these beliefs affect teachers' selection of learning tasks and classroom activities and also influence students' epistemological beliefs. Different categorization schemes for teachers' epistemological beliefs concerning the nature of mathematics have been developed (Ernest, 1989; Blömeke, Felbrich, Müller, Kaiser, & Lehmann, 2008; Felbrich et al., 2012) indicating that teachers differ regarding their epistemological beliefs. It is stated that beliefs have an experiential and context-bound nature and can be understood as socially and culturally shaped mental constructs (Felbrich et al., 2012; Schoenfeld, 1998). Thus, teachers' epistemological beliefs may differ according to background variables or across different countries.

This study focuses on mathematics teachers' epistemological beliefs about the nature of mathematics. Specifically, we investigated the structure and level of 10-12th grade mathematics

teachers' epistemological beliefs. Furthermore, we investigated for differences in these teachers' epistemological beliefs in relation to their background characteristics.

## **Theoretical framework**

### **Teachers' epistemological beliefs about mathematics**

Beliefs in a more broad meaning could be described as “psychologically held understanding, premises, or propositions about the world felt to be true” (Richardson 1996, p.103). In the domain of mathematics, teachers' epistemological beliefs refer to beliefs concerning the nature of mathematics or beliefs concerning the acquisition of mathematical knowledge (Depaepe, et al., 2016; Felbrich, et al., 2012).

Different categorizations for teachers' epistemological beliefs concerning the nature of mathematics have been developed in the respective literature. Ernest (1989) distinguishes between three views on the nature of mathematics: an instrumentalist, a Platonist and a problem-solving view. Another classification is that of Grigutsch, Raatz and Törner (1998) who developed four distinct views on the nature of mathematics: (a) the formalism-related view where mathematics is conceived as an axiomatic system, developed by deduction, (b) the scheme-related view, where mathematics is regarded as a collection of terms, rules and formulae, (c) the process-related view, where mathematics can be understood as a science which mainly consists of problem solving processes and (d) the application-related view, where mathematics can be understood as a science which is relevant for society and everyday life. The formalism-related view corresponds to Ernest's Platonist view, the scheme-related view parallels Ernest's instrumentalist perspective while the process view resembles Ernest's problem solving view (Ernest 1989; Felbrich et al., 2012). Grigutsch et al. (1998) suggested, based on empirical work, that these four views could be subsumed under two perspectives of mathematics: the formalism together with the scheme-related view, describing mathematics as a static science and the process together with the application-related view, characterizing mathematics as a dynamic process. In their study they found that application and process views positively correlated with each other and the same happened with formalism and scheme views. Moreover, they found that the process view correlated negatively with the formalism and scheme orientations whereas there could be no systematic correlation traced between the application view and any other view except for the process view.

Some empirical studies on teachers' epistemological beliefs regarding the nature of mathematics involved teachers at different levels of mathematics education. The study by Felbrich et al. (2008) investigated the structure and level of beliefs concerning the nature of mathematics of future mathematics teachers in Germany at the beginning and at the end of their education and also the epistemological beliefs of their educators in three academic disciplines (mathematics, mathematics pedagogy and general pedagogy). Using an adaptive version of the questionnaire developed by Grigutsch et al. (1998), they confirmed a solution of four factors corresponding to the four categories of teachers' epistemological beliefs. With respect to the overall level of beliefs, both groups of future teachers' highly agreed with process-related statements while they agreed less with scheme-related statements. Another interesting result was that future teachers' views seemed to be dominated by the dynamic aspect of mathematics (process and application) while the static aspect

(formalism and scheme) received less agreement; this pattern was more pronounced at the end of their education. In addition strong positive relations between formalism and scheme as well as between process and application views have been found for future teachers. The belief structure of these future teachers could not be characterized by the two poles as antagonistic (static and dynamic). Mathematics educators agreed with process, application and formalism related aspects and showed a less positive attitude toward scheme-related aspects while educators of mathematics pedagogy hold a more antagonistic view of mathematics as indicated by their preference towards dynamic aspects and relatively lower preference towards the static aspect.

As Depaepe et al. (2016) concluded, research has revealed that it is not easy to describe one's epistemological beliefs as either static or dynamic. Investigating the structure of epistemological beliefs with respect to a different population, this of mathematics instructors at university, Grigutich & Törner (1998) found that the antagonistic structure of beliefs regarding the nature of mathematics could not be found. The epistemological beliefs of these mathematicians could be described by both the dynamic and the static aspect co-occurring at the same time, revealing a complex understanding of mathematics. In the same vein, Roesken and Törner (2010) found that university mathematics teachers can hold simultaneously static and dynamic epistemological beliefs.

Regarding cultural differences, Felbrich et al. (2012) analyzed pre-service primary teachers' epistemological beliefs in mathematics in 15 different countries and investigated whether differences in these beliefs could be explained in terms of cultural differences between countries, using Hofstede's terminology, individualistic versus collectivistic orientation. For this purpose, they used instead of the four views of mathematics the two perspectives, mathematics as a static science and mathematics as a dynamic process, stating that this was more useful in quantitative large-scale research. In individualistic countries (e.g. Germany, Switzerland, Norway), learners are perceived as autonomous subjects acquiring knowledge mainly independently on their own. In collectivistic countries (e.g. Philippines, Thailand, Malaysia) the role of social relationships on the acquisition of knowledge is more prominent. Learners participate in learning due to an obligation towards their teachers, their families and society. The results of the study showed that the epistemological beliefs of these future teachers varied strongly within but also between countries. Future primary teachers from highly collectivistic oriented societies agreed more strongly with the static aspects of mathematics while in highly individualistic oriented societies, the future teachers more strongly stressed the dynamic nature of mathematics. Additionally, in countries that could not clearly be characterized as individualistic or collectivistic, future teachers emphasized both aspects of mathematics to the same extent. The data also showed that future primary teachers with high mathematical content knowledge conceived mathematics as more dynamic in nature compared to the future primary teachers with lower mathematical content knowledge.

Depaepe et al. (2016) stated that there is a strong need for a clear conceptualization of mathematics epistemological beliefs. In addition, the studies concerning mathematicians' epistemological beliefs are rare. Hence, the purpose of this study was to investigate (a) the structure and patterns of the epistemological beliefs for a different population, i.e. 10-12th Grade mathematics teachers in the Cyprus context and (b) the differences in these teachers' epistemological beliefs in relation to their background information.

## **Methodology**

### **Data collection, instruments and participants**

According to the Annual Report of the Cyprus Ministry of Education and Culture (2014), the Public Secondary General Education in Cyprus is offered to pupils between the ages of 12 - 17, through two three-year levels: the Gymnasium (Grades 7-9, ages 12-14) and the Lyceum (Grades 10-12, ages 15-17). Currently, there are 38 Lycea and 7 joined Gymnasias and Lycea in Cyprus, where approximately 280 mathematics teachers are employed. In Cyprus, there are minimum requirements to work as a mathematics teacher for grades 7-12 which refer to a bachelor's degree in mathematics and also to the attendance to a one year course in pedagogy offered by the Ministry of Education and Culture.

Data for this study was gathered from mathematics teachers working in Lyceum during the 2015-2016 academic year. The study was conducted in the context of the international comparative study New Open Research: Beliefs about Teaching Mathematics (NorBa) investigating mathematics teachers' beliefs in more than 15 countries. In the context of the NorBa project, a questionnaire was developed and culturally adapted in all participating countries. The questionnaire comprised of seven parts: one of them qualitative and six quantitative (86 items). The current study used data only from two parts (Part A and Part F) of the afore-mentioned questionnaire. Part A collected data on teachers' background variables (gender, age, their teaching experience and highest level of formal education) while Part F explored mathematics teachers' epistemological views on mathematics.

Part F included the shortened version (20 items) of an instrument developed by Grigutsch et al. (1998) also used in Mathematics Teaching in the 21<sup>st</sup> century (MT21) study by Schmidt, Blömeke and Tatto (2011). The formalism-related and scheme-related scales consisted of 5 items each whereas the process-related and the application-related scales were approached using 6 and 4 items respectively. Example items for all subscales are presented below: for the formalism scale, "Mathematical thought is characterized by abstraction and logic", for the scheme scale, "Mathematics is a collection of rules and procedures that prescribe how to solve a problem", for the process scale, "Mathematical problems can be solved correctly in many ways" and for the application scale, "Mathematics entails a fundamental benefit for society". Respondents were asked to show their agreement or disagreement to each item on a five-point Likert scale ranging from strongly disagree to strongly agree.

Informative letters along with the questionnaire and prepaid envelopes were sent to mathematics teachers in all Lyceums inviting them to participate in the study on a volunteer basis. A total of 147 mathematics teachers (44,8 % men and 55,2% women) completed and returned the questionnaire. The response rate was 52,5%.

Various statistical techniques were employed to analyze the data of the current study. Confirmatory factor analysis (CFA) and Exploratory factor analysis (EFA) were employed in order to investigate the possible underlying structure of mathematics teachers' epistemological beliefs, correlation analysis aimed to explore the relationship between the resulting factors and finally MANOVA was

employed to investigate for differences in epistemological beliefs in terms of background variables. Data was analyzed using the statistical packages AMOS and SPSS.

## Results

Regarding the first research question of this study, a CFA using AMOS was carried out in order to validate the suggested structure for the 20 items. By employing a maximum likelihood estimation method, three types of fit indices were used to assess the overall fit of the model: the chi-square index, the comparative fit index (CFI), and the root mean square error of approximation (RMSEA). Hu and Bentler (1999) suggest using a combination of cutoff values for the aforementioned fit indices ( $CFI \geq 0.95$  and  $RMSEA \leq 0.06$ ). The model fits the data sufficiently when the ratio of chi-square statistic over degrees of freedom does not exceed 3. A solution with four factors as suggested by the respective literature did not achieve satisfactory fit indices ( $\chi^2 = 272.334$ ,  $df=164$ ,  $p < 0.001$ ,  $CFI=0.847$ ,  $RMSEA=0.068$ ). Therefore, we proceeded with exploratory factor analysis in order to explore a new country-specific structure for the 20 items. EFA was employed on the 20 items of the epistemological beliefs using maximum likelihood with orthogonal rotation (varimax). The Kaiser-Meyer-Olkin statistic (0.77) and Bartlett's Test of Sphericity ( $\chi^2 (120) = 694.872$ ,  $p < .001$ ), indicated that data was suitable for factor analysis. To determine the number of factors to be retained two criteria were used: a) Kaiser's criterion for eigenvalues greater than one and b) the scree plot (Kaiser 1974).

	1	2	3	4
<b>Factor 1: Formalism</b>				
Essential for mathematics is definitional rigor, i.e. an exact and precise mathematical language.	.784			
Fundamental to mathematics is its logical rigor and preciseness.	.744			
Mathematics is characterized by rigor, namely rigor of definition and rigor of formal mathematical argumentation.	.673			
Mathematical thought is characterized by abstraction and logic.	.462			
Hallmarks of mathematics are clarity, precision and unambiguousness.	.410			
<b>Factor 2: Application</b>				
Mathematics helps solve everyday problems and tasks.		.776		
Mathematics entails a fundamental benefit for society.		.755		
Many aspects of mathematics have practical relevance.		.540		
Mathematics is useful for every profession.		.484		
<b>Factor 3: Scheme</b>				
To do mathematics requires much practice, correct application of routines, and problem solving strategies.			.877	
When solving mathematical tasks one has to know the correct procedure else one is lost.			.570	
Mathematics means learning, remembering and applying.			.565	
Mathematics is a collection of rules and procedures that prescribe how to solve a problem.			.517	
<b>Factor 4: Process</b>				
Mathematical problems can be solved correctly in many ways.				.762
In mathematics many things can be discovered and tried out by oneself.				.601
Usually there is more than one way to solve mathematical tasks and				.567

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problems.

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**Table 1: Factor analysis of the epistemological beliefs about mathematics**

In line with respective literature, only items with component loadings larger than 0.30 were retained (16 items). Both criteria supported a four-factor solution explaining 48.08% of the total variance. Factors were named according to the items included in each group. Items that loaded on Factor 1 represent the formalism-related view, items that loaded on Factor 2 represent the application-related view, Factor 3 items represented the scheme-related view and Factor 4 items represented the process-related view. All factors had sufficient internal consistency values (Formalism- $\alpha=0.78$ , Application- $\alpha=0.76$ , Scheme- $\alpha=0.72$  and Process- $\alpha=0.70$ ).

Table 2 presents the mean scores of mathematics teachers' epistemological beliefs for each dimension. Mathematics teachers' epistemological beliefs are characterized by a high level of agreement to statements representing the application and process view. Additionally, they display a strong agreement with statements representing a formalism view whereas they stand somewhere in the middle towards the schematic view.

Scale	Mean	SD
Formalism	4.08	.57
Application	4.46	.46
Scheme	3.47	.70
Process	4.30	.48

**Table 2: Mean estimates for mathematics teachers' epistemological beliefs**

Table 3 presents the correlations between the different dimensions of epistemological beliefs. According to Table 3, correlations were very low and statistically insignificant for the four dimensions concerning the nature of mathematics.

	Scheme	Application	Process
Formalism	.060	.051	.021
Process	-.026	.105	
Application	.040		

**Table 3: Bivariate correlations between the four beliefs concerning the nature of mathematics**

Finally, in order to explore for differences regarding mathematics teachers' epistemological beliefs in relation to various background variables (gender, age, their teaching experience and highest level of formal education), we employed a multivariate analysis of variance (MANOVA). However, we found no statistically significant differences between teachers' epistemological beliefs in terms of these background variables.

## Discussion

In this paper, we investigated the beliefs concerning the nature of mathematics of upper secondary Cypriot Mathematics teachers. Regarding the first aim of the study, the results confirmed a solution

of four factors corresponding to the four categories of teachers' epistemological beliefs, suggested by relative literature. These teachers highly agreed with application- and process-related statements, whereas schematics statements were less agreed with. Compared to the results of Felbrich et al. (2008), the ranking of the four views was slightly different. While in Felbrich et al. (2008) study, the process view received the highest agreement level, in this study the highest agreement level was received by the application view. Cypriot mathematics teachers strongly agreed with both application and process-related statements, perhaps due to the new curriculum which has been implemented since 2011 and follows current trends in education, approaching mathematics as a dynamic tool for thought (Cyprus Ministry of Education and Culture, 2014). More specifically, the new mathematics curriculum has been designed according to principles, such as that students should be involved in mathematical investigations, based on real life situations and that interdisciplinary questions and emphasis should be placed on problem-solving (Cyprus Ministry of Education and Culture, 2014).

The results of the current study showed no statistically significant correlation between the four factors contrary to the results of other studies (Felbrich et al., 2008; Grigutsch et al., 1998). No correlation between the dynamic aspects of epistemological beliefs was also reported in the study by Liebendörfer & Schukajlow (2017). Moreover, the two fundamental dimensions of mathematics, namely the dynamic and the static dimension could not apply to this sample of mathematics teachers, supporting Depaepe's et al. (2016) conclusion that it is not easy to describe one's epistemological beliefs as either static or dynamic. Furthermore, the results of this study supported the results of other studies (Felbrich et al., 2008; Grigutsch & Törner, 1998; Roesken & Törner, 2010) which suggest that mathematics teachers can hold simultaneously static and dynamic epistemological beliefs.

These mathematics teachers agreed with both the dynamic and static epistemological beliefs' dimensions, exhibiting, using Hofstede's terminology, characteristics of both collectivism and individualism aspects. Countries in which teachers adopted both dimensions were also mentioned in Felbrich et al. (2012), with a different population (i.e. future primary school teachers). However, further study is needed using related scales (e.g. Individualism scale of Hofstede) in order to substantiate this argument.

Finally, an interesting result worth mentioning is that no statistically significant differences were found regarding these teachers' epistemological beliefs in terms of background variables (gender, age, their teaching experience and highest level of formal education). These results do not agree with the results reported by Felbrich et al. (2012) in which future teachers with high mathematical content knowledge endorsed dynamic beliefs more strongly than static beliefs. We presume that background variables are shadowed both by the centralized nature of the educational system in Cyprus and the centralized in-service training these teachers' received regarding the new mathematics curriculum.

In conclusion, there is more to be done in this field concerning mathematics epistemological beliefs held by mathematics teachers in various grade levels. More research is also needed regarding the relationship between epistemological beliefs and other beliefs such as teacher authority and self-



efficacy beliefs. Moreover, mathematics teachers' epistemological beliefs and their instructional approaches worth more attention since empirical evidence in this direction remains scarce (Depaepe et al., 2016).

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