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Reflections on the development and application of an instrument to access mathematical identity

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Mathematical identity is understood in this paper as the multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others. The researchers developed a protocol for accessing mathematical identity, initially designed for use with initial teacher education (ITE) students. The protocol has been refined and its use extended to students of mathematics in other contexts in Ireland. This paper reviews the iterations of the protocol, its development into an instrument, and the context and use of such an instrument. It addresses the issue of robustness of the instrument when applying it to contexts within and outside ITE.

Keywords: Mathematical identity, personal narratives, online survey.

Introduction

Mathematical identity has been a topic of interest especially over the last twenty years, and many studies have addressed the mathematical identity of teachers or students in initial teacher education (ITE) programmes. The work described in this paper is intended to contribute further to exploring students' mathematical identity. Originally focused on ITE students, it now includes other cohorts.

In 2009, as part of the project MIST (The Mathematical Identity of Student Teachers), a protocol was developed that scaffolded students in writing about their encounters with mathematics. It was used with two groups of ITE students in the island of Ireland (Eaton & O'Reilly, 2009). The aim was to encourage them to reflect on their mathematical identity with a view to facilitating discussions around how that identity may impact on their own classroom practice. The students' narrative responses allowed certain themes to be identified that spanned the responses of both cohorts. These themes also sufficed to describe the results of using a very similar protocol with undergraduate mathematicians taking modules on mathematics education (Eaton, Oldham, & O'Reilly, 2011). For the next iteration, in the study Mathematical Identity using Narrative as a Tool (MINT), the protocol was developed into an instrument that could be administered online, with the aim of accessing further cohorts with greater ease and efficiency. It was used with six cohorts of students in both jurisdictions in Ireland, within and outside ITE (Eaton, Horn, Liston, Oldham, & O'Reilly, 2014, 2015). Details of these cohorts, and of the development of the instrument, are given below.

The main aim of this paper is to address the usefulness of the instrument in a variety of settings. The paper first situates the work of the projects MIST and MINT within traditions of research on identity as distinguished in recent reviews of that research, and then traces the evolution of the

original protocol into the instrument used in MINT. In particular, the authors focus on how the themes identified initially in MIST apply to other cohorts (even outside ITE), and reflect on how robust the outcome themes appear to be, regardless of the cohort of students addressed.

Literature review and outline of a theoretical framework

Research in mathematical identity has attracted, and received, overviews and critical analyses for at least two decades. Earlier syntheses are discussed and expanded in some more recent surveys (Darragh, 2016; Goldin et al., 2016; Hannula, Pantziara, & Di Martino, 2018; Lutovac & Kaasila, 2018). From these overviews, several aspects of relevance to this paper can be distinguished; they provide a framework within which the studies of mathematical identity can be addressed.

A notable aspect is that there is still no agreement on the *definition of mathematical identity*. Darragh (2016) highlights and critiques the differing and indeed conflicting versions, or, in some papers, the absence of any clear definition. In Lutovac and Kaasila's (2018) overview, the uncertainty created by multiple definitions is regarded as a given; however, following Sfard and Prusak (2005), they demand that studies should provide at least a working definition.

Approaches can also be categorized as *broadly social* or *broadly psychological*. Many of the studies on mathematical identity draw on Wenger's (1998) work on communities of practice. In this respect, they tend to follow what Lerman (2000) calls the social turn in mathematics education research that has been a prominent feature especially from the mid-1980s (Goldin et al., 2016). However, Lutovac and Kaasila (2018) argue that "neglecting the individual, i.e. how one thinks and feels and who one is, is at odds with the core concept of identity itself" (p. 767); they advocate for a more balanced psychosocial approach that allows for a focus on individuals' thinking and feeling.

Methodology constitutes another aspect. Qualitative approaches have dominated, with sample sizes typically being small, and just a few studies using coding and counting techniques (Darragh, 2016; Goldin et al., 2016; Hannula et al., 2018). The use of narrative has been particularly important; in a seminal paper, Sfard and Prusak (2005) actually take the narratives as constituting identities, hence opening the way to approaches requiring "close attention to the words used" (Darragh, 2016, p. 25).

The *mathematical identity of ITE students* has been a topic attracting considerable focus. The value of research in the area is highlighted, for example by adverting to "some evidence that encouraging pre-service teachers to narrate their own or listen to their peers' personal experiences with the subject makes them cope better, which may lead to the development of a more suitable identity for mathematics teaching" (Goldin et al., 2016, p. 17). Lutovac and Kaasila (2018) also note the process of change in ITE students' mathematical identity, for example by emphasizing affective facets.

Methodology and evolution from MIST to MINT

The *definition* of mathematical identity formulated for the studies considered here is the following:

Mathematical identity is considered as the multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others.

(See Eaton et al., 2014, for references.) This situated the work primarily in the *broadly social* tradition, with some psychological aspects. The *methodology* has chiefly involved students writing narrative accounts; analysis of the resulting qualitative data has been done by coding and counting as described below. Most work has involved *ITE students*; extension to other cohorts is a focus of this paper.

The protocols and instrument, student cohorts, and establishment of themes

The first study (of three), MIST, considered the identity of ITE students (pre-service primary teachers) who were at one of two institutions – one in Northern Ireland and one in the Republic of Ireland – and were specialising in mathematics (see, for example, Eaton and O'Reilly, 2009). Introductory demographic questions and Likert items were used along with a two-part protocol:

- P1. Students were asked to respond in writing to the prompt: “Think about your total experience of mathematics. Tell us about the dominant features that come to mind.”
- P2. They were then offered a more structured prompt: “Now think carefully about all stages of your mathematical journey from primary school (or earlier) to university mathematics. Consider:
- Why you chose to study mathematics at third level
 - Influential people
 - Critical incidents or events
 - Your feelings or attitudes to mathematics
 - How mathematics compares to other subjects
 - Mathematical content/topics

With these and other thoughts in mind, describe some further features of your relationship with mathematics over time.”

The narrative responses were used to provide guidelines for focus group discussions, one in each institution. Discussion transcripts, together with the original narratives, were analysed to identify common threads and themes (Clandinin & Connelly, 2000). Using an approach informed by grounded theory, seven themes were identified; their main features can be summarized as follows:

- T1. **Students’ self-reflection on learning and teaching** relates to how students’ exploration of their mathematical identity leads them to deepen their insight into learning and teaching mathematics.
- T2. **The role played by key figures in the formation of mathematical identity** focuses not only on teachers and family members, but also on peers and society at large.
- T3. **Ways of working in mathematics** explores what students find effective in learning mathematics and why, either through individual endeavour or through collaboration.
- T4. **How learning mathematics compares with learning in other subjects** considers the particular characteristics of learning mathematics, usually at school, that distinguish the process from learning in other subject areas.
- T5. **The nature of mathematics** draws from a broad range of students’ perceptions touching on the philosophy of mathematics and on what doing mathematics is about.

T6. **“Right” and “wrong” in mathematics** concerns students’ perception that what is important in mathematics is to find the correct answer, and also a more general notion around the unambiguous nature of mathematical truth.

T7. **Mathematics as a rewarding subject** examines the extent to which students enjoy the subject, often relating to how they persist with it or to significant moments of insight. (Eaton et al., 2011, pp. 156–157)

These reflect both social and psychological aspects. For instance, T4 involves both aspects, and T7 in particular includes affective issues, as highlighted by Lutovac and Kaasila (2018).

In the second study, the students were undergraduate mathematicians at another institution, had elected to take modules in mathematics education, and thus were considered as “prospective” teachers despite not being in an ITE programme. Students wrote responses to P1 and P2 in their own time as part of the module assessment. Analysis was based on the MIST themes, which were found to be adequate for describing these narratives (Eaton et al., 2011; Eaton et al., 2015).

In its third iteration, MINT, the research team of three people – the two from MIST, one from the second study – was augmented to five, with new members coming from two institutions in the Republic of Ireland. The protocol was extended (Eaton et al., 2015) to include a third part:

P3. What insight, if any, have you gained about your own attitude to mathematics and studying the subject as a result of completing the questionnaire?

The items, including introductory demographic questions and Likert items as used for MIST, were gathered into one instrument and made available online to six cohorts of students using SurveyMonkey, allowing students to respond at their chosen time and location. The students in four of the cohorts were in Initial Teacher Education (ITE) programmes. The other two cohorts were not; one comprised first year students of entrepreneurship (involving a one-year business mathematics module), while the other consisted of fourth year students of applied psychology (taking research methods and statistics-related modules in all of their four years of study). For the purposes of this paper, these two cohorts were denoted as Non Teacher Education (NTE).

Coding the MINT data

Of the 99 respondents in the MINT study, 86 provided data (narrative responses) for at least one of the three open questions, P1, P2 and P3; detail relevant to this discussion is provided in Table 1 below. The five members of the research team were assigned respondents, ensuring that the data from each respondent was studied by three people; each person coded the data using the MIST themes and also sought to identify any other themes. No other themes were found. Several meetings were held to compare the coding and reconcile differences; this entailed clarifying the scope of some of the themes to obviate confusion and/or overlap, as elaborated below. Consensus was reached, so the question of inter-coder reliability did not arise. Overall, the seven themes provided adequate descriptions and coverage of the data, though themes T4 and T6 were less prominent than the others. Once the coding was complete for each element (respondent and question, P1-P3) of data, the instances of each theme (T1-T7) were counted. Analysis of data coded T1 (**students’ self-reflection on learning and teaching**) from the 56 ITE students is reported by Eaton et al. (2015).

As regards clarification, when the MINT research team discussed the responses that were potentially to be coded T1, for example, the theme was refined as follows: T1 (i) applied when students explicitly reflected on their mathematical journey with reference to “then” and “now”; (ii) was extended to include reference to how students see themselves teaching and learning into the future; (iii) did not apply to data that comprised lists without context. There was also some development concerning potential overlap of themes, particularly T3 (**ways of working in mathematics**) and T5 (**the nature of mathematics**). The research team agreed that T3 applied to (i) what students did as they learned mathematics, inside or outside “the classroom”, and (ii) what students find effective in learning mathematics and why, either through individual endeavour or through collaboration. Moreover, it was agreed that T3 would include “rote learning” and “learning for understanding” where the learner is the agent (rather than experiencing teaching for rote learning or understanding, for example). In the summary description of T3, it was agreed that “effective” would be interpreted as anything on the spectrum of effectiveness (including ineffective); T5 would include (i) the “relevance versus abstraction” dimension and (ii) applications in the real world. After clarifying such matters in relation to coding, it was judged that the MINT instrument was appropriate for exploring characteristics of mathematical identity among different cohorts of students or in different settings.

Extending the instrument to NTE students

The application of the instrument to NTE students in MINT is considered in this section, and a comparison is made between the data from this cohort and the ITE data. It was found that the same themes could be applied satisfactorily across the entire MINT study; however, more detailed analysis is of interest.

Cohort (ITE or NTE); questions from protocol (P)	Number of respondents	Percentage of students in each cohort coded in each theme						
		T1	T2	T3	T4	T5	T6	T7
ITE; P1	52	29	44	58	4	17	6	83
ITE; P1, P2 & P3 combined	56	48	84	63	36	38	23	98
NTE; P1	27	22	19	30	4	52	4	89
NTE; P1, P2 & P3 combined	30	63	70	47	33	67	10	97

Table 1: Coding of themes in response to open questions

A feature of the protocol (for both MIST and MINT) is that respondents did not see P2 until after they had responded to P1. The importance of this open-ended and non-directive initial question is discussed in Eaton and O'Reilly (2009). Of the 56 ITE students who answered at least one of the three open questions, 52 responded to P1. Likewise, 27 out of 30 NTE students answered P1. On examination of the responses from all six cohorts (four ITE and two NTE), Table 1 summarizes the coding for the themes (T1-T7) for ITE and NTE students, drawing attention to the frequencies of

the codes in responses to P1 and to all three parts (P1-P3) of the protocol combined, as percentages of the number of respondents.

The table reveals that themes T6 and T4 are weakest for each cohort; their frequency is especially low in response to P1. The theme most related to affect, T7, is by far the strongest for each cohort. With the prompts offered by P2 and P3, the codes for each theme are more frequent, as might be expected. However the profile of the two cohorts differs between themes and according to the prompts given. From a qualitative perspective, we should expect that the narrative around mathematical identity would depend on the cohort to which students belong, reflecting factors such as programme focus, teaching culture or maturity of students. To test if there are any significant differences between the ITE and NTE cohorts, a Fisher's exact test was conducted for each of the themes. When all three questions in the protocol were considered together, the only statistically significant difference was for theme T5. NTE students were found to capture this theme significantly more frequently than did ITE students (with $p = .013$ in the Fisher test). For all other themes there was no statistically significant difference ($p > .15$ in all cases). However, when the responses to P1 alone are considered, the contrast between the two cohorts in relation to T5 is more stark ($p = .003$), while significant differences between the cohorts in relation to T2 and T3 are apparent (with $p = .027$ and $p = .032$, respectively). This indicates that prospective teachers are more likely to draw attention to the influence of others (such as their own teachers) and to the importance of ways of working with mathematics than are NTE students.

Focusing on T1, it was noted that ITE students' narrative relevant to this theme emphasized how the evolution of mathematical identity from second to third level education was intertwined with their commitment to teaching mathematics (Eaton et al., 2015). Naturally, such a perspective was found to be absent from the NTE cohort. However, significant evolution across the same educational transition was intertwined with the new focus that students appreciated regarding the applicability of the subject to the "real world". It is exemplified in the following quotation from one respondent:

Whereas in Maths in secondary once the problem was solved that was the end of the matter, in college we use the solution after its completion ... and apply them to everyday/business/social scenarios ... My experience of maths in 3rd level has given me a new opinion on the subject, I can clearly see its uses, and an understanding of where to apply them. (P1, NTE)

As far as T5 (the nature of mathematics) is concerned, the importance of application of mathematics to real-world problems comes out strongly for 17 of the 20 NTE students whose responses were coded for this theme. Here is an example of what one student wrote:

Mathematics, in particular statistics is vital within the area of Psychology. I really enjoyed it during third level because I could see what the analyses were being used for. ... Also, while I really enjoy maths now, I feel that the style of teaching through primary and secondary school had given me a complete misconception into what maths really is. (P2, NTE)

In summary, the data show that the NTE students in MINT emphasise the practical nature and importance of mathematics and its relevance to business or psychology. It is reasonable to argue that precisely this aspect gives rise to the much higher instance of T5 among the NTE students than among ITE students.

Robustness of the instrument

Two aspects of robustness are considered here. They are *robustness of the themes* (their adequacy in describing data for different cohorts), and *inter-coder reliability* in using the instrument.

The seven themes identified in MIST, using a grounded approach following the steps of thematic analysis, were re-evaluated and basically endorsed in MINT by the members of the larger research team. The argument so far has shown that these themes appear to be robust in facilitating access to the mathematical identities of differing cohort of students: ITE students in different types of ITE programme, and in two different (yet neighbouring) countries; and also NTE students who had some element of mathematical work in their courses. No new themes have emerged. While variations are evident in the frequency with which the themes figured for the two cohorts, these could be related to the backgrounds and interests of the cohorts and are of intrinsic interest. It can be noted that placing the instrument online for MINT, using SurveyMonkey, reduced the influence of the researcher and strengthened anonymity for the respondents (as well as allowing them to respond where and when suited them); it also facilitated data collection for the research team.

With regard to ensuring inter-coder reliability, considerable work was needed. Some of this involved clarification of the borderlines between themes, as described above. Future coders might need training, as well as time to grasp the definition or description of each theme. However, while the “coding and counting” approach was important for research, the narrative data elicited can, in any case, be of value for lecturers in appreciating their students’ preoccupations. As judged by answers to P3, it is also of value to at least some of the respondents themselves, in line with the original aims for the studies (and perhaps aligning with the work of Goldin et al. (2016), cited above). In fact (though details are outside the scope of this paper), two of the authors have used the instrument with several cohorts of students, and have found that their own teaching was enriched by what students wrote and that many of them reported finding the exercise helpful.

Conclusion

The importance of mathematical identity is well established, particularly in teacher education, notwithstanding the challenges in establishing clear definitions and a shared understanding in the academic community. This paper has drawn attention to the development of one protocol and instrument that have proved stable when working with a range of students both within and outside teacher education and in different countries. The online instrument used has elicited both social and psychological aspects of identity. It can be easily administered to students, and allows teachers and even students themselves to gain insight into mathematical identity, which can be beneficial for learning and teaching mathematics. The value of the data collected, for research and also for teaching and learning, promises well for further use in both similar and different contexts.

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References

- Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. San Francisco, CA: Jossey-Bass.
- Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, 93, 19–33.
- Eaton, P., Horn, C., Liston, M., Oldham, E., & O'Reilly, M. (2014). Developing an instrument to explore mathematical identity: A study of students from several third level institutions in Ireland. In E. Arntzen (Ed.), *Educating for the future: Proceedings of the ATEE 38th Annual Conference, Halden 2013* (pp. 280–297). Brussels, Belgium: ATEE.
- Eaton, P., Horn, C., Liston, M., Oldham, E., & O'Reilly, M. (2015). From learner to teacher: Using reflection to develop mathematical teacher identity. In L. Leite, M. A. Flores, L. Dourado, M. T. Vilaça, & S. Morgado (Eds.), *ATEE Annual Conference 2014: Transitions in teacher education and professional identities* (pp. 367–375). Brussels, Belgium: ATEE.
- Eaton, P., Oldham, E., & O'Reilly, M. (2011). Looking back to look forward: The mathematical identity of prospective mathematics teachers. In T. Dooley, D. Corcoran, & M. Ryan (Eds.), *Proceedings of Fourth Conference on Research in Mathematics Education (MEI 4)* (pp. 155–167). Dublin, Ireland: St. Patrick's College.
- Eaton, P., & O'Reilly, M. (2009). Who am I and how did I get here? Exploring the mathematical identity of student teachers. In D. Corcoran, T. Dooley, S. Close, & R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education (MEI 3)* (pp. 228–237). Dublin, Ireland: St. Patrick's College.
- Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., ... Zhang, Q. (2016). *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions*. New York, NY: Springer.
- Hannula, M., Pantziara, M., & Di Martino, P. (2018). Affect and mathematical thinking: Exploring developments, trends, and future directions. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 128–141). Oxford, England: Routledge.
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex.
- Lutovac, S., & Kaasila, R. (2018). Future directions in research on mathematics-related teacher identity. *International Journal of Science and Mathematics Education*, 16, 759–776.
- Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34(4), 14–22.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, England: Cambridge University Press.