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# On the role of multicriteria decision support in mathematical modelling

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*Modelling is one of the key skills in applied mathematics. This is reflected by the Standing Conference of the Ministers of Education in Germany, and, consequently, mathematical modelling is one of six competences mentioned in the Educational Standards. There is a vast amount of literature representing, discussing, and teaching modelling. In practice, a mathematical model serves the purpose of making fast, cheap and – probably most importantly – reliable decisions. Having this in mind, one should reconsider the way mathematical models are treated in activities with students. In this paper, we advocate a ‘multicriteria’ perspective on mathematical modelling, optimization and decision making. We present several examples from real-world problems illustrating the need of multicriteria decision making and give some first hints toward incorporating this multicriteria perspective in the well-known mathematical modelling paradigms.*

*Keywords: Mathematical models, decision making, multicriteria optimization, STEM education.*

## Introduction

Mathematical modelling is considered to be an important topic in our daily life. The usage of mathematical models is often not apparent, but they are omnipresent. Moreover, modelling and applications have been an important topic in mathematics education with growing importance since the late 1990s (Blum, 1993; Blum & Borromeo Ferri, 2009). Showing the applicability of mathematical ideas to real-world problems increases the motivation of students to study mathematics as well as their knowledge of several mathematical concepts (Borromeo Ferri, 2009). Furthermore, modelling provides opportunities to integrate mathematics in the curriculum that allow multidisciplinary teaching. Therefore, the need and the benefits of teaching mathematical modelling competences at school have already been recognized several years ago (Blum, 1993; Blum & Borromeo Ferri, 2009). Students should be able to set-up, simplify, analyse and compare mathematical models (KMK, 2012). The mathematical modelling competences of students should be stimulated with regard to other mathematical and general competences: the ability of communicating and discussing mathematical problems as well as designing and applying problem solving strategies. Moreover, the methodological, social and personal competences are increased (KMK, 2012). Obviously, mathematical modelling becomes a more and more important skill for students.

## A general view of mathematical modelling

### Mathematical models and modelling

Having stressed the necessity of mathematical models and modelling, we now review these concepts briefly. Though the notions of ‘modelling’ and ‘mathematical model’ show overlaps, differences are observable: Modelling means to construct a mathematical model, so we first discuss

the mathematical modelling process before we consider modelling cycles. There is a broad consensus in the international discussion, that mathematical modelling is a process which translates between reality and mathematics in both directions (Blum & Niss, 1991). Pollak (1979, p. 233) defines mathematical modelling as a special aspect of applied mathematics. He distinguishes between several definitions: The “classical applied mathematics” and the applied mathematics with focus on a “significant practical application” are part of the mathematics as a whole. Furthermore, applied mathematics implies going around the cycle once: “Beginning with a situation [...] in real life, making a mathematical [...] model, doing mathematical work within that model, and applying the results to the origin situation”. Another definition of applied mathematics refers to the preceding definition, but means going around the cycle more than once. The last two definitions are process-oriented and focus on the real world. This implies that modelling is a process for solving real-life problems. Borromeo Ferri and Blum (2009, p. 45) define a mathematical model as “the process of translating between the real world and mathematics in both directions”. Reit (2016) states, in the tradition of Pollak (1979), that mathematical modelling is not a process in both directions: The translation from reality to mathematics is a mathematical model, the reversal, the process from mathematics to reality, is an application. Thus, modelling emphasizes the process – the implementation of real problems in mathematics – while applications focus on the mathematics. The aim of a mathematical model is, as already stated by Blum (1993), the comprehensible simplification of the real problem for enabling the application of mathematical methods. Just a few details from reality are considered. The fundamental properties will be extracted and translated into mathematics; the simplification of the problem usually results in a solution. These models are not unique and non-linear processes; there often exist several mathematical models for a real-life situation (Greefrath et al., 2013). For Blum (1993, p. 4), the mathematical modelling begins with a situation in the real world. The problem has to be structured and simplified, so the solver gets “a real model of the situation”. Then, the real model will be translated into mathematics. The result is a mathematical model. In the next step, the model is solved and translated back into the real model. Often, the first model-based attempt to solving a problem is not satisfying. Thus, more attempts might be necessary which is why mathematical modelling should be regarded as a cyclic task.

### **The concept of modelling cycles**

As Blum (1993) illustrated, the process of mathematical modelling is often depicted in terms of modelling cycles – a representation which is widely accepted. Modelling cycles are useful to describe a mathematical process. A huge number of such cyclic representations are discussed in the literature: They mainly differ in the concept of the sub-steps, the number of sub-steps or the chronological order of the sub-steps. We are geared to the categorization due to Borromeo Ferri and Kaiser (2008), which characterizes the modelling cycles by their phases: The easiest example of a cycle has a real-life problem at the beginning. This problem leads to a real model, which has to be translated into a mathematical model. This model is treated mathematically, and the result is transferred to the real-life problem. Greefrath et al. (2013) define three categories of modelling cycles with the focus on mathematisation: the direct, the two-step and the three-step mathematisation. For the directed mathematisation, it is typical that there exists only one step from the real-life problem to the mathematical model. The direct transition between real-life problem and

mathematical model refers to the definition of applied mathematics. In contrast to the directed modelling, the modelling cycles in the two-step mathematisation include an additional step between the real-life situation and the mathematical model. The most famous modelling cycle of this category is from Blum (1985). The simplification of reality is called the real model and is a phase of its own. This kind of modelling cycle is typically used for modelling in school. The three-step mathematisation is characterized by the cognitive process. In modelling cycles, this cognitive process is described by an own phase: the situation model, i.e. the mental representation of the situation (Borromeo Ferri, 2006). For Blum and Leiss (2007) the phase of the situation model is the most important one during the modelling process. This phase characterizes the translation between the real situation and the situation model, so it can be seen “as a phase of understanding the task” (Borromeo Ferri, 2006, p. 87).

### **Research question**

Literature on mathematical modelling, especially on modelling cycles, has not focused much on aspects of discussing of a solution provided by using a mathematical model. Multicriteria decision making as an element of mathematical modelling has not been addressed at all to the best of our knowledge. Our proposal is to take multiple criteria and perspectives into account and to research how the multicriteria perspective can be incorporated in mathematical modelling. In this paper we examine how decision making affects mathematical modelling in schools and extracurricular activities with students by elaborate and discuss two example cases (see below).

### **Multicriteria decision making (MCDM)**

Decisions are an important part of our life, they consider multiple, often contradicting criteria. Thus, solving decision problems is difficult: The simultaneous consideration of different alternatives and their consequences often prevents us from making a good decision. We all strive for a “good” decision and frequently “solve” MCDM problems. From a mathematical point of view, trading-off between contradicting criteria needs the integration of multicriteria optimization and decision-making methods into modelling cycle.

### **Mathematical background**

One of the main characteristics of multicriteria decision making is the presence of several, non-commensurable objectives to be pursued by an entity (e.g. a person, a group or a company) called Decision Maker (DM). It is generally assumed that a DM tries to maximize his/her utility while choosing one out of a set of alternatives. Utility is a theoretical concept which describes the value of some alternative for the DM; however, it is assumed that the DM is not able to describe ‘utility’ in a closed-form function. Instead, a DM may only be able to provide partial information about specific preferences or tendencies. Depending on how much information is provided by a DM before or during the establishment of a mathematical model, one can distinguish three basic approaches in multicriteria decision making: ‘a priori’, ‘interactive’ and ‘a posteriori’ approaches (T’Kindt & Billaut, 2002).

In ‘a priori’ approaches, the DM is able to express preferences before a model is built. These preferences are taken into account during a process e.g. as parameters, as weights or as ordering

relations. The result is a model which reflects the DM's utility as well as possible, and the set of 'reasonable' alternatives delivered by the model or the decision support is consistent with the DM's preference information.

In 'interactive' methods, the DM takes part in the modelling process by providing some initial, yet vague information about preferences. A first rough model is built and treated mathematically, thus showing consequences and possibilities of different alternatives to the DM. Based on this information, the DM is able to specify preferences more precisely, since he/she is now more educated. The model is refined, the consequences are shown to the DM who then refines the information provided and so on. This process is iteratively repeated until a satisfactory model is set up. It is then treated in detail and, as a result, a set of reasonable alternatives is presented to the DM, among which a final preference has to be chosen when making a decision.

In 'a posteriori' approaches, it is assumed that the DM is not able to specify any kind of preference information beforehand. This may be due to the complexity of the real-world problem and the fact that certain meta-information or expert knowledge which may be important in decision making can often not be captured by a mathematical model. Moreover, a DM often makes decisions based on 'experience' or 'depending on the specific case' and, again, these issues are hard or even impossible to model. Under such circumstances, a mathematical model and the mathematical methods for dealing with such model (i.e., simulating, optimizing, etc.) often rely on the concept of Pareto dominance: an alternative is preferred to another, if it is at least as good in any of the objectives.

### **Multicriteria decision making and mathematical modelling**

Methods for solving decision problems are helpful to support students within the modelling process, especially they motivate to remodel the mathematical model. In this section, we point out the relation of the three basic approaches in multicriteria decision making to perspectives on mathematical modelling. In the 'a priori' approach, the simplifications are done in the real model from the DM, before a mathematical model is developed. The preferences made by the DM are implemented in the mathematical model in terms of parameters, weights, etc. The set of alternatives which are found in the mathematical model, correspond to the DM's preference information. No further loop in the modelling cycle is needed. In the 'interactive' approach, the real model has to be formulated and then translated into the mathematical model. The DM can specify preferences based on the results of the mathematical model, since he/she has more information. The real model will be adapted and a suitable mathematical model is (iteratively) solved. More attempts might be necessary to find a final preference for making a decision. In the 'a posteriori' approach the DM does not specify any preference before the modelling process. Therefore, the real model is set up and then translated into a mathematical model. After solving the mathematical model, a set of alternatives can be presented after the modelling process. The DM can choose the alternative which fits best to his/her most important criteria. Because of the complexity of the problem and the huge set of alternatives, the DM might need additional support in choosing the finally preferred solution.

### **Cases and experiences**

In the following subchapters, we report about several cases in which we extended mathematical modelling perspectives by aspects of multicriteria decision making.

### **Case 1: *Stiftung Warentest* – Analysis of product tests**

*Stiftung Warentest* investigates and compares objects with multiple, often contradicting criteria such as functionality, price or usefulness. It supports consumers in making their decision between alternative products. Ratings are issued and supplemented by rankings. Thus, these rankings are used for a first orientation and support the decision-making. Products are usually tested in a certain way: There are multiple categories, i.e. criteria, which are important for the rating of the product. In a further step, several methods for the product rating will be developed. Next, the products will be tested, and the results will be evaluated. The method for establishing a ranking can be implemented in school, since it uses various techniques dealing with topics in the range of analytic geometry and linear algebra (points, planes, distances) and stochastic (mean, variances). The central content of the teaching unit is to develop a method for ranking objects with the means of the multicriteria optimization (Ruzika, Klöckner, & Gecks, 2018). This teaching unit was situated within a classroom context in three units of 90 minutes. The execution of the teaching unit was conducted in a 12th grade class with 17 students at a *Gymnasium*. The teaching unit is opened by demonstrating examples of rankings from several areas, e.g. electronics or restaurants. The students are motivated to discuss about attributes of rankings and point out the difficulties of such rankings. It is obvious that the multiple criteria which influence the rating are contradicting, for instance price versus quality. Then, the students investigate how the multiple, contradicting criteria are treated by a weighted sum aggregation and how this method of establishing a ranking of products can be analysed geometrically. Students work out how the rating is compounded and how the best product can be determined. To simplify the problem, the number of categories of the ranking will be reduced, so that the students can develop how the test of a product can be implemented in mathematics. Using the weighted average of the rating in each objective, the ranking can be established and analysed with the help of a coordinate system. This leads to the observation that several products can make the top of the list depending on the chosen weights. More surprisingly, it can be observed that under some circumstances, ‘reasonable’ products which a customer would intuitively prefer, are not rated best in any scenario. But what if the test includes more than two categories? In a next step, the students have to revise and extend their mathematical model, which considers now more categories. With the help of a spreadsheet, they simulate the influence on the ranking, while different weights for each of the categories are chosen. Just as in the mathematical model with two categories, the application of different weights causes different rankings. After this teaching unit, it can be investigated how a ranking can be done in a fair and consumer-friendly way. The goal will be to find a product which is robust in a certain sense.

### **Case 2: *Navigation for electrical vehicles***

Decision making in the context of finding routes for electrical vehicles requires the consideration of multiple, often conflicting criteria. Thus, the selection of a route involves the consideration of several aspects: The battery consumption should be as small as possible while simultaneously the travel time should be as short as possible. The determination of efficient and optimal routes can be based on analytic geometry. In an extracurricular activity, we have worked on this problem for three days with a group of girls aged 16 to 18. The course was opened to introduce the students into the basic problem: Finding optimal routes for electrical vehicles. In a first step, the students simplified

the problem and focused for the beginning on one criterion only – the shortest path for a route. For solving such a problem, they obtained a theoretical input in form of a short teaching unit about network optimization containing the basics about (directed) graphs and Dijkstra’s algorithm. In order to achieve a motivating connection to the real world, students applied their knowledge about finding a shortest route on a selected example themselves. The goal was to find optimal routes for electrical vehicles, so students were motivated to discuss about several criteria which should be considered: price, energy consumption, speed, location of charge stations and travel time. After observing that, for an electric vehicle, energy consumption is increased by driving with higher speed and, that energy consumption is directly connected to the cruising range, the students realized that one important problem is the selection of a route that satisfies both criteria – consumption and distance. But how to compare routes with two objective functions? Next, students received a little more input – this time in multicriteria optimization. They learned about methods for the computation of efficient routes. With the knowledge of these methods, they extended their mathematical model from the beginning. The students chose two solving strategies: One group focused on the implementation, e.g. with GeoGebra. They plot the objective values for the different routes for the criteria travel time and energy consumption. The students recognized that efficient routes can be determined by considering the dominance cones of each route and that an efficient route is never contained in the dominance cone of another route. The second group implemented a multicriteria Dijkstra variant, e.g. using Python, in order to find an efficient route. In conclusion, the students compared their methods: The implementation in GeoGebra is descriptive but has the disadvantage that this method can only be used for given routes. This is the advantage of the implementation in Python: This method can be used for arbitrary areas and routes; the graph of the road network can be imported.

## **Conclusion**

The cases *Stiftung Warentest* and *Navigation for Electrical Vehicles* follow the ‘a posteriori’ approach. In the teaching unit *Stiftung Warentest*, students discuss attributes of a ranking. They point out that it is very difficult to express preferences about the most important criteria. Thus, in a first step, they simplify the problem because of its complexity. After solving the mathematical model, which is based on the weighted-sum method, they observe that one alternative is preferred to another. In a further step, they extend the mathematical model and include more than two criteria. Based on a method from multicriteria optimization, they reach a solution which is at least good for one of the criteria. It is obvious that the students remark any preference for the criteria before they build the mathematical model. In the extracurricular activity *Navigation for Electrical Vehicles*, the students are not able to make a ‘good’ decision for finding an optimal route, since several aims are contradicted. Because of the complexity of the problem, they cannot weigh a criterium more than another. Thus, they simplify the problem in a first step and solve it for one criterium. It is very hard to establish a mathematical model based on their experiences and their meta-information. Thus, they apply methods from the multicriteria optimization to solve the mathematical problem, e.g. solving the real-life problem. Students are not able to express preferences before they establish the model. Hence, the usage of the Pareto concept of dominance helps modelling and solving the problem.

The input about multicriteria optimization, especially about methods for solving decision problems, support the students in their modelling process. The usage of multicriteria optimization methods supports the establishment of the mathematical model and the extended remodelling of the real model. Furthermore, they obtain an overview on how the process of multicriteria decision making is staged. They learn how to handle multiple, contradicting criteria and find a trade-off between them. We support their modelling process with input for several reasons: Using the ‘a priori’ approach, preferences are taken into the model, e.g. criteria are weighted. The result is a model which reflects the student’s, i.e., DM’s utility. In contrast, in the ‘a posteriori’ approach, no criteria are weighted before the modelling, and several alternatives can be presented after the modelling process. Thus, the DM can choose the alternative which mostly reflects their most important criteria. With regard to the competences, it is apparent that, apart from the modelling competence, other competences are promoted and extended as well: They use mathematical tools, such as GeoGebra and Python, to solve problems, they communicate mathematically using terms like ‘efficient’ or ‘dominating’, and they apply solving problem strategies.

We have seen that methods of the multicriteria optimization can be didactically reduced for students. The focus on the most important aspects and their proper visualisation lead to an adequate implementation in a school context. Both examples show a contribution to authenticity – real world problems reinforce the students’ motivation obviously. Students get to engage in critical thinking: They can ask provocative questions and concentrate on finding their answers. Based on the answers well-founded decisions are made. By giving them additional input, we enable the students to work with these methods on their mathematical model. We call this process modelling: We influence the modelling process in a way, but – what is more important in this case – we support the establishing of a mathematical model, so that students are supported to make decisions and solve problems with multiple, contradicting aims. The students go through the modelling cycle more than once: They simplify the real-life situation to a real model, establish a mathematical model, improve the real model, extend the mathematical model and solve this with the help of given methods. These results are applied to the origin problem. The emphasis lies on the “last node” of the modelling cycle: the application of the mathematical results to the real world problem. After the modelling process the used mathematics are discussed, reflected and justified. In conclusion, these two cases show how the multiple perspective can be incorporated in mathematical modelling. We have shown that decision making affects the mathematical modelling in a school context and also an extracurricular activity. Our aim is to evaluate the benefits and the drawbacks of this approach rigorously in future research.

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