

Supplementary material 1. Using a proposal distribution based on the multiplication by a log-normally distributed variable.

We consider the relation between two random variables X and Y , $Y = X \times LN(0,0.1)$. We need to compute the ratio

$$\frac{\Pr(Y|X)}{\Pr(X|Y)}$$

which is used in several acceptance ratios of the MCMC developed for this study (e.g. Step 1 in the Appendix “*Bayesian estimate through a MCMC algorithm*”).

Because the ratio Y/X follows a log-normal distribution with parameters ($\mu=0$, $\sigma^2 = 0.1$) we have

$$\Pr_Y(y|X)dy = \Pr(Y = y \pm dy|X) = \Pr\left(Z = \frac{y}{X} \pm \frac{dy}{X}|X\right) = \Pr_{LN}\left(\frac{y}{X}|X\right)\frac{dy}{X}.$$

Using the PDF of a log-normal, then

$$\Pr(Y|X) = \frac{1}{\sqrt{2\pi\sigma Y/X}} \exp\left(-\frac{(\log Y/X - \mu)^2}{2\sigma^2}\right) \frac{1}{X},$$

and the ratio simplifies to

$$\frac{\Pr(Y|X)}{\Pr(X|Y)} = \frac{X}{Y}.$$

Now, this expression is used to compute the acceptance ratio, r_σ , defined in Step 1 of the Appendix “*Bayesian estimate through a MCMC algorithm*” that becomes

$$r_\sigma = \frac{\Pr(\mathbf{F}|\sigma_t^*) \sigma_t^* \text{Prior}(\sigma_t^*)}{\Pr(\mathbf{F}|\sigma_t) \sigma_t \text{Prior}(\sigma_t)}.$$

Because we chose a prior distribution defined as $\text{Prior}(\sigma_t) \propto \frac{1}{\sigma_t}$, the acceptance ratio r_σ simplifies to

$$r_\sigma = \frac{\Pr(\mathbf{F}|\sigma_t^*)}{\Pr(\mathbf{F}|\sigma_t)}.$$