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► **To cite this version:**

Marta Pla-Castells, Irene Ferrando. Downscaling and upscaling Fermi problems. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02408969

HAL Id: hal-02408969

<https://hal.science/hal-02408969v1>

Submitted on 13 Dec 2019

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Downscaling and upscaling Fermi problems

Marta Pla-Castells and Irene Ferrando

Universitat de València. Facultat de Magisteri. Valencia, Spain; marta.pla@uv.es
irene.ferrando@uv.es

The introduction of modeling tasks in the first grades of primary school requires a deep reflection from the point of view of both the design and the implementation. In this paper we describe an empirical exploratory study developed with the objective of designing a sequence of modeling tasks focused on solving a big numbers estimation problem that, in origin, is inaccessible for students. The key to the design of our sequence is to use a technique that we have called downscaling - upscaling, inspired by heuristic strategies. So, by resizing the initial problem, 2nd grade students are able to tackle a problem that seemed very big using complex mathematical procedures.

Keywords: Problem solving, measurement techniques, early childhood education, real context problems, heuristic strategies.

Introduction

In the last years, tasks based on real contexts are becoming increasingly important in primary and secondary school curricula. Even so, due to the complexity of carrying out this type of tasks in a classroom, both by teachers and students (Cabassut & Ferrando, 2017), there are few studies that analyze the implementation of this kind of task in the first years of primary school. All this makes of special interest the study of the concepts related to the measurement of magnitudes and the estimation of large quantities in a real context, using the so-called Fermi Problems.

Theoretical framework

Fermi problems

Jonas Ärlebäck (2009) defines Fermi problems as “open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations”. According to the research presented by Ärlebäck and Albarracín in the previous edition of CERME, this definition is the most used in mathematics education research literature. Here we will focus on a particular kind of such problems: *Big numbers estimation problems* (Albarracín & Gorgorió, 2013).

Twenty years ago, Clements (1999) already claimed that children, in their first experiences with length measurement, should be given a variety of experiences in order to develop measurement strategies. According to Hogan and Brezinski (2003), an important part of teaching measurement concerns the development of estimation. Nevertheless, there are still few studies that refer to the way estimation is worked in primary school classroom (Pizarro et al., 2015).

In previous works, other authors have made experiences based on the use of Fermi problems with students of the higher levels of primary education (Peter-Koop, 2009) and very recently with students of the first cycle of primary education (Gómez & Albarracín, 2017). In the present work we have tried to take advantage of environments close to the students to introduce some elementary

concepts related to the calculation of a magnitude that is often complicated and leads to confusion: the perimeter of a polygon (Mason, Stephen, & Watson, 2009).

Modelling sequences

Ärlebäck, Doerr and O’Neil (2013, p. 316) gave importance to the development of sequences of modeling tasks so that through these sequences students “engage in multiple cycles of descriptions, interpretations, conjectures, and explanations that are iteratively refined while interacting with other students”. In this way, by constructing a sequence of modeling tasks, we can deepen the study of complex concepts in the first years of primary school. Based on the previous works of Pólya (1973) and Stender and Kaiser (2017) we have also focused on the use of heuristic strategies directed by teachers when solving complex modeling tasks. We propose a design of a sequence of tasks based on the heuristics recommendations provided by Pólya (1973, p. xvi): “If there is a problem you can’t solve, then there is an easier problem you can solve: find it.” Stender and Kaiser (2017) affirm that, ideally, students should use heuristic strategies during the modeling process that teachers should convert into strategic interventions. Since our experience is addressed to very young students, our aim is to allow them to approach a complex problem using techniques based on downscaling and up-scaling the original problem as a heuristic strategy.

Research objectives

Our work is in an initial phase. We will give some details about the first phase of an empirical exploratory study where we intend to describe a technique, called downscaling-upscaling, to design sequences of Fermi problems in the first years of primary school. Since this is an early-stage investigation, our objective is limited to: (1) Describe in detail the aspects considered in the design of the sequence of tasks; (2) Describe a qualitative exploratory analysis of the results of implementation with a natural group of second grade students.

Methodology. Design of the empirical study

Sample

Our study focuses on a natural group of 21 second grade students of primary education who have not yet worked on the concepts of area and perimeter nor have ever faced a modeling task. They have already worked on arithmetic concepts like addition and subtraction but they have very little notions about multiplication. The activity was implemented in May 2017.

Treatment: downscaling – upscaling design of tasks sequence

The starting point of the activity is a very general question that, a priori, the students are not able to answer. The objective of establishing an initial problem is to observe that a complex problem that seems difficult to tackle – estimate the number of people necessary to surround the town – can be simplified if we try to solve similar smaller problems where the resolution strategies are equivalent. Following Pólya’s heuristics, we propose them to “solve a simpler problem”. In this case, what we try is to reduce (downscale) the scope of the initial question, first to the schoolyard and secondly to their classroom, giving them the opportunity to make measures in a very close context. Once they have attempted a solution of the simpler problem, we foster the generalization of the procedures

developed in the first part of the sequence (there the upscaling process starts). In the following, we give the details of the activities developed during each session of the experience.

1st session: A “Big Problem”. The introductory session aimed exclusively to present to the students the problem to be solved. Thus, this first activity started with an open question that students did not know how to answer: “*If we asked all the inhabitants of Sueca¹ to surround the town by standing next to each other, could they do it or would they have to ask for reinforcements from the people of the next town?*” At this point we just wanted to catch the attention of the students and identify some ideas about perimeter and big numbers conceptions.

2nd session: Progressively reducing the “Big Problem”. The first step was to propose to the students to make a smaller human chain. In this case, they were asked to think about how to estimate the number of students needed to surround the schoolyard. At this point, we just ask them to think about possible strategies to obtain, graphically, a procedure to get an estimation, not to measure or even compute. The second step was to answer to the smallest question: “*How many small wooden cubes do we need to surround a large wooden board?*” Here, we try to show them that the previous problems were the same as this one. In this case, we provide them with manipulative materials in order to foster measure strategies based on the iteration of the unit as explained in Barrett et al. (2011).

3rd and 4th session: We solve the problem of the schoolyard. During the third and fourth sessions, the students worked on the rectangular schoolyard. So, based on the strategies worked on in the previous session, the students would try to solve a bigger problem. They had to estimate the number of students needed to surround the perimeter of the yard. In these sessions, we provide students with one-meter-long ropes.

5th session: We work on scaled ground plans. In order to answer the “Big problem”, it is necessary to introduce the perimeter measurement on scaled plans. Therefore, the fifth session was devoted to work on a scale plan of a relatively small space close to students; a garden close to the school. We asked them to use the ruler and, using the scale and the estimations made in the schoolyard in the previous session, to estimate the number of people needed to surround the yard.

6th session: Solving the big problem. During this session each group of students was given a map in A3 format of the town of Sueca in a grid in order to finally give an answer to the “Big problem”.

Data collection and data evaluation

The experience was organized sequentially through five activities that were carried out over six sessions of 50 minutes. Students worked sometimes individually and sometimes in 4 groups of 4 children and one group of 5 (we will specify this in the following section). The teacher who conducted the sequence had previous experience in modelling activities and was working under the supervision of two researchers. During the group work sessions, we video-taped two groups, and we audio-taped the other groups. In Session 1 and at the end of the Session 6 there were whole group

¹ A Spanish town with around 28000 inhabitants.

discussions that were also video-recorded. In our analysis we have also considered the written productions (individual and grouped ones).

Since this is an exploratory study, we are limited to analyze the actions of students' groups to each of the tasks. In fact, the sample is not big enough to establish a grounded analysis. That is why we will not include any quantitative analysis; we'll just describe some aspects that we have considered important for further research. Our objective, at this point, is to identify whether the design of the sequence allows students to face, without blockages, the resolution of each task. We want also to identify, if possible, different strategies for solving equivalent problems and their evolution throughout the sequence. In the following section we summarize the description of the results for each session.

Description of the results

First session. The objective of this session was to collect general impressions of the students about a problem of estimating a big number. It is not expected that they will base their answers in mathematical arguments. Here are some of the answers collected: *"In the village there are too many people (S1)"*, *"There won't be enough people (S2)"*, *"There are many people who work (S3)"*, *"The planet is very large and we would not be enough (S4)"* and *"If we put one next to the other and put also the dogs and cars, yes we will be enough (S5)"*.

Since students were not asked for more than an unjustified spontaneous response, none of the reflections contain references to neither calculations nor mathematical concepts. This first activity allows us to detect difficulties linked to the understanding of the question (it is not clear in the case of the student S5), problems associated with the concept of perimeter (this we observe, for example, in the answer of the student S4) and the process of obtaining an estimation (response of student S3).

Second session. 1st part: Let's reduce the problem: the schoolyard: *"To surround the yard, are there enough children in this class? Or would we need to ask for help from children in the other classes?"* The students, working individually in the classroom, represented graphically how they would make the estimation. In the productions shown in Figure , different schemes are observed. In the third one there is some confusion between the perimeter and the area since the student tries to cover all the space of the schoolyard with children and not just surround its perimeter.

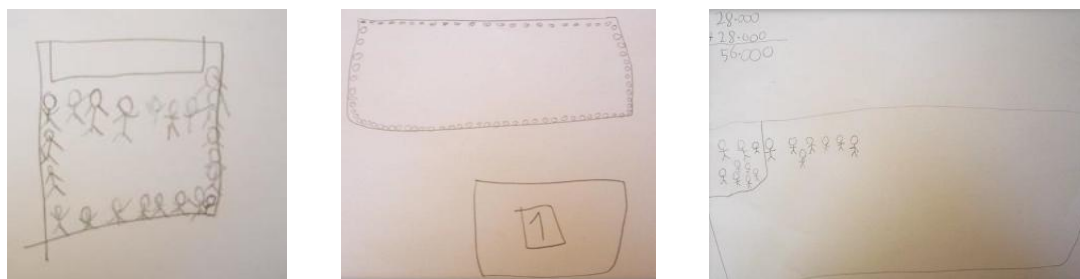


Figure 1: Some of the schemes made by students in the first part of Session 2

Another element detected in this part of the experience is the difficulty of the students to simultaneously handle elements of very different dimensions (the sides of the yard and the linear space occupied by a person). Indeed, when we asked the children to explain their answer from their

drawings, the unanimous answer was that they had to count the drawn “dolls”. Anyway, this part of the experience was rich to detect difficulties (linked to the difference between perimeter and area) and different levels in the resolutions.

As we have already noticed, the problem was still big for the students. Once all the students had graphically described their strategy to estimate the result of the problem of the schoolyard, workgroup began. At each group, students shared their ideas they previously represented. In order to help them better understand what they had done, they were encouraged to estimate the number of students needed to surround their tables. Here we observed that some students infer the most effective way to find the perimeter of a rectangular surface from the measurement of two unequal sides: add them and then make double. Once the notion of perimeter has been introduced intuitively, the second part of the session aimed to further reduce the initial problem.

2nd part: Deduction from manipulation. To answer the question “*How many small wooden cubes do we need to surround a large wooden board?*” only 5 small cubes were provided to the children. The boards provided to each group differed in their dimensions. In some of them the solutions were whole values, while in others there was too much or too little space, in order to observe how the students faced the problem.

Some groups proposed strategies based on the cyclic movements of the group of 5 cubes in order to count the number of cubes needed to cover one side. This strategy is interesting because the idea of an effective five-in-five counting is introduced and this improves the strategy of exhaustive counting. The concept of the measurement and estimation of the perimeter of a polygon was now formally introduced to the students.

3rd and 4th session: Each group was provided with a pair of ropes of the same length (one meter) in order to solve the problem of the number of children needed to surround the schoolyard. To clearly describe the two strategies observed, we will focus on the description of the groups that we will call Group A and Group B. One of the groups was totally lost and was not able to finish this activity.

The students of Group A counted effectively (making repeated sums of four-by-four) the number of children needed to cover the four sides of the rectangle that makes up the schoolyard using their bodies as a unit. They measured two unequal sides of the yard (in which they fit 29 and 38 children), then they doubled the values and then added both values. This group of students obtained as a result that 134 children were needed to surround the schoolyard.

Group B, although they obtained the measure in “children” of one of the sides of the yard, chose to measure the uneven side using the ropes. The students measured first with the ropes, and then they were urged to find out, from that measurement, the measure in “children”. Thus, during the development of the activity, Group B students obtained a measure of “9 ropes and 3 children” for the minor side of the schoolyard. In this way, from their first measurements, the teacher asked the students to try to find out how many children fit in each string. This part is particularly interesting because two complex aspects are worked on in a simple manipulative way: the notion of linear density (*how many children complete a rope?*) and the change of units (*how to transform a measure given in big unit –rope– in a smaller one –child–?*). The students estimated that they could cover each rope with 7 children. Thus, they faced the question: “*If we know that 7 children fit in each*

string, how many children do we have in 9 strings + 3 children?” After discussing this problem, the students came to the conclusion that the operation to be performed was to multiply 9×7 and then add 3. In this way, measuring the two unequal sides of the schoolyard, they infer the total of children that they needed to surround it: they estimate 292.

5th session: Work on scaled ground plans. During this session the students worked individually, each of them was given a scaled ground plan, a 15 cm graduated ruler and a calculator. Since they had already worked with the concept of perimeter, the first part consisted in answering the question: *“How many centimeters does the perimeter of the garden measure in the ground plan?”*

We did not identify any problem in the use of the graduated ruler (each side had a whole number measure), but we observed different strategies to obtain the total perimeter in centimeters. Some students followed the strategy of measuring each of the sides and adding the successive measurements of the sides as they got them (using the calculator). Others preferred to take note of the partial measures and, finally, add them. Once they had obtained the perimeter of the garden on the ground plan, they had to infer the real perimeter from the use of the scale (each centimeter of the plane represents 5 meters of reality). With the help of the calculator and after a whole group discussion, the students deduced that they should multiply the measurement found in the plan by 5. Next, they were asked to estimate, based on the measure of the perimeter in meters (approximately 360 meters), the number of children needed to surround the garden. Since in the previous session they had found, experimentally, that in each one-meter string 7 children can fit in, they simply had to 360 by 7. The children were very surprised by the result: they could not imagine that 2520 children were needed to surround the entire perimeter of the garden!

6th session: Solving the big problem. Before starting to do the calculations, the difference between a map and reality was explained again. Some students had difficulties with this concept, but we observed that those that had understood this idea were able to explain it to their classmates. Anyway, the intention of using a grid was to facilitate the use of scales, so the students were told that each side of each square of the grid corresponded to 250 people in line. Figure shows a picture of the students working on the map. First, the students obtained an approximation to the perimeter of the town using the side of the square as unit. Since we told them that around 250 people fit in one square side of the grid, they just had to make one operation: multiply (using calculator) their previous result by 250. The obtained results varied between 51500 and 64500 people.

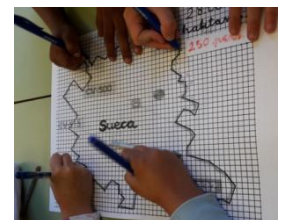


Figure 2: Working on the gridded map

At the end of the activity, it was important that the students could validate their solution. For this aim, we used the Google Maps tool that allows to measure distances on a plan. **Erreur ! Source du renvoi introuvable.** The perimeter of the town of Sueca measures approximately 10.5km, that is, 10500 meters. Since the students knew –from the resolution of the previous task – that 7 children fit in each meter, it was enough to multiply by 7, being the final result 73500 children. Since the number of adults that fit online in a meter is surely lower than the number of children, the result obtained by the students working on the grid was quite approximate. Therefore, the inhabitants of Sueca (around 28000 inhabitants) would not be enough to surround the perimeter of the town.

Conclusions

The procedures associated with the measurement of magnitudes, particularly the measurement of lengths, are strongly linked to estimation processes. The choice of the appropriate unit, the use of a specific unit, the strategy of iterating a unit, more than one unit, changes of scale (moving from one unit to another), etc., are, among others, procedures associated with the act of measuring. But in addition, in real life problems, these procedures also appear when making estimations. That is why, in our current research -which begins with this work- we intend to design a sequence of tasks whose starting point is a problem of estimating large quantities that allows us to introduce, at an early age, procedures linked to the length measurement.

We have presented here the design of a sequence of tasks that starts from a Fermi problem that is unapproachable for students. Thus, from two elementary heuristics of Pólya's work, a sequence of tasks is constructed. The sequence begins with the simplification (named downscaling in our context) of a large problem. In this way, when students are able to develop a procedure in a reduced context, we encourage them to generalize (upscaling process, in our context) that procedure to apply it in a larger context. We have implemented the sequence with a natural group of 21 second grade students. A first qualitative analysis of this intervention allows us to identify some aspects of interest to work on in the future. The following are the most interesting ones. At the start of the sequence, in the second session, we have detected that the children have difficulties in differentiating the reality from the representations of the real world. Thus, they try to find out the number of children that fit on the perimeter of the track by simply counting the children (or their abstract representations) in their own drawings. This reminds us of the importance of introducing, even in a very simplified way, the idea of scale. We have also observed some difficulties in the process of using physical elements as units of measure. This is why we think that our proposal, and the future versions we'll make of it, has to foster students to make use of different units (cubes, children, ropes and, finally, graduated rules). In this sense, we have observed that, in the third and fourth session, a group of students was totally lost. The problem was not, apparently, the mathematical activity itself, but the fact that they were not used to work outside the classroom. This is an aspect that should be considered in our future work.

The proposed sequence has also allowed us to observe the evolution of the students. Given that this is not an isolated activity but a sequence, some of the students had the opportunity, in the next activity, to assimilate or discover the procedures they were working on. In addition, they were able to put them into practice in another context. This seems to us a strong point of our sequence, but we will have to test its efficiency by working with more students.

Furthermore, the sequence design aims to support the teacher's task that, in many cases, feels overwhelmed to bring this type of activity to the classroom. In this way, its own design serves the teacher as a guide to be able to offer what Stender and Kaiser (2017) call strategic interventions. Finally, we want to highlight that the designed activity allows working the validation phase that, from our perspective, is fundamental when solving any problem of real context.

Acknowledgements

Research supported by the project EDU2017-84377-R, Programa Retos de la Sociedad of the Agencia Española de Investigación and FEDER.

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