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► **To cite this version:**

Diego Lieban, Zsolt Lavicza. Instrumental genesis and heuristic strategies as frameworks to geometric modeling in connecting physical and digital environments. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02408923

HAL Id: hal-02408923

<https://hal.science/hal-02408923>

Submitted on 13 Dec 2019

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Instrumental genesis and heuristic strategies as frameworks to geometric modeling in connecting physical and digital environments

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This study aims to offer frameworks for exploring geometric modeling while combining physical and digital resources in teaching and learning of mathematics. Originally designed as a task to tackle pre-service teachers' learning transition from 2D to 3D under the lens of instrumental genesis approach, our study revealed how heuristic strategies from Polya spontaneously emerged through the process of digital representations. The use of physical simulation was helpful in supporting participants to better comprehend and describe action of joints and to (re)interpret their mathematical behaviour, which was not possible when they were working only in the digital environment. The combined use of both physical and digital resources appears to bring a relevant contribution for refining students' thinking and enhance their mental schemes or strategies through of the modeling processes.

Keywords: Geometric modeling, teacher training, GeoGebra.

Introduction

In this work, pre-service teachers, students in a teacher training program, were assigned to represent the kinetic simulations based on real objects' movements through physical and digital resources. Geometric modeling of seesaws using GeoGebra was analysed with a special focus on the use of perpendicular lines, symmetry features and circular movements. This was mainly related to handling geometric concepts while transitioning between both 2D and 3D representations. The aim of observing the process, including integrated and multiple representations, was to identify the benefits coming from either physical and digital prototypes and how they can support one another. Such experience fed us with some elements to our posed research question:

In a geometric modeling approach, how can the combined use of physical and digital resources to represent joints and their movements support the 3D geometric knowledge of prospective teachers?

Following our research approach, we observed that on the one hand, the modeling task as such played a crucial role together with the software and enhanced participants' geometric understanding; prospective teachers' behaviour could be examined and understood through the Instrumental Genesis theory (Bussi & Mariotti, 2008). On the other hand, procedures adopted by participants were aligned with Heuristic Strategies proposed by Polya for problem solving. Hence, we decided to share excerpts where these evidences were more apparent, above all the strategies related to *backward thinking, generalization, specialization and decomposing* (Polya, 1973).

While working with geometric modeling in technology-enhanced environments, students are faced with different construction procedures, compared to their physical modeling experiences, which are mainly caused by the features and constraints of the software. The different approaches to physical and digital modeling could offer opportunities for teachers and students to further their learning. In addition, modeling real world problems could enable connections and new perspectives between science and mathematics. Before discussing the analysis of these examples¹ we will outline the theoretical frameworks used for this work.

Theoretical framework

The idea for this study was originally to develop geometric modeling from real and physical daily objects to investigate the ways they could foster mathematical exploration. As claimed by Carreira and Baioa (2015), students' behaviours in class usually do not reflect experimental sciences since they are not in the position to examine and interpret their surroundings mathematically.

Through this geometric modeling approach, we intended to enhance students' modeling competencies and mathematical thinking:

[...] to interpret a situation mathematically from iterative cycles of describing, testing, and revising mathematical interpretations as well as identifying, integrating, modifying, or refining sets of mathematical concepts drawn from various sources. (English, Lesh, & Fennewald, 2008, p. 8)

Considering the recent significant changes in mathematical thinking led by new technologies as well as social aspects such as communication and collaboration led by new technologies (English, Lesh, & Fennewald, 2008), we started this investigation under the lens of the instrumental genesis approach. We examined the transition of GeoGebra from an artifact to an instrument and later we witnessed practices emerged that could be explained by Polya's strategies of problem-solving, then how these strategies further assisted students in their instrumental genesis.

Instrumental Genesis

According to Bussi and Mariotti (2008), Rabardel's instrumental genesis framework is based on the distinction between artifact and instrument. While the artifact can exist by itself, the instrument is a mixed entity with two components: the artifact produced for the subject, and the associated schemes of use. They are the results of a construction of the subject itself or of an appropriation of already existing schemes of use. The instrument describes an artifact and it is constituted in the use(s) that the subject develops. In this way, the uses of the artifact also depend on the needs and objectives of the user. According to Stormowski (2015) the importance of transforming a software, in our case GeoGebra, into an instrument is an evolution of the reorganization and modification of users' schemes, structuring of teachers' actions, and relations to mathematical concepts. In our particular case, participants had their strategies influenced by software outcomes even when such results were not exactly as they expected. As pointed out by Sinclair and Robutti (2012), if participants 'internalize' the use of a Dynamic Geometry Software (DGS) the artifact becomes a mean that offers

¹ The mentioned students' constructions are available online and can be utilized for other researches and teaching activities: <https://www.geogebra.org/m/Tng4JXDk>.

opportunities to solve problems. We observed that GeoGebra contributed to participants' reflections on simulation tasks and reorganization of their thinking schemes, as well as forming mathematical concepts. Nevertheless, these transformations were occasionally need to be assisted by teachers' external minimal guidance and supported by physical models.

Heuristic Strategies

Polya's (1973) work on problem-solving listed a range of heuristic strategies that can be used to tackle a diversity of problems, not only restricted to mathematics. In our research, we consider "*the problem to be solved*" as the challenge to develop physical and digital representations as equivalent as possible. In this case, the seesaws' digital simulation was in the main focus of our investigation as we were already considering students' instrumental genesis with GeoGebra. For this purpose we have identified different strategies that were strongly connected to the geometric modeling approach: *guess and check*, *use of symmetry*, *draw a picture* and *be creative* to mention some of them. In the results section, we aim to outline more explicitly how some of these strategies connected the instrumental genesis with Polya's problem-solving. Stender and Kaiser (2017) state that there is little empirical research exploring how these heuristic strategies can be implemented in classroom teaching. They suggest that heuristic strategies can be transferred into strategic interventions by tutors. Through our current research and further results we intend to share geometric modeling tasks designs beyond teacher training programs. Once prospective teachers have the opportunity to experiment with different strategies with DGS they can feel more confident to reapply them in their teaching and encourage their students to carry out similar tasks.

Methodology and Methods

The explorative nature of the study demanded an interpretative approach comprising qualitative methods as observation, data collection, and analysis (Cohen, Manion, & Morrison, 2011). Our experiments were carried out with 19 volunteer prospective mathematics teachers in Brazil and lasted a period of four months with meetings in every second week on average, to follow the evolution of the modeling process. All participants had already been introduced to GeoGebra during an introductory ICT course, where they explored the main software functions and features. For this experiment, students worked mostly in pairs and were assigned the task to build a seesaw using materials (e.g. wood, plastic, metal, etc.) of their choice. Then, they were asked to digitally model the same seesaw using a Dynamical Geometry Software.

The data collected in the initial part of the research came from three different sources: i) the digital modeling process of individual students or pairs was video-recorded; ii) the GeoGebra files were shared by students with the research team; and iii) an explanatory section, where participants presented their solutions based on their previous constructions was also video-recorded. In addition, all participants attended other four meetings with the whole group in order to expose their progress, exchange their ideas and discuss exercises with geometric modeling and joints.

Data Analysis and Results

For the purposes of this paper, we will outline steps of constructing digital examples that we consider valuable to be supported by physical prototyping for learning. Through the connections of

both modeling approaches, participants first *worked backwards* to define the order of dependencies and the sequence of elements that they had to create. After students' trials they had to occasionally redefine the sequence of their constructions in order to obtain better results. This indicated that participants were constrained by the development processes and by their interactions with the software to rethink their strategies or "schemes of thinking". Both unexpected outcomes and inactivity from the software, due to for example inconsistent inputs, led students to rearrange their strategies of constructions. Valuable discussions arose from these experiments and often students had to re-evaluate their ideas and return *back to* basic mathematical *definitions*. Problem-solving strategies such as *generalization* and *specialization* were also recurrent in students' practices while sharpening their solutions². The following examples aim to highlight the nature of the heuristic strategies utilized by students. They also offer some insights into how the use of these strategies makes physical and digital artifacts become instruments. In our first example, we examine the importance of using physical prototyping to trigger conceptual discussions about circles and tangents while students are digitally modeling the joints' movements. Such approach in this experiment encouraged students to make connections between the physical and digital models, using one of them to better understand the other. Figure 1 illustrates parts of this process.

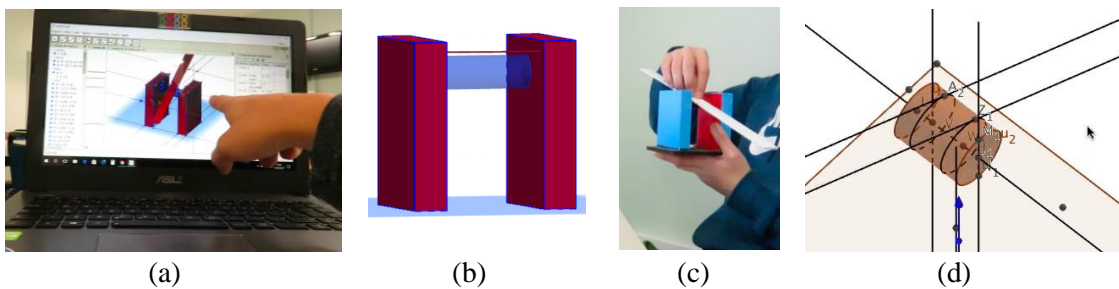


Figure 1: Physical and digital resources supporting participants' rethinking of their solution strategies

After the first digital prototype of the seesaw was modelled, students were asked about the accuracy of their constructions. They were asked to identify and explain in which aspects their two models were, or were not, representatives of each other. Students identified that the ratio of the elements in the physical prototype was preserved in the digital model (Figure 1a). They also reported that the movements of the main seesaw board and the cylinder, on which the board should rotate, did not represent exactly the wanted physical movements. In their digital model, the board was intersecting the cylinder or (depending on how it was moving) the board and the cylinder did not even touch each other (Figure 1b)! While reviewing their digital model students felt the necessity to explore their physical model again (Figure 1c) to more concretely visualize movements and connections of the elements. In their words, they explained that: "ideally the main board is supposed to be connected to the cylinder, which should rotate around its axis of revolution." In continuity they should move from this "optical trick solution" to a most realistic representation. After few days they presented a new version (Figure 1d), in which they fixed the problem described above. They used

²*generalization*: circles and lines before arcs and segments;

specialization: board in a parallel position to the basis plane to rotate afterwards;

tangent lines to particular circles, which were obtained as a section of the cylinder and rotating the cylinder as the last step of the process. Therefore, the main board would stay connected.

The second excerpt brings another example of how the combination of digital and physical resources contributed to rethink geometric definitions in 3D space.

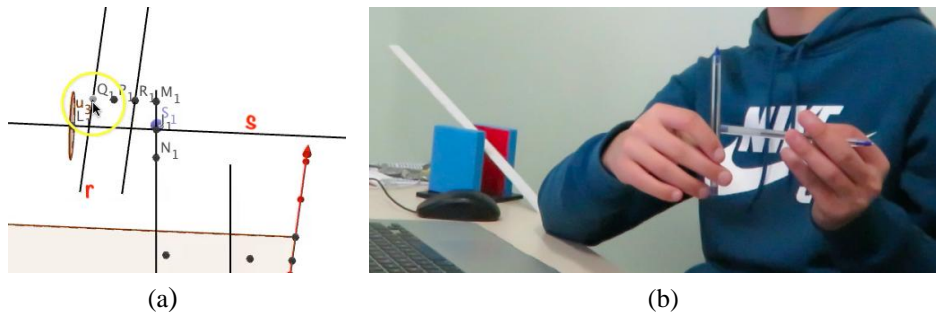


Figure 2: Physical and digital resources enhancing proper meaning in 3D

In a particular step of the construction³ (Figure 2a), students needed to trace a perpendicular line passing through a point in 3D space and belonging to a given line (r). However, while doing this, students were puzzled why GeoGebra did not trace such perpendicular line. When asked to explain why the software was inactive, they assumed that it was an operational problem. They repeated the same procedure twice without any result. Finally, students solved their practical problem using another line (s). However, this was not enough for them to realize that they were skipping a “conceptual action mistake”. In order to give students some insights, the researcher suggested the use of concrete objects (Figure 2b). Part of this approach is registered in the following conversation:

Student: What I can do is a perpendicular line to this one that passes here (the point Q_1 that belongs to r). [*There is no effect in GeoGebra.*]

Researcher: Why do you think nothing happened?

Student: I guess is because I’ve clicked on the wrong spot. [*He repeats exactly the same procedure and again achieved no usable results.*]

As a third attempt, they considered a different line (s) that was convenient for his purpose and finally the perpendicular line emerged. Then the student continued:

Student: Actually I was taking the wrong line, I guess.

Researcher: What did you intend to do in the beginning?

Student: Exactly like that, a perpendicular line passing through Q_1 .

Researcher: Ok, that is what you wanted as a final goal and you finally got it, but why it didn’t work on your first attempt?

After some new trials similar to the preceding ones and some vague conjectures, he finally said:

³ Note that this screenshot is from a 3D representation, that means the lines r and s are not in the same planes. In this construction, Q_1 belongs to r but not to s .

Student: I really don't know.

Researcher: Ok, go ahead. Let's discuss it later [...]

Student: The issue is that GeoGebra is quite intuitive, so you can get results through trial and error. Sometimes there is a hidden fundamental math principle, but you have been making so many constructions that you might eventually miss some math behind these constructions.

Interesting to note that even though the student was not able to identify the real reasons why he was stuck at this time, he recognized a consistent performance of GeoGebra and as a benefit for his learning due to GeoGebra's intuitive interface. At that moment the researcher felt the necessity to intervene with some pens to simulate, using physical objects, how the software was "thinking".

Researcher: Do you want to see something? [*The researcher hands him the pens and asks to represent a perpendicular line passing through a point belonging to another line*]

The student then immediately presented a possible solution.

Researcher: Is this one? [*Referring to the solution presented by him*]

Student: In order to obtain a perpendicular line it might need to pass through this point and make a right angle with the first one... [*He slightly rotates the second pen keeping the right angle*]

Student: [...]or like that maybe. [*He presents another solution*]

Researcher: But is there only one solution?

Student: Actually, not. I have thought about that. Somehow that is the reason it worked with the second line (the line that the point did not belong to), but not with the first one (the line that the point belonged to). If I had done it in 2D the software would understand that there is only one option (whether the point belongs or not to the first line).

The student's observation of linking 2D and 3D representations confirms that he realized that there was a significant difference between both 2D and 3D spaces. This is because the conditions to define a perpendicular line through a given line with a point lying on it were not properly the same. The transition from plane to space was quite recurrent among all participants and it happened as a natural step in the geometric modeling process. In particular cases, it was a bit conflicting when students kept the 2D concept, but ignored the new dimension in 3D. If something worked in 2D it did not necessarily work in 3D as well. In the previous example, the student realized the subtle difference caused by the new dimension in his solution after such interventions. Similarly as in the previous example, another challenging situation happened while the student was working with symmetry in 3D space. For example, students tended to use lines instead planes to reflect solids, which was misleading. This example was presented with further details in a previous paper (Lieban & Lavicza, 2017). The use of 2D geometry was useful in general as an auxiliary intermediate step in the construction of the seesaws. This process is what Polya labelled as *decomposing*, or splitting the problem into simpler steps. For instance, when students were struggling with reflecting solids,

they “reduced” the problem to a planar case and then continued with the extrusion afterwards. This change in strategy exemplifies the connection between the two theoretical frameworks. First students utilize the tool, which respond with an unexpected result by them, but through this process students operationalize the tool and it became an instrument. However, to be able to solve the problem they need to change their solution strategies. This is when heuristic strategies emerge, students start to decompose the problem and develop new solutions by using their instruments, but with subsequent steps from their decompositions. Another situation to exemplify the connection between theories happened when in the digital environment students had to define two points of a line as the ends of a segment in 3D space. They could do this construction directly with 3D features of the software, for example, spheres or circles with the normal (perpendicular to a plane) directions. While encountering such difficulties, they chose to use an auxiliary plane to continue the construction from this planar view as an alternative strategy. We present these steps in Figure 3, in which an auxiliary plane was used to construct a circle on it.

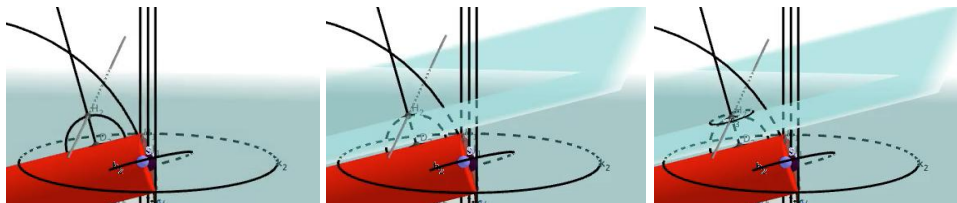


Figure 3: Circles into 3D space as a tool to define distances

Based on these examples, it can be assumed that the instrumental genesis and heuristic strategies can be combined to better understand student thinking. We observe that participants’ progress in their solution strategies and their schemes of using the software were triggered by the challenge to represent objects in 3D space and fixing this process through Polya’s *decomposing* strategy. The problem-solving approach by Polya seems to assist in the transition of GeoGebra from a tool to an instrument.

Conclusions

Geometric modeling was applied in this research to investigate perspectives in teaching and learning of mathematics from real-life objects combining physical and digital resources. Several of Polya’s strategies are intuitively used when solving mathematics problems with DGS. To take further advantage of using the software it is important not only apply these strategies, but also identify, practice, and organize them. Nevertheless, it is important to emphasize that using the digital tools without constant reflections on various responses of the software can be difficult to achieve valuable learning outcomes. Thus, the assistance and supervision of teachers in such activities is crucial. In this study we observed the value of the use of physical simulation in supporting students to better comprehend and describe the action of joints as well to (re)interpret some mathematical behaviours often hidden while interacting with a software tool. The combined use of both physical and digital resources seem to bring a relevant contribution for refining students’ thinking and enhance their mental schemes or strategies during their geometrical modeling. With this approach, while hands-on prototypes were bound to physical barriers, digital representation helped to reinforce either geometric relationships or algebraic descriptions. The

outlined examples are only parts of a wider set of exercises and activities developed in our study. In future papers, we will report additional insights into connecting physical and digital modeling approaches.

Acknowledgments

This work was supported in part by Federal Institute of Education, Science and Technology of Bento Gonçalves, Brasil, and Johannes Kepler University, in Linz, Austria.

References

- Bussi, M. G. B., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: artifacts and signs after a Vygotskian perspective. In L.D. English, M.B. Bussi, G.A. Jones, R.A. Lesh, B.Sriraman, & D. Tirosh (Eds.), *Handbook of International Research in Mathematics Education* (pp. 746–783). New York, NY: Routledge.
- Carreira, S., & Baioa, A. M. (2015). Assessing the best staircase: Students' modelling based on experimentation with real objects. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 834–840). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.
- Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th edition). London, UK: Routledge.
- English, L., Lesh, R. A., & Fennewald, T. (2008). Future directions and perspectives for problem solving research and curriculum development. In 11th International Congress on Mathematical Education. Monterrey, Mexico. Retrieved from <https://eprints.qut.edu.au/28450/1/c28450.pdf>
- Lieban, D., & Lavicza, Z. (2017). Geometric Modeling as Spatial Thinking Approach among Prospective Teachers. *Electronic Proceedings of the 22nd Asian Technology Conference in Mathematics*. Chungli, Taiwan: Mathematics and Technology, LLC. Retrieved from http://atcm.mathandtech.org/EP2017/contributed/4202017_21497.pdf
- Polya, G. (1973). *How to solve it. A new aspect of mathematical methods* (2nd ed.). Princeton University Press.
- Sinclair, N., & Robutti, O. (2012). Technology and the role of proof: The case of dynamic geometry. In M. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 571–596). New York, NY: Springer
- Stender, P., & Kaiser, G. (2017). The use of heuristic strategies in modelling activities. In T. Dooley & G. Gueudet (Eds.), *Proceedings of CERME 10* (pp. 1012–1019). Dublin, Ireland: DCU Institute of Education and ERME.
- Stormowski, V. (2015) *Formação de professores de matemática para o uso de tecnologia: uma experiência com o GeoGebra na modalidade EAD* [Mathematics teachers training for the use of technologies: An experience with GeoGebra through online courses]. PhD Dissertation. Universidade Federal do rio grande do sul, Porto Alegre, Brazil