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Creating contact points between empirical modelling and theoretical modelling in teacher education: The case of pendulum problem

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Both empirical modelling (EM) and theoretical modelling (TM) are essential in mathematical modelling. This study explored how graduate students promoted teaching competencies for mathematical modelling by conducting EM and TM with the pendulum task. Through the analysis of modelling lessons, we found that the experiences of both EM and TM with the same material were important for the students to understand a modelling cycle and the aims and perspectives of modelling.

Keywords: Mathematical modelling, teacher education, empirical modelling, theoretical modelling.

Introduction

Educating teachers about teaching mathematical modelling is crucial, because the instruction is demanding for teachers (Blum, 2015). Several courses, programmes and materials for supporting pre-service and in-service teachers in teaching modelling have begun to be developed around the world including CERME community (e.g., Cai et al., 2014; Barquero, Carreira, & Kaiser, 2017). Various competencies are required for teaching modelling. Borromeo Ferri (2018) elaborates four competencies for teaching modelling: *Theoretical competency (incl. modelling cycles or aims and perspectives of modelling as background knowledge)*, *Task competency (incl. multiple solutions or cognitive analyses of modelling tasks)*, *Instruction competency (incl. interventions, support and feedback)*, and *Diagnostic competency (incl. recognising students' difficulties and mistakes)*. She mentions that these four competencies are the basis for the structure of teacher education on teaching modelling. Although these four competencies are related each other strongly, theoretical competency is necessary and important background for teachers' practice on teaching modelling (Borromeo Ferri, 2018).

Some of the common modelling approaches include empirical modelling, theoretical modelling and so on (Ang, 2018). Berry and Houston (1995) mention that both empirical modelling and theoretical modelling are essential in mathematical modelling. Similarly, it might be valuable for teachers to experience both modelling and understand their features in order to foster Borromeo Ferri's (2018) teaching competencies for modelling (e.g., theoretical competency). Limited research exists, however, on teacher education involving both empirical modelling and theoretical modelling. In this study, we explore how teachers promote Borromeo Ferri's (2018) teaching competencies for modelling through teacher education addressing both empirical and theoretical modelling.

Study Background

Empirical modelling (EM) and theoretical modelling (TM)

Berry and Houston (1995) define data-driven modelling cycle as *empirical modelling (EM)*. EM cycle includes collecting data through experimentation and measurement, making sense of data, and making graphs and regression equations that fit to the data. Berry and Houston (1995) mention that “[e]mpirical models are fairly easy to find providing that we are given or can collect the data from appropriate experiment.” (p. 10), and that “[e]mpirical modelling] has severe limitations in the validity of our interpretations from the graph.” (p. 10), and that “[t]he mathematical answer might be perfectly correct but the interpretation in the context of the real world is meaningless.” (p. 12). On the other hand, Berry and his colleague call theory-driven modelling process a *theoretical modelling (TM)*. They recognize TM process as a different one from EM process. They conceive TM process as mathematical modelling including understanding the problem, identifying the important features, making assumptions and simplifications, interpreting and validating the model, and improving the model and explaining the outcome. Both EM and TM are required to understand and describe the real world, and predict results (Berry & Houston, 1995, p. 23). This paper focuses on both EM and TM as components of mathematical modelling.

Teacher education on empirical modelling and theoretical modelling

We reviewed how EM and TM were addressed in pre-service teacher education on modelling from the International Community of Teachers of Mathematical Modelling and Applications (ICTMA) (see table 1). We focused on six chapters mentioning the concrete teaching tasks and materials. One chapter addressed only EM task; four chapters addressed only TM tasks; one chapter addressed both EM tasks and TM tasks. None of chapters used same material throughout EM and TM. If pre-service teachers tackle both EM task and TM task, they can understand modelling in a multifaceted manner. Furthermore, they can grasp the connection between EM and TM by experiencing EM and TM using the same material.

Authors	EM Task	TM Task
Tan & Ang (2013)	Car stopping distance task	Fuel tank calibration task
Winter & Venkat (2013)		Contextual word problems
Widjaja (2013)		Parking space task
Hagena (2015)		Modelling task “Brickhouse”
Villarreal, Esteley, & Smith (2015)	Trash and recyclable collection	
Villa-Ochoa & Berrio (2015)		Planting trees problem

Table 1: EM and TM tasks in pre-service teacher education from ICTMA-book chapters

This study addressed the research question: *How did graduate students promote teaching competencies for modelling (e.g. understanding of modelling cycle and significance of mathematical modelling) by experiencing both EM and TM with the same material?*

Design and Methodology

A pilot study of mathematical modelling lessons for teacher education which comprised four lessons (90 mins each) was designed and implemented for two Japanese in-service teachers and two pre-service teachers. Student A had 16-years teaching experience in lower-secondary schools, whereas student B had 20-years in upper-secondary schools. Student C and B were pre-service teachers and graduate students in mathematics education at the first level. The instructors were the first and second authors. The participants had little knowledge about modelling.

Berry and Houston (1995) illustrate two examples of EM and TM (i.e., audio cassette recorder and pendulum). We consider the pendulum example a better case for novice teachers of modelling and chose that. From the viewpoint of EM, the pendulum example is easy to image its movement, to realize important variables, and to collect data. From the viewpoint of TM, the needed knowledge of mathematics and physics for the pendulum is less than the recorder example. Hence, in the pendulum phenomenon is easy to connect EM with TM. The pendulum phenomenon sometimes induces students' misconceptions and their own ideas about matter (Osborne & Freyberg, 1985) through EM, and students' cognitive conflicts result from the misconceptions (Saeki, Ujiie, & Tsukihashi, 2001). TM, explaining the mechanism of the phenomenon, becomes inevitable activities to resolve the cognitive conflicts. The main theme of the lessons and main pendulum EM tasks and TM tasks are shown in Figure 1.

<p>Main Theme of the Lessons</p> <p>To consider and clarify the key points for the development of teaching materials and problems that are related to the daily situation like the pendulum.</p> <p>Main Pendulum EM Tasks</p> <p><u>Let's find the relationship between the pendulum length and period empirically.</u></p> <ul style="list-style-type: none">• There is the scene of a girl rowing a huge swing in the introduction of the Japanese famous animation video. How long is the huge swing length when the swing period is twelve seconds?• Let's confirm your conjectures of the huge swing length based on the result of the pendulum empirical experiment. <p>Main Pendulum TM Tasks</p> <p><u>Is the empirical result of the huge swing length true? Let's verify it theoretically.</u></p> <ul style="list-style-type: none">• Let's find theoretical formula with the knowledge of mathematics and physics.• Compare the regression equation elicited from EM with the formula elicited from TM.

Figure 1: Main theme of the lessons and main pendulum EM and TM tasks

The activities of modelling lessons for teacher education are shown in Table 2. In the first lesson, the students elicited and choose variables concerning the pendulum empirically through graphing calculators and sensor kits. In the second lesson, they drew graph using data from experiments and performed regression. In the third lesson, they conjectured and verified graphs of Displacement-Time, Velocity-Time, and Acceleration-Time about pendulum based on the physical knowledge. In addition, they elicited the formula about the proportional relationship between displacement and acceleration. In the fourth lesson, they elicited the formula $T=2\pi\sqrt{l/g}$ and confirmed the consistency between the

theoretical model and empirical model in the first lesson. At last, they looked back on the learning processes in the lessons from the viewpoint of modelling cycle.

Sequence of Lessons	Activities in Lessons	EM	TM
1 st lesson	<ul style="list-style-type: none"> ● Eliciting variables concerning the pendulum (e.g. mass of bob, amplitude, and pendulum length) ● Finding that the pendulum period depends on the pendulum length through teacher's demonstration ● Collecting data about the pendulum length (l) and pendulum period (T) through graphing calculators and sensor kits 	●	
2 nd lesson	<ul style="list-style-type: none"> ● Plotting data and draw the graph from the data ● Eliciting the regression equation $T=2\sqrt{l}$ ● Checking the equation using data from experiment ○ Drawing hypothesis graphs of Displacement-Time, Velocity-Time, and Acceleration-Time about pendulum 	●	○
3 rd lesson	<ul style="list-style-type: none"> ○ Verifying students' graphs of Displacement-Time, Velocity-Time, and Acceleration-Time about pendulum based on the physical knowledge ○ Drawing hypothesis graphs of Displacement-Restoring force, and Displacement-Acceleration ○ Verifying students' graphs of Displacement-Restoring force, and Displacement-Acceleration based on the physical knowledge ○ Eliciting the formula $F = -Kx$ about the proportional relationship between restoring force and displacement based on the graphs ○ Eliciting the formula $F = ma$ about the proportional relationship between restoring force and acceleration based on the graphs ○ Eliciting the formula $-Kx = ma$ about the proportional relationship between displacement and acceleration 		○
4 th lesson	<ul style="list-style-type: none"> ○ Eliciting the formula $T = 2\pi/\omega$ between pendulum period and angler velocity based on the definition of angler velocity ○ Reformulating the relationship $F = -m\omega^2x$ between restoring force and displacement by using angler velocity and the mass of the bob ○ Reformulating the relationship $T = 2\pi\sqrt{m/K}$ between pendulum period and the mass of the bob by using $T = 2\pi/\omega$ and $F = -Kx$ ○ Eliciting the formula $F = -mgx/l$ about the relationships between restoring force and pendulum length by using the mass of bob, gravity and displacement ○ Eliciting the formula $T=2\pi\sqrt{l/g}$ by using $T = 2\pi\sqrt{m/K}$, $F = -mgx/l$ and $F = -Kx$ ●○ Confirming the consistency between the theoretical model and empirical model, the regression equation, in the first lesson ●○ Looking back on the learning processes in the lessons from the viewpoint of modelling cycle 		○

Note: ●=EM activity; ○= TM activity

Table 2: Outline of modelling lessons

Our data collection comprised students' descriptions in the post-lesson questionnaires about material development, teaching design, and teaching implementation with real-world context. Their descriptions were coded and categorised according to Borromeo Ferri's (2018) teaching

competencies for mathematical modelling. In this study, three of the four abilities were confirmed as shown in Table 3.

Competencies and focused contents	Examples of students' descriptions
<i>Theoretical competency</i>	(A) Descriptions about process and phases related to the transitions between real world and mathematical world
(A) Knowledge about modelling cycles	(B) Descriptions about the significance of modelling and modelling tasks
(B) Knowledge about goals/perspectives for modelling and modelling tasks	
<i>Instruction competency</i> (Knowledge about interventions, support and feedback)	Descriptions about the necessity of the interventions for connecting between the real world and the mathematical world
<i>Diagnostic competency</i> (Knowledge about students' difficulties and mistakes)	Descriptions about designing teaching modelling according to students' abilities

Table 3: Students' descriptions according to Borromeo Ferri's (2018) teaching competencies for modelling

Results

This section illustrates the findings about students' understanding of modelling cycle, aims and perspectives of modelling, and beginning of understanding related to teaching modelling.

Students' understanding of modelling cycle including EM and TM

Student B summarized the modelling cycle including EM and TM (Figure 2).

1. The contents of the task are interesting for students.
2. Students can experiment and analyse phenomenon in real society.
3. Students can elicit mathematical expressions like equations from the experimental results.
4. Phenomenon in real society can be translated into mathematical contents.
5. The mathematical contents can be solved with students' mathematical knowledge.
6. The mathematical results can be validated and be applied to the phenomenon in real society.
7. Students can realize the usefulness of mathematics.

Figure 2: Student B's description to post-lesson questionnaires

Student B clarified the modelling cycle through the modelling lessons. Description related to the modelling cycle was only one; however, whole students discussed about the modelling cycle in the reflection of the lessons. They all might understand the modelling cycle including EM and TM.

Students' understanding of aims and perspectives of modelling and modelling tasks

Three students described aims and perspectives of modelling and modelling tasks. Student C understood the difference between EM and TM as perspectives of modelling: "Modelling with experiments was concrete for me and was pleasant to imagine the result of pendulum. But, I wondered if the result is right or not", "Modelling without experiment was difficult for me, but the process and the result of formula was right", and "The modelling with the experiment of pendulum was very interesting. The pendulum experiment deepened the understanding of the period of

pendulum”. Student A and Student B identified the usefulness of mathematics as an aim of modelling. Student A described that the lessons of modelling could convey the usefulness of mathematics to school students: “Through the lessons that connected between mathematics and the real world, students would be surprised that mathematics is hidden in real-world situations and they might be able to notice the usefulness of mathematics”. Student B described the role of mathematics in real world and society: “Mathematics is used at various places in the real world. For example, the cord, GPS, and statistics. I would like to develop the lessons emphasizing to realize that mathematics might be powerful and useful for students to solve real world problems”.

Beginning of understanding related to teaching modelling

In the fourth lessons, four students reflected the learning processes from the viewpoint of modelling cycle. From these experiences, they found the importance of observing students’ situation and of preparing teaching materials for EM and TM. Students A, C, D realized the necessity of the interventions in EM and TM. Student A emphasized that teachers should grasp students’ mathematical knowledge before modelling lessons, especially TM activities: “I would like to design my lessons that the students are able to consider real world problems and to grow their viewpoint for consideration using previous mathematical knowledge. Student D described similarity as follows: “I experienced EM and TM. Through my experience, I thought that TM was difficult for students who did not get some knowledge of mathematics and physics. I think it is important to consider students’ situation and to prepare for teaching materials to diverse students”. Student C described the importance of teachers’ pre-experiment in EM: “For modelling with experiment, I must make plan to have included time for carrying preparations of experiments and means thoroughly”. Student A understood the necessity of the intervention for connecting between the real world and the mathematical world. He described as follows: “The facilitation of connecting the mathematical world and the real world is necessary”. He realized the needs of preparation for mathematical modelling. Student B found the importance of modelling lessons according to students’ abilities: “Through the lessons for modelling, I should plan the lessons flexibly and diagnose students’ situation and understanding in case of the difficult modelling with mathematics for students”.

Discussion and Conclusion

This paper has addressed some aspects of students’ understanding about modelling cycle and significance of modelling through the experiences of both EM and TM with the pendulum task. From the results of the transcripts and the descriptions in the post-lesson questionnaires, three results became clear. First is students’ understanding of modelling cycle including EM and TM. Second is students’ understanding of aims and perspectives of modelling and modelling tasks. Third is beginning of understanding related to teaching modelling that included the necessity of the interventions for connecting between the real world and the mathematical world, the necessity of the facilitation in EM and TM, and designing teaching modelling according to students’ abilities.

These results demonstrated the development of Borromeo Ferri’s (2018) theoretical competency, instruction competency and diagnostic competency. For example, the first and the second results correspond to the theoretical competency. Student A’s description on the necessity of the intervention for connecting between the real world and the mathematical world and students A, D, C’s

descriptions on the necessity of the facilitation in EM and TM correspond to the instruction competency. Student B's description on designing teaching modelling according to students' abilities corresponds to the diagnostic competency. However, Niss and Højgaard (2011) point out that competences are difficult to measure and should be done in a 3D model with technical level, degree of coverage, and radius of action. This study focused on the technical level and degree coverage; hence, the in-depth assessment of competency is our future work. We believe that experiencing EM and TM with the same material (Berry & Houston, 1995) worked for the above results. The students confused the difference between the misconception (Osborne & Freyberg, 1985) and the result of experiment in EM. In fact, Student D answered this situation: "Findings from my experience of pendulum were uneasiness". This uneasiness born in EM became the driving force for TM elucidating mechanism inherent in pendulum phenomenon mathematically and physically. Through the experiences of both EM and TM using the same material, the students were able to consider real-world problems deeply connecting between inductive and deductive viewpoints. They were able to recognize that EM and TM were like opposite sides to the same coin and essential components of modelling cycle through the reflection of learning processes via a diagram of modelling cycle (Kawakami, Saeki, & Kaneko, 2018). In this study, EM produced intellectual curiosity and uneasiness and elicited inevitability of the TM. However, the previous study on teacher education on modelling did not focus on the relationships between EM and TM. We need rethink the role of experiencing both EM and TM with same material in developing teachers' teaching competencies for modelling in other cases.

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