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Pre-service mathematics teachers' learning through designing modelling tasks

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The results of analysing the projects for the teaching of mathematics and modelling developed by future mathematics teachers are discussed in this paper. The participants were part of a seminar in which the objective was to develop their mathematical and pedagogical content knowledge related to the teaching of mathematics through the design of modelling tasks. With a qualitative methodology the written reports of and interviews with the pre-service teachers were analysed. The results bring to light some elements associated with their knowledge and how this knowledge is constructed in their experience with modelling activities and then reflected in the creation of teaching tasks.

Keywords: Modelling task, future teachers, teacher training.

Introduction

The design of novel tasks and teaching activities represents a challenge for teachers and future teachers. They must be able to involve students in meaningful activities and create environments in which students construct and reflect on their own learning. In designing modelling tasks, the future teachers' knowledge about the context and the mathematics related to it, joined with the knowledge about the teaching and learning of the students becomes structured and shapes the tasks. The design and implementation of modelling tasks has been subject of innumerable studies with attention to the characteristics of the final product; but few investigations consider the processes that are favoured in this activity (Czocher, 2017), for example the construction of the teacher's pedagogical content knowledge.

When modelling activities are introduced to the classroom, teachers must adopt an adaptive view of new teaching methods. This involves a challenge that requires the mastery of mathematical knowledge associated with the context of the task and adapting that knowledge to the environment in which students will work with the task according to the learning objectives (Doerr & English, 2006). In the setting of a teachers' training seminar, we analyse: 1) how future teachers, when designing teaching tasks, construct and transform their mathematical knowledge associated with the modelling of population growth and 2) how this knowledge is adapted for teaching in a middle school mathematics course. The research question is: *What elements associated with pedagogical content knowledge emerge in the design of modelling tasks for teaching mathematics?* An important aspect in this study is the design process where participants learn to model to teach modelling. This fact is evidenced in the exploration of several situations and in how future teachers simplify and idealise the initial situations to adapt the task to the knowledge of the students.

Conceptual framework

When studying task design in mathematics education two perspectives can be identified: the design as implementation or the design as intention. The task design analysed here is characterized as

design as intention since it addresses the initial formulation of the design (Czocher, 2017) and the analysis is supported by constructs for studying teacher's pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Doerr & English, 2006; Carrillo-Yañez et al., 2018). By teaching modelling it is intended that teachers become competent at modelling activities, they also should be able to link knowledge about modelling cycles, about goals and about types of tasks. Solving, analysing and creating tasks as well as planning and performing modelling lessons are important aspects here (Borromeo Ferri & Blum, 2010; Huincahue-Arcos, Borromeo Ferri, & Mena-Lorca, 2018). Designing and modifying tasks to be easier or harder for presenting mathematical ideas constitutes a part of the teacher's knowledge (Ball, Thames, & Phelps, 2008). When designing tasks for learning mathematics and modelling, the teacher's pedagogical content knowledge comes into play in teaching organization, in the understanding of how to present the concepts to favour learning and when understanding the mistakes associated with the domain (Doerr & English, 2006). In addition, the teacher must know the specifications of the curriculum and maintain a critical position to organize the learning and management the class (Carrillo-Yañez et al., 2018). The practice of modelling leads to the development of modelling skills (Blomhøj & Kjeldsen, 2006), so that when the teachers design a task, they structure their specialized content knowledge related to models with the intended knowledge for the students. On the other hand, when modelling students build knowledge depending on the characteristics and cognitive demands of the tasks (Henningesen & Stein, 1997). Therefore, it is expected that the tasks created by future teachers support progress in the modelling abilities of the students. According to the model of three dimensions (Blomhøj & Kjeldsen, 2006), the modelling competences can be distinguished as: (1) Technical level, related to the kind of mathematics the students use and to how flexible they are in their use of mathematics. (2) Radius of action, related to the domain of situations in which students can perform modelling activities and (3) Degree of coverage, according to the phase of the modelling process the students are working in, and the level of their reflections.

Context and method

In most Chilean teacher training programs, modelling has not been explicitly part of the education for future teachers. The future teachers take part in several courses of mathematics, didactics and pedagogy. They learn mathematical concepts and theories of education, but they have few experiences with modelling situations; as Blomhøj and Kjeldsen (2006) pointed out modelling is different than the nature of mathematics. In this context, it is described and analysed the process developed by a group of pedagogy in mathematics students, when they designed an activity for teaching. The experimentation was carried out with a group of six students, who participated in a research seminar for future teachers. Here, the case of a group with two participants is presented. They were selected because of their willingness to participate in the research. In the seminar some topics of differential (and integral) calculus and differential equations were discussed and among these functions, derivatives, logistic equations and systems of differential equations. Some perspectives of modelling (Blomhøj & Kjeldsen, 2006; Borromeo Ferri, 2006) were also reviewed. During the seminar several discussions about the use of technology and the content knowledge for teaching (Ball, Thames, & Phelps, 2008) were carried out. The students explored modelling different phenomena and proposed a teaching activity for middle school level (15 years old). The

process of designing a task (for approximately three months) was documented, where participants studied how some growth situations are described in reality. Then, after simplifications and idealizations their initial study took shape of a teaching activity. The design phase considered aspects such as, idealization and exploration of the initial conditions, parameter values and the domain of the solutions. Meetings with students once a week during one semester were developed in where the design process was followed. At the end of the course, they presented the teaching task and a description of the process of designing it in a written report.

Data analysis was carried out following a qualitative perspective (Cohen & Manion, 1989). First the episodes related to the modelling phases along the process of designing the task were identified. Then, units of meaning associated with mathematics, modelling and pedagogical content knowledge were identified. At the end of the seminar participants were interviewed by two researchers to verify the first interpretations of the author. Finally, the units of analysis identified in the written report and those from the interview were contrasted and discussed in the modelling seminar, where the author took part. Thereby, the validity of the results was verified.

Discussion and analysis of results

In this section, the study carried out by the participants related to the phenomenon in question is initially presented. It is not the intention in this paper to inform about a common process of population growth modelling, however, in order to analyse the knowledge that emerges when future teachers design a modelling task, we present a summary of the facts that they recognise as relevant. It should be noted that most researches on teacher's knowledge do not explicitly address the role of modelling (Ball, Thames, & Phelps, 2008; Carrillo-Yañez et al., 2018) but consider it as an element of mathematical practice. Here modelling is posed as a precursor element of the development of future teachers' knowledge. Furthermore, the experiences with modelling activities can be related to the improvement of the mathematical knowledge of the future teachers. Subsequently the teaching task proposed by the participants was analysed where some elements from the pre-service mathematics teachers' pedagogical content knowledge were identified. Three episodes were recognized: first when participants explored the growth of penguin populations reported by the research literature, where the case was analysed by statistical methods. Second, when the relevant information was identified and linked with an idealised mathematical model. Third, when information gathered in the first and second episodes was adapted to create a task for teaching.

Study of real situation

After consulting literature related to different growth phenomena, a growth situation of Magellanic penguins was selected (Pozzi et al., 2015; Valdebenito, 2013). The habitat places of this type of penguin were located on a map. In addition, the total population of penguins in the world was identified (1.5 million pairs of reproductive age). This led the future teachers to focus their attention on the population of penguins in Chile (700 pairs), particularly a population that is located on an ecological reserve that hosts around 400 specimens. Here participants observed the existence of infrastructure designed to allow tourism without affecting the reproduction of the penguins. This fact allowed them to transit from the study of a more general situation to the case of a growing population limited by a border. When doing the first exploration participants observed that studying

a population in freedom, implies considering predators and death of penguins due to the fishing process. By exploring the reproduction of the population, they identified the nesting seasons, the number of eggs per pair (2) and the places where the nests were located. They also noticed the existence of other predators like seagulls that attack eggs. Considering this information, participants associated the successful breeding with the number of chicks that survive per nest on each breeding season. These considerations were initially associated with logistic equations and systems of ordinary differential equations (predator-prey model).

Then, an idealisation of the situation was made, when participants saw the case as population growth in a limited place where the necessities of feeding (anchovies, sardines and some crustaceans) also influence the size of the population. Thereby, pre-service teachers made relationships between the competition for limited resources in the environment and the population size; this led them to model the situation using the logistic equation of population growth. The elements used to determine the parameter values in the equations were: the reproduction age (the females reproduce between 4 to 5 years old, while the males reproduce when they are 6-7 years old), and the penguins are monogamous. The average life expectancy is about 20 years. With this information students calculated an approximation of the growth rate.

This phase was a support for pre-service teachers to: a) recognise the reasons why some simplifications are necessary, b) recognise and explain the frequent difficulties students find in mathematizing a situation and c) identify and explain the behaviour of some situations. In addition to the ideas of Ball, Thames, and Phelps (2008) it is assumed that those elements must be part of the teacher's mathematical knowledge for teaching modelling.

Simplification of the real situation and modelling

When simplifying the real situation, the participants made decisions that were reflected in the design of the task as will be discussed below. In summary, the literature gives information about the penguins' birth and mortality rates per year. Then by calculating an average students approximated this values by a constant (intrinsic rate of increase) to be used in the expressions to describe the population growth. From here, by comparing continuous and discrete exponential growth future teachers were aware of the complexity of modelling realistic situations. They also realised the limitations of modelling population growth with simple models. Hence, when pre-service teachers explored open modelling situations, they built their mathematical knowledge associated with the situation (Carrillo-Yañez et al., 2018), and when they used that knowledge to design the task, they strengthened their pedagogical content knowledge related to modelling practices (Doerr & English, 2006). Thus, the experience they got by exploring the characteristics of the phenomena served as the basis for the design of the teaching situation.

Below some of the arguments from the participants regarding to the relevant information and the simplification they made are shown:

Initial population – the literature mentions that in 1993 a population of 400 penguins was estimated in the reserve.

Birth- [...] paired penguins lay 1 to 2 eggs, but not all penguins are in pairs and not all couples get two eggs, for this reason the birth rate can be considered to be 50%. However not all chicks are born, which leads us to consider a birth rate of 40% of the total population. This rate is consistent with the data in the literature [...].

Mortality- [...] the longevity due to natural death of penguins is almost 20 years, in addition there are other factors that cause the death of this species [...], the spilled oil and the fishing nets, that add up to 16% mortality [literature information]. If [mortality] is adapted only to natural causes of death (Diseases, poisonings, etc.) an estimate of 14% can be made since the reserve has the necessary care to maintain the health of the penguins.

Predators- The presence of predators was not taken into account for the design of this task, since the penguins are in a reserve and are protected from these threats.

With this information the participants analysed two cases described by the equations $\frac{dN}{dt} = kN$ y $\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)N$, getting the solutions $N(t) = 400e^{kt}$ y $N(t) = \frac{Ze^{rt}K}{Ze^{rt} + 1}$. When studying the graphs of these expressions they identified the behaviour of a population modelled by the logistic equation and described it as below:

[...] if the population is less than the carrying capacity, the graph behaves increasingly, but when it approaches (K) the graph begins to stabilise, instead if the population was greater than the capacity of load the graph behaves decreasingly and begins to stabilise when it is closer to K .

In this episode, stand out identification and simplification processes of relevant information and the participants' mathematical work using analytical and graphical representations. The experience gained studying the real situation helped future teachers to identify and anticipate appropriate mathematical representations to be used in the teaching activity. Those elements are related with the Specialized Content Knowledge (Ball, Thames, & Phelps, 2008).

The teaching task

In this section the task designed by two participants is analysed. The learning objectives were defined as: Analysing the behaviour of a population based on birth and mortality rates in relation to the time. The task is the following:

In 1991 a massive death of penguins of the Magellanic species was evidenced. To revert the situation a group of people in 1993 decided to create a Natural Reserve of Magellanic Penguins called "Seno Otway". The reserve has capacity for hosting 6500 penguins. At the beginning the reserve started with an initial population of 300 Magellanic penguins. From a census they calculated a mortality rate of 70 per 500 penguins and a birth rate as 200 per 500 penguins.

Some of the questions that the future teachers posed are:

- 1) What do you think would happen if the population exceeds the capacity of the reserve?
- 2) Use the data for graphing the situation.

3) Analyse what should happen when reaching the load limit.

Those questions show that teachers focused on the carrying capacity as the relevant aspect that affects the growth of the population. The first question demands an intuitive answer from the students, with the third question they are challenged to prove or reject their initial conjecture by studying the data. To answer the second question, students were encouraged to organizing the data about the time and the population in a table, using this information they could build a graphical representation. Here, the future teachers' knowledge related to the different registers in which a mathematical content can be represented (Carrillo-Yañez et al., 2018) and their strength to interpret situations becomes evident. This is also related to the knowledge of some theories of learning as it is shown in the next fragment related to the characteristics of the task:

The activity is designed for the students to articulate the same information in different semiotic representations, and to look for a generality that represents the behaviour of the population [...]

The teachers' knowledge associated with the characteristics of modelling was also evident when they described the processes through which the student should transit when answering the questions. They anticipated that students would go through a spiral-shaped cycle increasing as they require more knowledge to answer the questions. This is shown in an extract of the written inform:

In order to solve successfully the task, it is necessary to carry out several modelling processes that respond to different situations, we could group the questions into three categories, easy (1, 2), medium (3, 4) and hard (5, 6); each of them corresponds to a separate modelling cycle that adds more difficulty as the students respond.

In the previous transcript the knowledge of pedagogical content was evidenced in the design of the task when teachers anticipated the students' reaction in relation to the difficulty in answering the items (Ball, Thames, & Phelps, 2008).

While designing the task, future teachers recognised the importance of students being aware of the idealization processes involved in the analysis of a phenomenon. In the following fragment, when justifying the pertinence of the question number three, they referred to the idealization that implies modelling the situation through the differential equation $\frac{dN}{dt} = kN$ and compared it with the case

modelled by the equation $\frac{dN}{dt} = r\left(1 - \frac{N}{k}\right)N$:

[...] the students should reflect on the exponential growth of the population, because so far we have only analysed an ideal case, where no carrying capacity or predators in the area have been taken into account.

In relation to the graphical representation of the behaviour of the population, future teachers pointed out that working with graphics could generate some errors of interpretation when students superficially see the generated curves; this is associated with the knowledge of the structure of mathematics (Carrillo-Yañez et al., 2018). They point it out as follows:

They [students] may build a more flattened exponential, arguing then that the growth population should be slower.

In the previous analysis a strong relationship between the (mathematical and extra-mathematical) knowledge constructed by future teachers when experiencing modelling to design a teaching task was observed. And the pedagogical content knowledge was evidenced when they were able to anticipate the students' actions with the modelling task. The task was designed to cross through the modelling cycle in different ways. However, the activity was subordinated to only working with the data in the tasks (Blomhøj & Kjeldsen, 2006).

Comments and conclusions

It was evidenced that in the challenge of designing teaching tasks the future teachers experiment with mathematical and extra-mathematical knowledge and bring to light what they learn for planning the teaching. The future teachers' learning endured a transformation from studying pure mathematics to study the mathematical and extra-mathematical content associated with several phenomena. When exploring the situation, the participants' work highlighted activities as identifying relevant information, simplifying the situation, formulating a task, systematising, mathematising and analysing mathematically. Those activities reinforce their knowledge about the mathematical characteristics of the models. They also had the opportunity to link concepts, understand procedures, rules and meanings. The knowledge constructed in this way was a support for designing the task, so that it strengthens their pedagogical content knowledge. In relation to the progress of mathematical competence (Blomhøj & Kjeldsen, 2006), it was observed that the future teachers' performance with the modelling activity and the task designed by them are situated in the technical level, related to the use of mathematics and its representations. However, it is desirable that future teachers widen their radius of action.

The experience in this research showed that – in addition to the knowledge pointed out by Ball, Thames, and Phelps (2008) – for implementing modelling as a vehicle for teaching mathematics, the teacher should be able to: recognize the reasons why some simplifications are necessary, recognize and explain the frequent difficulties students find in mathematizing a situation, identify and explain the reasons for the behaviour of some situations, identify and anticipate appropriate mathematical representations, identify and enhance the transit through the different phases of a modelling process. On the other hand, some tension between developing modelling competences and teaching the content was identified. That is, although the task could be more open allowing students to build their own strategies, the final characteristics of the task limit the actions of the students to whom it will be directed. This situation has a strong relationship with the educational system because most institutions expect that teachers taught the content as it is described by the curriculum where modelling is considered closer to word problems. In the same way, Chilean teachers training programs are more directed toward the teaching of content rather than developing skills. Therefore, our challenge is to encourage pre-service teachers to explore innovative ways for teaching mathematics.

Finally, the construction of pedagogical and mathematical knowledge was analysed when pre-service teachers design a task; this fact does not allow us to know how future teachers put into play their pedagogical content knowledge in the practice. This aspect is been explored in parallel studies.

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