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► **To cite this version:**

Rita Borromeo Ferri. Assessing Teaching Competencies for Mathematical Modelling. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02408728

**HAL Id: hal-02408728**

**<https://hal.science/hal-02408728>**

Submitted on 13 Dec 2019

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# Assessing Teaching Competencies for Mathematical Modelling

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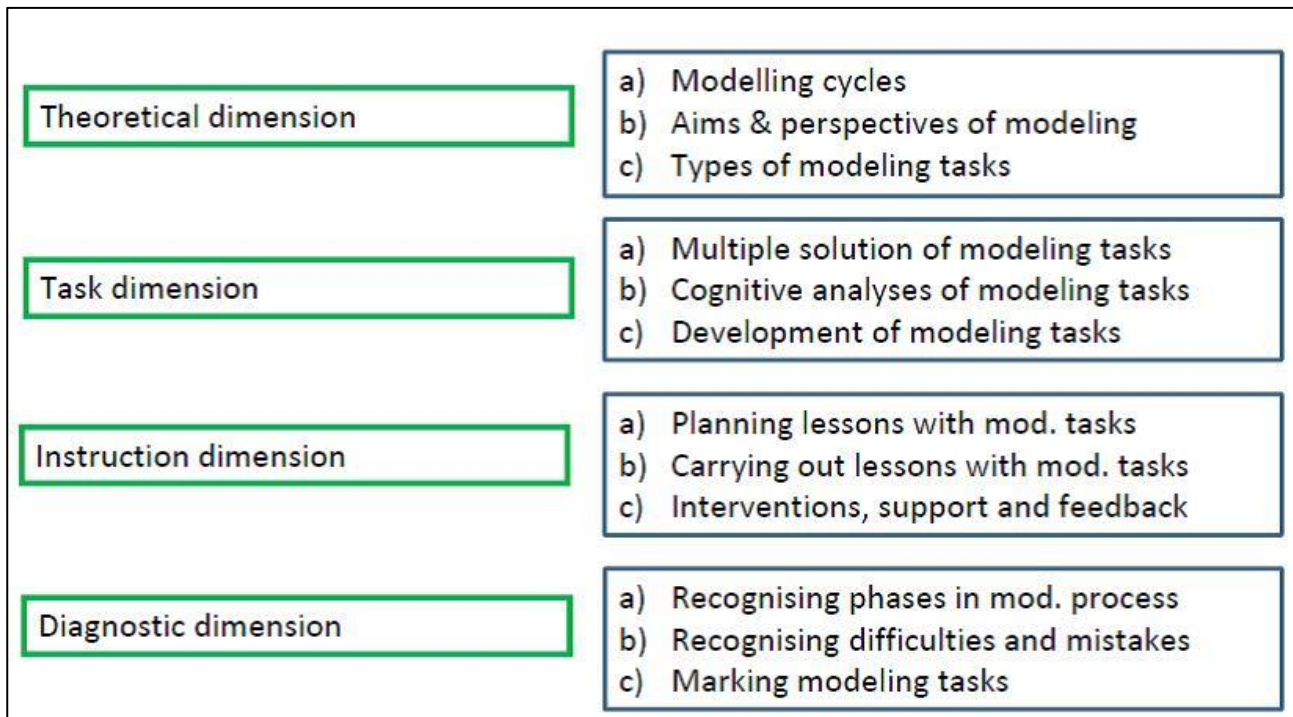
*The question of how mathematical modelling competencies can be assessed has been well investigated and discussed in the last decades within the modelling research field. The test instrument, originally from Haines and Crouch, was adapted for learners in secondary and high-school and as well for teachers in many empirical studies. The results of these investigations also helped to conceptualize the term of modelling competencies. Far more at the beginning of research efforts is the question of how teaching competencies for mathematical modelling, based on the model of Borromeo Ferri, can be assessed. The complexity of the four dimensions is challenging in order to develop a test instrument with the goal to see an increase of teaching competencies after a course on ‘learning and teaching of mathematical modelling’. The paper describes recently developed test items used in an intervention study with prospective teachers, and presents first results of this pilot study.*

*Keywords: Mathematical modelling, teaching competencies, assessment.*

## Teaching competencies for mathematical modelling

Since the beginning of research in the field of mathematical modelling education it is of great interest to get insight into how teachers in school, but also lecturers at university, should teach mathematical modelling to their learners in the best way. Many best-practice university courses or seminars can be found in the literature concerning this aspect (e.g., Lesh & Doerr, 2003; Schwarz & Kaiser, 2004; Borromeo Ferri & Blum, 2010; Maaß & Gurlitt, 2011; Burkhardt, 2018; Vorhölter, 2018). The cited authors describe in their courses or training workshops for mathematical modelling in teacher education several activities and thus competencies, which teachers should learn for teaching modelling successfully. The variety of didactical approaches, contents and aims in these course descriptions are very broad. Thus the conceptualization and operationalization of teaching competencies in mathematical modelling was necessary from the author’s perspective in order to get also a theoretical framework.

Based on a Designed-Based Research (DBL) approach, Borromeo Ferri (2018b) developed, within the last 10 years, a module for a modelling course for university and also for teacher training, which led to a model for competencies needed in teaching mathematical modelling (Borromeo Ferri 2014, 2018, Borromeo Ferri & Blum 2010). The goal is that the lecturer of a university seminar for prospective math teachers or a teacher trainer covers all competencies of the model in a balanced way and sometimes with different emphasis due to the timeline of the course or the workshop. This works very well, because I practiced and investigated these courses since a long time. Figure 1 shows the model with its 4 dimensions and sub-competencies:



**Figure 1: Model for competencies needed in teaching mathematical modelling (Borromeo Ferri, 2018)**

The structure of my modelling courses is based on this model, which means going through each dimension starting with the theory and ending with discussing about assessment in modelling. Step after step the participants gain more competencies and understand how everything is linked together. Due to the aspect that participants should develop a modelling problem, teach it at school and finally reflect on it, a Theory-Practice Balance is given. This greatly helps to understand what modelling means in the school context and in particular that teaching modelling is possible. For more details concerning the full module, the used of modelling problems, materials or teaching methods, see Borromeo Ferri (2018a). The continuous modification of the modelling course is one aspect, a further aspect is of course to measure systematically how teaching competencies of the participants increase during and after the modelling course, which means at the end of a semester or after a teacher training of two full days. Based on the observations and evaluation sheets of the university students and teachers, I can definitely argue that an increase of teaching competencies takes place after these courses, but the goal is to assess this from a scientific and empiric point of view. This is a challenge when recognizing the complexity of the model. Klock et al. (in press) and Klock et al. (2018) also developed a test instrument for facets of a modified version of the model of Borromeo Ferri. This test instrument is piloted and used already in intervention studies.

However, I developed items for the four dimensions of mathematical modelling teaching competencies based on my model in order to get a test instrument. These items were recently applied and piloted with a pre- and post-test design for a university modelling course for prospective secondary teachers, in order to get knowledge on how their teaching competencies increase. In the following, the item development is briefly described, which shall show the difficulties of building scales, but also some examples of final items are presented. Furthermore first results of the pilot study are shown and discussed.

## Development of test items for the teaching competencies

When looking at the model for teaching competencies it becomes clear that items are needed, which should test both declarative and conceptual knowledge. The four dimensions build on each other and are connected. The diagnosis of mistakes in students' modelling process for example is much easier, if the teachers have knowledge about the modelling phases and about modelling tasks. Learning to diagnose does not only mean to give a definition of several types of diagnosis (product oriented/process oriented), but also to practice it with school students and also with concrete examples presented in video clips. The same applies for the topic teacher interventions while modelling. Although this is a strong part within the modelling course, it is not easy to test these competencies.

Within the first approach of test development and evaluation, most of the items were open. This test version was used for 24 prospective secondary and high-school teachers in their fourth semester at university. A prominent open question for the theoretical dimension is: "What is mathematical modelling?", because on the one hand one would expect some kind of definition and on the other hand it is interesting to see what prospective teachers really have understood, when they describe it before and at the end of the course. According to this, very broad answers were given at the end of the course. Often, modelling was described as a process referring to the modelling cycle, for example: "Mathematical modelling means to follow the steps of a modelling cycle." This is not wrong, but also not completely right, because the nature of mathematical modelling is not emphasized. In a further open item the students should draw a modelling cycle, write down and describe the several steps. This is a nice activity, but difficult for coding the results at the end. How to code only partly answers or not precise descriptions? In sum, this kind of test items by using open questions for the teaching dimensions offered a lot of insight of university students' increase of their teacher competencies only on the basis of their elaborate answers and statements from the pre-test to the post-test, but not in the sense of a classical test. Furthermore, the test time was about 45 to 60 minutes, because answers needed intensive writing and the test construction with its complex items does not allow to cover all sub-competencies within the dimensions. Thus the decision for a multiple choice format with closed items was made for assessing declarative and conceptual knowledge in a balanced and economic way for all teaching dimensions.

Like for the previous version, the construction of new test items was based on the content of the modelling course. All items of the new test instrument were developed in such a way that one has to choose a correct answer according to a statement or after the analysis of a written dialogue from learners while modelling. Options between two or five possibilities, depending on the item, are given to pick out the right answer or one has to write down the correct term for which is asked. Thus, this answering format offers the possibility for dichotomous coding. The table gives an overview about the number of items per scale (teaching dimension):

	<b>Theoretical D.</b>	<b>Task D.</b>	<b>Instruction D.</b>	<b>Diagnostic D.</b>
<b>Number of items</b>	26	11	14	14

**Table 1: Number of items per scales (4 teaching dimensions)**

Within the theoretical dimensions the testing of declarative knowledge with 26 items was in the foreground. In the following three examples items are shown.

Here, your theoretical knowledge is asked. Choose, if the statement is right (yes) or wrong (no):

	yes	no
The basis of mathematical modelling are problems from real life	<input type="checkbox"/>	<input type="checkbox"/>
The "Complexity" describes one criterion of a modelling problem	<input type="checkbox"/>	<input type="checkbox"/>

**Figure 2: Example items of the scale "Theoretical dimension"**

For the scale "Task dimension" 11 items were developed, which tested declarative and conceptual knowledge. In order to assess the conceptual knowledge, a modelling problem was given with corresponding solutions of learners. It was asked to choose the right phase of the modelling process, where the solution belongs to. Because of the fact that only one answer from five options is right, one has to analyze the solution more in detail. Not only the knowledge about the terms and the phases is of importance, but to apply it based on own experiences during the course while solving and reflecting modelling problems. The "Instruction dimension" is, compared to the other dimensions, quite hard to assess. First of all it is not possible to test if an individual is really able to teach modelling in the class, nor if the lesson is good or bad. However items were developed, which asked for basics of quality teaching concerning modelling and furthermore about the aspects of teacher interventions as well. Hence 14 items are in the scale "Instruction dimension", which cover declarative and conceptual knowledge. Two items of this scale are shown in Figure 3.

Here, your knowledge about teaching modelling is asked. Choose, if the statement is right (yes) or wrong (no):

	yes	no
The introduction of modelling activities works with over-determined problems	<input type="checkbox"/>	<input type="checkbox"/>
Responsive interventions lead back to the teacher	<input type="checkbox"/>	<input type="checkbox"/>

**Figure 3: Example items of the scale "Instruction dimension"**

In particular the teacher interventions were tested along different short written conversations of learners while working on a modelling problem in the classroom. The levels or activators of the interventions, for example, had to be determined correctly.

The last dimension and scale is the "Diagnostic dimension" with 14 developed items. Like in the task and instruction dimension, both declarative and conceptual knowledge was tested with several items. To diagnose in which modelling phase learners worked and also problems, mistakes and misconceptions were in the focus for the item development. Again, items were used, which showed a conversation of learners working on a modelling problem. Then one right answer has to be chosen.

**Diagnose the modelling phase, in which the learners work primarily. Choose one option!**

understanding the problem	<input type="checkbox"/>
simplifying/using extra-mathematical knowledge	<input type="checkbox"/>
mathematizing	<input type="checkbox"/>
working mathematically	<input type="checkbox"/>
interpreting	<input type="checkbox"/>
validating	<input type="checkbox"/>

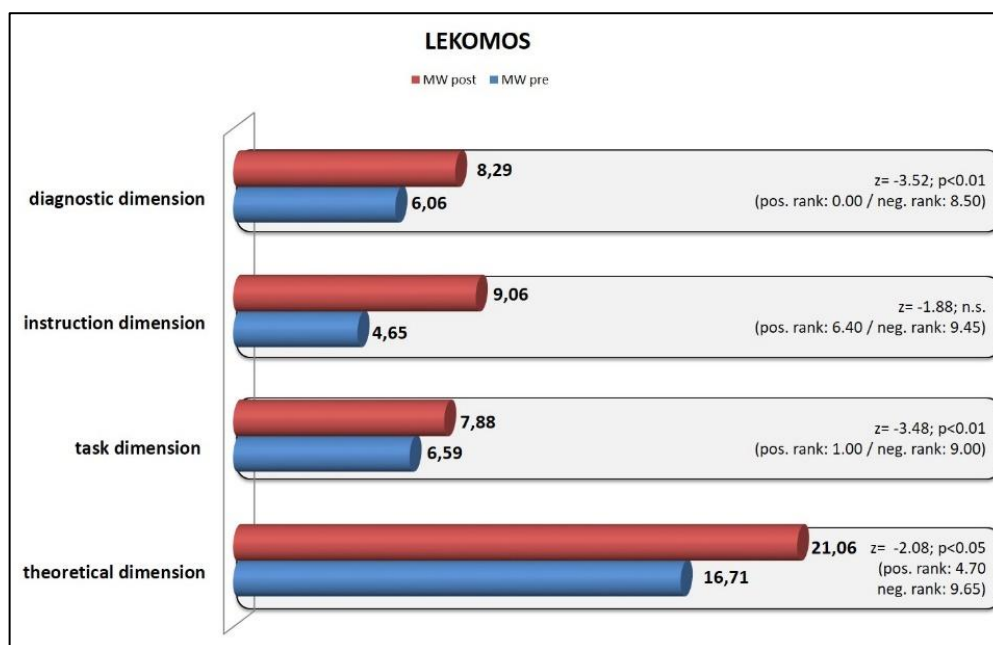
**Figure 4: Example item of the scale “Diagnose dimension”**

A crucial part of the modelling course is the development of students’ own modelling problems, which are finally taught in school. This includes besides planning, executing and reflecting on the lesson also to diagnose learners while they work on the problem. This kind of real life observation is connected with the usage of video-clips of learners while modelling within the course. Thus the diagnostic dimension is stressed as very important in the course, because it gives a résumé and the possibility to apply all the competencies learned within the previous dimensions.

### **What you test is what you get! – First Results of the Pilot Study**

The test instrument was piloted in September 2018 in a modelling seminar with 17 prospecting secondary teachers at university in their third semester. This course was a blocked seminar with four sessions each three hours. Between the third and fourth session, the students taught their developed modelling lesson in school and observed learners during modelling activities. The pre-test was done at the beginning of the first session and the post-test at the end of the fourth and last day of the course. In contrary to the first test version with open items, the new test time was now 25 minutes. All participants already had basic knowledge about mathematical modelling from one lecture in the first semester within the lecture-series with the topic “Introduction in Mathematics Education”. In this lecture the university students learn about the modelling cycle and they also solve a modelling problem. However, only basics about modelling are stressed in this lecture and of course one goal of introducing modelling in this lecture-series is that the prospective teachers are interested to choose a modelling course in the upcoming semesters.

Due to the fact that  $n=17$  is a very small sample in the current pilot study, however, the very first results are promising concerning the quality of the items and  $\alpha$ -values. Thus for the upcoming testing in the winter term with a larger population, the test instrument will be checked and if needed, modified again. In the following descriptive analysis the results of the pre- and post-tests are presented. The results are based on a dichotomy coding procedure, showing the means of the dimensions before and after the treatment. Because of the fact that  $n$  is smaller than 21, a test of Gaussian distribution was necessary. Not all scales had Gaussian distribution and in order to test the level of significance, the Wilcoxon-Rank test (Bortz et al., 2010) was necessary. In Figure 5 the significance values become evident:



**Figure 5: Results of pre- and post-test**

The results show statistical significant differences after the treatment for three of the dimensions. Although no significance value could be stated for the instruction dimension, an increase of teaching competencies becomes evident like for the rest of the dimensions as well. The modelling course as the dependent variable can be seen as an effective predictor, so that teaching competencies for mathematical modelling increase. The mean of several items also makes clear that the treatment and thus the structure and content of the modelling course focuses on the central aspects of teaching and learning mathematical modelling. The last item within the Theoretical Dimension for example shows two tasks and one has to decide, which of them is a modelling problem. This item could also be put in the Task Dimension, but in this case, this item focuses more on the question of what mathematical modelling means and what characterizes it. In the pre-test the mean was 0.65 and in the post-test 1.00. Although students had basic pre-knowledge about modelling, it was not a trivial question at the beginning. During the modelling course, the university students again recognized several criteria of (good) modelling problems and that a real context leads not automatically to the conclusion that one can speak of a modelling task.

The Diagnostic Dimension contains many items for testing the declarative knowledge. As described in the previous section, items were developed, which show a short conversation of learners working on a modelling problem. One right answer had to be chosen according to the modelling phase the learners in the presented scene worked primarily. An increase of the means becomes visible from pre to the post-test from 0.65 to 0.88 for such a diagnostic item or from 0.41 to 0.71 for a similar item. Beneath this testing, the university students also evaluated the course separately, which is a normal procedure in my seminars. The aspect of the Theory-Practice-Balance is always strongly emphasized from the prospective teachers (see Borromeo Ferri, 2018). In particular the development of an own modelling problem and teaching this at school is for most of the students crucial for understanding the nature of mathematical modelling.

## Summary, Discussion and Outlook

The model of teaching competencies for mathematical modelling is based on a long-termed Design-based research approach. Until now a test instrument for the four teaching dimensions did not exist in order to assess the increase of these competencies in a modelling course by using a pre- and post-test design. In fact it became evident that a test instrument with open items offered a great insight into prospective math teachers thinking concerning their knowledge about mathematical modelling, but it was difficult to evaluate and the test time was too long. The goal was to develop items and scales in order to grasp each of the teaching dimension by reducing the test time.

The presented results are only based on a pilot-study with a small population of students and thus no generalization is possible at this time. However the results allowed an insight into the quality of the items on the one hand and on the other hand statistically significant differences can be stated for three of the four dimension concerning an increase of the teaching competencies on a descriptive level. The pilot study focuses on the test instrument and also on the course as a dependent variable with the hypothesis that prospective teachers, who participate in this seminar, get an increase of their teaching competencies. Hence, a control group is needed. These university students could attend for example a seminar on mathematical problem solving. Then both groups can be compared and a further hypothesis is that mathematical modelling needs more specific teaching competencies than mathematical problem solving, although both topics are very close. Definitely it is clear that the sample size must be enlarged anyway, in order to guarantee the reliability of the test instrument.

Normally the goal of a developed test instrument is that it is universally applicable. The test of modelling competency from Haines and Crouch is such an example. The idea of developing tasks for the sub-competencies of the modelling cycle was great and in the last decades, several researchers adapted these tasks for learners in secondary or high-school. That requires that learners at school or university students were taught in modelling in order to assess them. The same idea is behind the test instrument for the four dimensions of teaching competencies. You can only get what you test, when the contents of the modelling course are in line with the test items, if you want to measure for example an increase of these competencies. This means that the test instrument can, in principle, be used worldwide, but it will certainly work best if the contents of the modelling courses are very close to the one presented here. This is also a big limitation for this test instrument at this stage, but the goal is to use this test instrument together with colleagues from Germany and further countries in order to see if a transfer is possible.

## References

- Borromeo Ferri, R. (2018a). *Learning how to teach mathematical modeling – In school and teacher education*. New York, NY: Springer.
- Borromeo Ferri, R. (2018b). Universitäre Lehrerausbildung zum mathematischen Modellieren – Innovationen durch Design-Based Research. In R. Borromeo Ferri & W. Blum (Eds.), *Lehrerkompetenzen zum Mathematischen Modellieren – Konzepte und Transfer* (pp. 153–170). Wiesbaden, Germany: Springer.



- Borromeo Ferri, R., & Blum, W. (2010). Mathematical modelling in teacher education – Experiences from a modelling seminar. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of CERME 6*, (pp. 2046–2055). European Society for Research in Mathematics Education.
- Bortz, J., Lienert, G., & Boehnke, K. (2010). *Verteilungsfreie Methoden in der Biostatistik* (3rd edition). Berlin, Germany: Springer.
- Burkhardt, H. (2018). Ways to teach modelling—A 50 year study. *ZDM Mathematics Education* 50(1–2), 61–75.
- Kaiser, G., & Schwarz, B. (2010). Authentic modelling problems in mathematics education – Examples and experiences. *Journal für Mathematikdidaktik*, 31(1–2), 51.
- Lesh, R., & Doerr, H. (Eds.) (2003). *Beyond constructivism: A model and modeling perspective on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Maaß, K., & Gurlitt, J. (2011). LEMA – Professional development of teachers in relation to mathematics modelling. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in the teaching and learning of mathematical modelling* (pp. 629–639). New York, NY: Springer.
- Klock, H., Siller, H.-S., & Wess, R. (2018). Adaptive Interventionskompetenz in mathematischen Modellierungsprozessen – Erste Ergebnisse einer Interventionsstudie. In *Beiträge zum Mathematikunterricht 2018*, (pp. 991–994). Münster, Germany: WTM-Verlag.
- Klock, H., Wess, R., Greefrath, G., & Siller, H.-S. (in press). Aspekte professioneller Kompetenz zum Lehren mathematischer Modellierung bei (angehenden) Lehrkräften - Erfassung und Evaluation. In T. Leuders, E. Christophel, M. Hemmer, F. Korneck, & P. Labudde (Eds.), *Fachdidaktische Forschungen zur Lehrerbildung*. Münster, Germany: Waxmann.
- Vorhölter, K. (2018). Notwendige Kompetenzen zur Betreuung von Modellierungsaktivitäten aus Sicht von Studierenden – Evaluation und Weiterentwicklung eines Seminars zur Vermittlung modellierungsspezifischer Lehrerkompetenzen. In R. Borromeo Ferri & W. Blum (Eds.) (2018), *Lehrerkompetenzen zum Mathematischen Modellieren – Konzepte und Transfer* (pp. 203–215). Wiesbaden, Germany: Springer.