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► **To cite this version:**

Berta Barquero, Marianna Bosch, Floriane Wozniak. Modelling praxeologies in teacher education: the cake box. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02408705

HAL Id: hal-02408705

<https://hal.science/hal-02408705>

Submitted on 13 Dec 2019

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Modelling praxeologies in teacher education: the cake box

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This article presents a teacher education activity with pre-service Primary school teachers who are asked to address modelling problems before describing their activity using new discursive and conceptual tools. The terminology of modelling and the questions-answers maps are proposed to enrich the logos of teachers' modelling praxeologies, that is, to provide future teachers with a discourse—a logos—to explain and analyse the modelling praxis.

Keywords: Modelling, teacher education, mathematical discourse, anthropological theory of the didactic, praxeologies.

Introduction: Teacher difficulties with the modelling discourse

When teaching mathematics is considered as the teaching of already-produced knowledge (concepts, procedures, properties, etc.), teachers dispose of discursive resources to talk about the content at stake: “Here we can use proportionality”, “Be careful with the distributive law”, “The problem leads to a quadratic equation”, etc. All these resources have been elaborated during long processes of didactic transposition (Chevallard, 1985) and can be found in textbooks, syllabi, curriculum guidelines, etc. They belong to what is called the *knowledge to be taught* and appear as crucial tools for teachers and students to manage learning processes, to talk about what is already done, what is still to be done, what mathematical tools can be used, what difficulties might appear, what mistakes have been identified, and so on.

In contrast, in the case of teaching modelling, the content at stake is not only a set of mathematical contents that are to be known and used to solve problems. Modelling is also taught as a process to follow, where all the steps—and not only the final result—are equally important. In this case, teachers and students are not always provided with the appropriate words or discursive elements needed to describe the different steps of the process.

The importance of language in teaching and learning modelling has been pointed out by many researchers (Barbosa, 2006; Borromeo Ferri & Blum, 2010; De Oliveira & Barbosa, 2013; Doerr, 2007). For instance, focusing on the students' performances, Redmond, Brown and Sheehy (2013) propose to use “the principles of collective argumentation to enable students to analyse mathematical contexts, to synthesise strategies to mathematize these tasks, and to communicate solutions and conclusions to others.” (p. 13). This paper addresses the problem of the teachers' lack of discursive tools to teach modelling processes and how it can be addressed through teacher education processes.

Research problem: mute modelling praxeologies

In this section, we analyse the lack of words in teaching modelling using the anthropological theory of the didactic (ATD). In spite of a diversity of representations (Perrenet & Zwaneveld, 2012), there is a widespread consensus about the modelling process, and its decomposition in different steps synthesised in the diverse versions of modelling cycles, where models are built and used to produce knowledge about a given piece of reality considered as a *system*. As Serrano, Bosch, and Gascón (2010, p. 2193) explain: “the productivity of the model, that is, the fact that it produces new knowledge about the system, requires a certain ‘fit’ or ‘adaptation’ to the system. This process is rarely done once and for all. It requires a forth and back movement between the model and the system, in a sort of questions-answers or trial-error dynamics.”

In the ATD, mathematical activities are described in terms of praxeologies. A praxeology is the connection between know-how (*praxis*), made of types of problems and techniques to solve them, and a discourse about the praxis (*logos*), composed of descriptions, justifications and theoretical organisations of the problems and techniques. Any human activity (solving an equation, driving a car, combing one’s hair, etc.) can be assigned to a praxeology or a set of praxeologies, with a more or less developed praxis and logos. We use praxeologies to describe both the activity of modelling and the teaching of modelling. In a teaching situation, the mathematical knowledge at stake is commonly revealed through the teacher’s discourse, which updates, describes, explains, justifies, questions and validates the knowledge aimed for. Wozniak (2012b, pp. 65–66) considers three kinds of mathematical praxeologies in teaching situations: “mute”, “weak” and “strong” praxeologies. A “mute” praxeology is one with its *praxis* component visible only, that is, the types of tasks and techniques used to carry them out without any explicit discourse to describe them. In a “weak” praxeology, the *logos* component is visible through the parts of the technique used, but the discourse is limited to the description of the technique through an incomplete formulation. Finally, a “strong” praxeology links its two components, the *praxis* and the *logos*, dialectically.

In the case of modelling praxeologies, Wozniak (2012a, 2012b) speaks of *mute* praxeologies when the hypotheses about the initial system are not explicitly stated (delimitation of the system), the model used is prebuilt and the work within the model gives results that are not validated against the system. *Weak* modelling praxeologies appear when the hypotheses are formulated but their validity domain is not clearly discussed. In this case, the model is not necessarily prebuilt but will not be validated. To develop them into *strong* praxeologies, all steps of the modelling process should be present throughout a discourse that explains their roles and validates the whole process. Wozniak (2012a) uses the three kinds of praxeologies to analyse five primary school teachers working with a modelling problem. Students are presented the photograph of the boot of a giant’s sculpture with some people around (Figure 1) and are asked to find the approximate giant’s height. The main phenomena observed—and which can be found in other investigations about teaching modelling activities—can be summarised as follows:

- The modelling activity is not identified as knowledge at stake and the modelling process is never questioned or taken as an object of study.

- The delimitation of the system remains implicit and is not discussed; the hypotheses are not explicitly formulated and only the missing data are identified.
- The discussion focuses on the choice of the values of the system variables to apply a pre-established model (proportionality).
- All 5 teachers lead the study towards their own solution, and then reduce the study to the same kind of techniques.
- In all but one case, a single model is used without discussing its legitimacy and validity. Teachers do not use the properties of the system (realism) to evaluate the answers obtained and even directly propose the model to be used.
- The fifth teacher includes discussions about the legitimacy of the measures estimated but does not talk about the domain of validity of the model used.



Figure 1: Picture of the Giant activity

Wozniak (2012a) concludes that the modelling praxeologies used in all these classrooms correspond to *mute* or *weak* praxeologies, which emphasizes the teachers' need of new knowledge and tools to deal with this kind of activities. In the following section, we propose an example of teacher education activity that aimed at transforming the *mute* praxeologies into *voiced* ones, that is, modelling praxeologies with a *logos* that give them voice to “talk about” the modelling praxis.

How to turn mute praxeologies into voiced ones: the cake box

The teacher education activity was designed and implemented by this paper's first author. It took place during eight 2-hour sessions with a group of 40 pre-service teachers in their 3rd year of a double university degree in preschool and primary school education. Although this activity has been tested since the academic year 2012-2013, we here focus on its last implementation in 2017-2018. Its design follows the first three modules of the proposal of *study and research paths for teacher education* (SRP-TE) (Barquero, Bosch, & Romo, 2018, pp. 3–4).

In the first module, pre-service teachers were proposed a general teaching question: “*How can modelling be introduced in Primary school education? Through what kind of modelling activities? With what kind of shared discourse about modelling?*” In this particular implementation, we used the giant height activity as an introduction, to make the students familiar with a modelling activity. The activity was also used to establish some common terms to refer to modelling: system, model, hypotheses, variables of the system, mathematical results, interpretation, model validation, etc. Some of the main results described in Wozniak (2012a) were also explained to the students. Then, in the second module, we started with the implementation of another modelling activity: the *cake box* (an adaptation of the activity described in Chappaz and Michon (2003)). The following three

sessions were devoted to implement the cake box activity with the pre-service teachers assuming the role of *students*. The main goal was to make teachers carry out an unfamiliar mathematical activity that could, to a certain extent, exist in an ordinary classroom. The third module covered three sessions, in which students were asked to analyse the modelling process as it was experienced.

The cake box and the different modelling phases

The modelling activity starts by presenting the case of a baker who needs help for packing her cakes in boxes. She wants to use the same type of boxes she has been using (Figure 2). The following question opens the activity: “How can we build boxes to help the baker packing the variety of cakes she offers? Which relation does exist between the sizes of the initial material (paper or cardboard) and the dimensions of the resulting box?” From this initial question, the activity is structured in three phases depending on what is given as known (the sizes of the paper, box or cake) and what remains unknown.

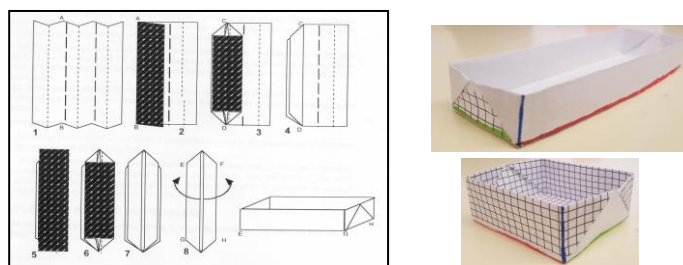


Figure 2: Instructions to build the box and examples of the resulting boxes

First phase: We considered that the paper sizes (width and length) are given and focused on question Q_1 : Which are the dimensions of a box resulting from a paper whose sizes are fixed? Students started by considering some particular cases, such as:

- $Q_{1.1}$: What dimensions of the box resulting from an A4 paper? Is there only one possible result?
- $Q_{1.2}$: If we take an A5 (half A4), do we obtain a box measuring half of the previous one?
- $Q_{1.3}$: Which box sizes do we get from a squared sheet? Do we get a squared-based box?

At this phase, all the students worked using *manipulative* and *measuring techniques*, by concretely building the box and measuring the sizes of the box using different instruments (paper grid, ruler, etc.). This experimental work facilitates to *delimitate a rich system*, which had many possibilities to be extended by considering other paper dimensions and raising new questions about the sizes of the resulting boxes. Students also agreed on which variables had to be considered, for the paper (width and length) and for the box (width, length and height), and how to *agree on a common notation*. Some of the more advanced questions came from the arithmetical comparison of the papers and resulting boxes' sizes: students started formulating the first hypotheses about the likely relationships between the different variables. Most of them referred to relationships of proportionality between the sizes of the paper and the box. Although, in most of the occasions, none of the groups working with these initial *arithmetic-geometric models* could validate their first hypotheses, the possibility to experimentally build new boxes helped them to refuse or refine the hypotheses, but not to validate them.

At the end of this first phase, some of the questions students raised were about a possible relationship of proportionality between the paper and the box sizes, and about how to get a particular sized box. When these types of questions appeared, the modelling activity moved to a second phase.

Second phase: It was assumed the baker gave the box sizes she needed and the question Q_2 was stated: *Which are the initial paper sizes needed to build a box with some specific dimensions?* Students propose to consider new auxiliary questions with particular sizes of boxes, special forms (squared-based boxes) and possible changes in specific dimensions of the box:

$Q_{2.1}$: What initial paper sizes do we need to get a box with base of 6cm x 13cm?

$Q_{2.2}$: How to get a box with a base of 8cm x 10cm and 3cm high?

$Q_{2.3}$: How to get boxes with a square base (such as: 5cm x 5cm, 8cm x 8cm, etc.)?

$Q_{2.4}$: How to modify the sheet of paper to get a box with the same base and different height?

Students started trying to use the same *manipulative-measuring* techniques developed in the first phase, by selecting box sizes close to the ones they looked for. Most of them proposed to apply *models of proportionality* to deduce the sizes of the paper, in many occasions expressed by a rule of three (Figure 3, left-hand side). When they estimated the sizes of the paper, they built the box to check if their deductions were true. When students checked that the “rule of three” did not work (as not all variables have relationships of proportionality), they looked for new models. The most common models that appeared were *pre-algebraic* ones obtained by opening the box, analysing the 2-dimensional geometrical pattern and describing the relationships (Figure 3, right-hand side). Some other groups proposed *algebraic models*, by assigning algebraic values to the different dimensions of the paper-box and looking for the algebraic equations synthesizing these relationships.

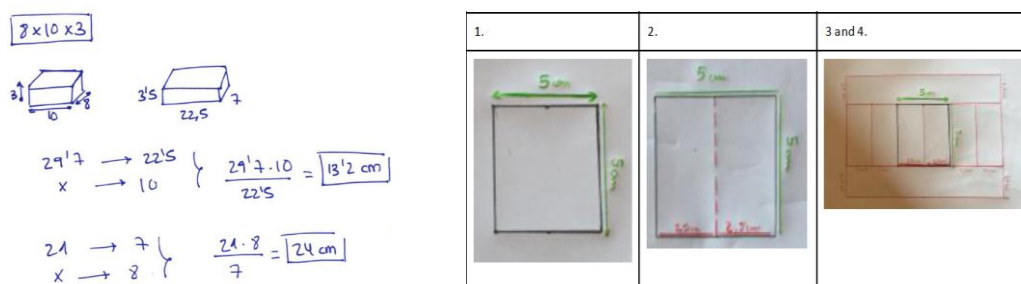


Figure 3: Models of proportionality (left-hand), pre-algebraic-geometric models (right-hand)

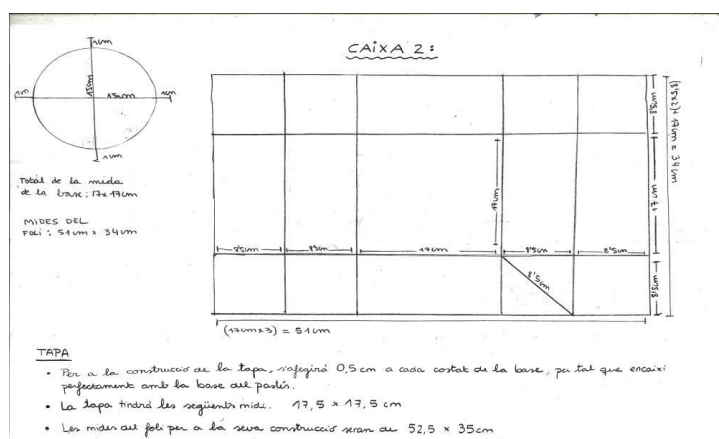
Concerning the modelling process, two traits may be stressed. First, the modelling work developed in Phase 1 constitutes a productive system from which Phase 2 starts. That is, thanks to the experimental work developed, students have many paper-box sizes collected, which constitutes a rich experimental *milieu* from where some first hypotheses about the relationships were explicitly formulated and against which students could evaluate their answers. Second, in this phase, more advanced models were built by students given the failure of the proportionality model. Several questions about how to reformulate the hypotheses (*Do any of the box or paper variables maintain a relationship of proportionality?*), about the comparison of models (*Are the pre-algebraic and*

algebraic models expressing the same?) and about the scope and limitations of the models (*Can we follow working with the pre-algebraic models for all the box cases?*) could be explicitly stated.

This second phase finished when students were able to predict the sizes of the paper/box, without manipulating the paper/box. The following questions were posed by the teacher:

$Q_{\text{final.1}}$: How can we predict the size(s) of the box (without building it) from the dimensions of the initial paper? $Q_{\text{final.2}}$: How to predict the size of an initial paper sheet given the dimensions of the box (without building it)?

Third phase: The aim of this phase was to prepare a letter-report as an answer to the baker's demand. A list of the cakes the baker wanted to pack, and their sizes, was provided. Students were asked to prepare the final report for the baker by letting her know about the paper sizes she had to order to pack all her cakes. Moreover, students were asked to add a tray for each box, following the same pattern of construction. At this stage, students had no necessity to build new models, but the main issue was to agree about the way to build the tray (*How many cm do we have to leave between the box and the tray (margins)? Where to add these differences?*). At this stage, they could better formulate the paper-box relationships, and took advantage of using the model (see Figure 4). Furthermore, some time could be devoted to work on how to report the results, given the extent of numerical work and results, with many variables, they had to communicate and organize.



Translation of the students' proposal:

BOX 2:
Base of the box: 17 cm x 17 cm
Size of the paper: 51 cm x 34 cm

TRAY:

- To build the tray, we add 0.5cm to each side of the box base to guarantee that it fits correctly.
- The tray base will be 17,5 cm x 17,5 cm

Figure 4: Pre-algebraic-geometric models for the box and the tray

A progressive construction of the modelling discourse

When experiencing the cake box activity, participants managed to assume the role of students and carried out the proposed modelling activity although, as they stated, they were not used to this kind of long and open activities. After each session, the working teams (3-4 persons) had to deliver a written report about the modelling activity developed. Participating in the classroom debate and writing the reports was not an easy task for them, as they were conscious they needed new terms to describe the process. At this stage, the previous work with the giant activity was very helpful, since modelling terminology was then introduced. Still, students had many difficulties with, for instance, formulating questions, describing and categorising the kind of models they were suggesting, accepting different possibilities of models and answers, reformulating hypotheses and models, etc.

The instructor decided to follow three main strategies during Module 2. First, she fixed the structure of the reports in terms of: questions studied, data and variables selected, mathematical models considered, answers obtained, new questions opened. This helped participants explicitly describe all these elements. Second, during the debates, she asked participants to explain their proposals and to *create names* to designate the models they were using (arithmetic, geometric, pre-algebraic, algebraic, etc.). This illustrated that there was not only one-way to work on this activity and that several valid models could cohabit. Third, in the debates and reports, the instructor paid special attention to the questions the working teams were posing to help them in their formulations and in relating them to each other.

In Module 3, students were asked to provide an analysis of the modelling work previously carried out, using the session reports and class debates. The instructor proposed to do this work using questions-answers maps (Barquero et al., 2018). She proposed an initial map with the first questions and answers (Q-A) that appeared in the classroom and asked the students to complete it with the description of their own work. This way of describing the modelling process not only provided students with new terminology. It also appeared as an alternative way to talk about doing mathematics, breaking with the usual and “static” way of describing school activities, more focused on concepts, notions and techniques to the detriment of questions, models and provisional answers. Students could also use the Q-A maps to distinguish between their own modelling trajectories—the path followed—and the more complete one they could describe after the work done in class (see Figure 5), thus evidencing the possibility of finding several possible solutions to the same problem.

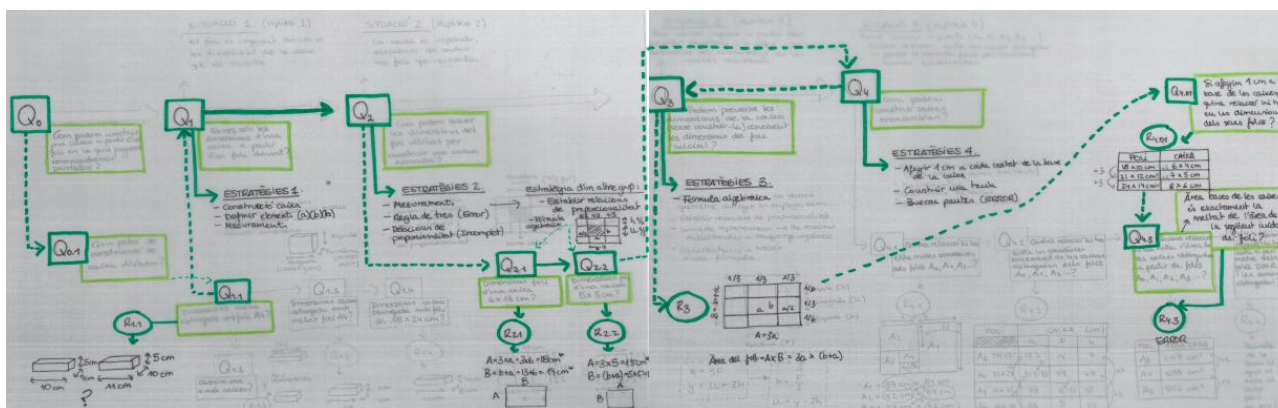


Figure 5: Example of questions-answers map used to analyse a modelling trajectory of a group

Conclusion

We have seen how Wozniak (2012b) describes the problem of the teachers’ lack of discursive tools to talk about modelling processes in terms of *mute* praxeologies, that is, praxeologies where the *logos* component remains implicit. This *logos* is part of the mathematical praxeologies to be taught related to modelling activities. However, it also seems to be a crucial didactic tool to teach modelling—therefore, part of the teacher’s didactic praxeology. It is difficult for teachers to highlight some important modelling procedures when they are not familiar with any specific mathematical terminology to talk about models, systems to be modelled, possible fits (and misfits) between models and systems, etc. To start addressing this problem, we propose to use *study and research paths for teacher education* to elaborate new epistemological tools, such as the questions-

answers maps, with the teachers as part of the modelling praxeologies to later on develop them as didactic tools to better describe and analyse the modelling activities carried out in classrooms. This strategy needs to be developed, but at the present, it seems helpful to fill in the gap between the traditional way of describing—and conceiving—school mathematics and the new epistemological and didactic necessities raised by the teaching of modelling processes.

Acknowledgments

Funded by Ministerio de Ciencia, Innovación y Universidades with the research projects Retos Investigación of the Programa Estatal de I+D+I: EDU2015-69865-C3-1-R; RTI2018-101153-A-C22.

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