



**HAL**  
open science

## Simulations and prototypes in mathematical modelling tasks

Ana Margarida Baioa, Susana Carreira

► **To cite this version:**

Ana Margarida Baioa, Susana Carreira. Simulations and prototypes in mathematical modelling tasks. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02408688

**HAL Id: hal-02408688**

**<https://hal.science/hal-02408688>**

Submitted on 13 Dec 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Simulations and prototypes in mathematical modelling tasks

Ana Margarida Baioa<sup>1,3</sup> and Susana Carreira<sup>2,3</sup>

<sup>1</sup> Group of Schools D. Manuel I – Tavira, Portugal; ambaioa@gmail.com

<sup>2</sup> Faculty of Sciences and Technology, University of Algarve, Portugal; scarrei@ualg.pt

<sup>3</sup> UIDEF, Institute of Education, University of Lisbon, Portugal

*This paper aims to explore and discuss the use of simulations with physical models and prototype construction in a mathematical modelling activity in a context of science, technology, engineering and mathematics (STEM). This study aims to understand the role of simulations and prototypes in student activity and how the use of prototypes and simulations modifies the mathematical modelling cycle. The study is based on a teaching experience in two 9th grade classes. From the analysis we can say that the integration of simulations and prototype construction in an educational STEM context is an important factor for students in assigning meaning to the various phases of the mathematical modelling cycle and in working effectively through those phases.*

*Keywords: Mathematical modelling cycle, prototype, simulation, STEM.*

## Introduction

In the last decade, both in conference papers and in articles published in various journals on science and mathematics teaching, the emphasis has increased regarding the educational plea for the integration and connection between knowledge, methods and concepts of different disciplines in school. This movement, in particular, embraces the perspective of an interdisciplinary teaching of Science, Technology, Engineering, and Mathematics, known today as STEM Education (acronym for Science, Technology, Engineering, and Mathematics).

The drive towards interdisciplinary teaching has been a constant trend both in reports from the PISA studies (OECD, 2016) and in official curricular guidelines from different countries, endorsing more relevant, less fragmented, and more stimulating learning opportunities for students (Furner & Kumar, 2007). STEM education represents an effort to combine science, technology, engineering and mathematics in a single classroom or lab or in more open and interconnected multipurpose school facilities, based on the relationships that can be generated between contents and real-life problems, in which the four areas of knowledge may be relevant, although they do not have to be present at the same time (Moore, 2008; Zawojewski, 2010; Michelsen, 2006).

Many examples of studies within a STEM perspective involve experimental work using simulations and physical materials where the engineering design process is put into practice in the learning activities and projects (Stohlmann, Moore, & Roehring, 2012). In some cases, the activities combine engineering and mathematics and the mathematical modelling cycle is used as a reference for the organization and orientation of the work to be developed with the students (Mousoulides & English, 2011; Gallegos, 2018). The aims may include learning and exploring new aspects involved in a real system, or designing new products and processes.

In this article, we consider the combination of engineering, biology, and mathematics, in a modelling task where the simulation of the situation and the mathematical modelling of real-world

systems are involved. The three research questions underpinning a larger research study, which is the basis of this partial report, are the following:

- (1) What is the role of simulation in the students' mathematical modelling activity when solving problems taken from a STEM context?
- (2) What is the role of a prototype in a mathematical modelling activity within a STEM context?
- (3) How does the use of prototypes and simulations impact the mathematical modelling cycle phases?

## **Theoretical Framework**

The theoretical underpinnings of the study are here generically presented, under the intention of making salient a pedagogical perspective on the use of simulations with physical models, aligned with an engineering and mathematical modelling approach for teaching and learning mathematical concepts and ideas that are interconnected with other areas of knowledge, like hand biometry, for example.

### **Engineering Modelling Eliciting Activities (EngMEAs)**

An engineering model eliciting activity (EngMEA) is formulated from a real-world problem, which is usually framed as a response to a customer's request. Its design and implementation is supported by the broader framework of the Models and Modelling perspective (M&M) (Lesh & Doerr, 2003; Diefes-Dux & Imbrie, 2008). The accomplishment of the task usually requires one or more mathematical concepts as well as engineering models and processes, which are not openly specified in the problem (Mousoulides & English, 2011). The fact that there is a client to whom a response or result should be provided seems to be a relevant element in this type of task, since it contributes to make the context of the problem more credible for the student, and the solution to be found is not abstractly required, but rather has a presumed recipient (Carreira & Baioa, 2017).

In this perspective, when solving the task, students should bring about their knowledge to formulate a mathematical model that can be used or applied by the client for the solution of a given problem or other similar problems (Diefes-Dux & Imbrie, 2008). An EngMEA type of task may, under adequate settings, promote the ability to develop conceptual tools and to construct, describe and explain complex systems; it also brings forth communication and collaborative work. An EngMEA should be elaborated according to the six principles stated below (Table 1).

<b>Principle</b>	<b>Description</b>
Construction of the model	Requires the construction of an explanation, description or procedure leading to a meaningful mathematical model development according the user's needs.
Generalization	The model must be useful to the user, sharable, re-usable and modifiable.
Documentation of the model	Students should document their activity and their thinking explicitly in the form of a report in some communicational setup.
Reality	The context and form of the task must be credible in order to promote a meaningful interpretation by the students, according to their levels of

	mathematical knowledge and general knowledge.
Self-assessment	The task should contain criteria that students can consider for testing and reviewing the model found.
Effective Prototype	In addition to the mathematical model, a physical/metaphorical prototype may be produced as a real outcome and as a response to a customer.

**Table 1: Principles for the development of an engineering model-eliciting activity (adapted from Diefes-Dux & Imbrie, 2008)**

In the present study, a simulation of the situation, by means of physical models, is expected to be carried out as a support for the production of the effective prototype.

### **Relation between simulation and prototype**

Pollak (2003) argues that a major obstacle related to the integration of mathematical modelling in the classroom is the need for students to understand how to connect mathematics with the rest of the world. One difficulty has to do with the lack of a clear formulation of a conceptual model (which implies an understanding of the problem, of the relevant conditions, and forms of mathematization). A way to overcome this obstacle is giving students the opportunity to engage in performing real simulations, since a simulation is an imitation of the real situation, is closely linked to experimenting, predicting, testing, solving problems, imagining alternative solutions, making the situation real, visible and tangible through representations conveyed by physical models.

In production processes, the simulation with physical models allows observing the system, predicting, supporting decisions, modelling, verifying product quality and performance prior to marketing, and refining production processes (Landriscina, 2013); such physical models offer means that produce behaviours, reproduce some aspects of a system, create a variety of reactions that depend on actions and assist on testing hypotheses in a research problem.

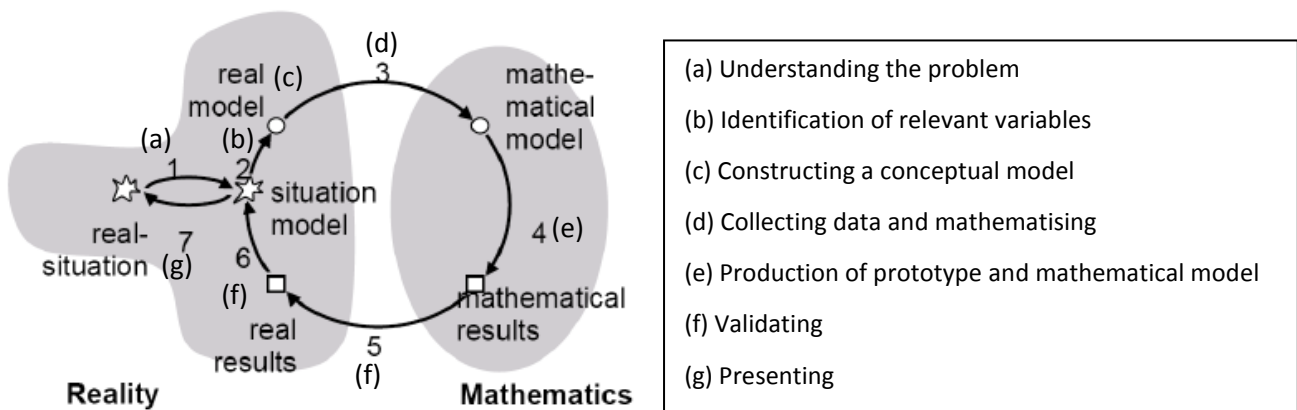
One of the fundamental aspects of a simulation lies on its close relationship with a model, in the sense that it establishes a representation of the real world, whether the model is an image or scheme, an object built to scale, a formula that can be used during a simulation, or even the product of the simulation (Gilbert & Boulter, 2000). The use of a physical model supports the exploration and manipulation of the situation until the creation of a final model (physical or conceptual) that can be called a prototype. The concept of prototype is the basis of the engineering design process where simulations and mathematical modelling are abundantly present, being the prototype a solution to the original problem.

### **Mathematical modelling and engineering design process**

According to Birta and Arbez (2007), the modelling and simulation process in an engineering context, where the objective is the creation of a prototype, consists of six essential steps: (1) The description of problem/project; (2) The construction of a conceptual model of the system; (3) The simulation with physical models; (4) The creation of a prototype and associated mathematical model; (5) The prototype validation in terms of credibility and stability; and (6) Obtaining an end product. These steps are repeated until a satisfactory result is obtained.

Something very similar is typically found in the description of the mathematical modelling cycle (Blum & Leiß, 2007), where the main steps are described as: (1) Understanding the problem; (2) Simplification of the original situation; (3) Mathematizing; (4) Working in the mathematical domain; (5) Interpretation of the result obtained; (6) Validation; and (7) Presentation of results.

The common points between these two forms of conceptualizing the processes that involve reality and the search for a solution to a given problem are evident. Thus, it seems reasonable to admit that both descriptions can be conformed to the other as suggested in the following sequence (Figure 1): (a) Understanding the problem; (b) Identification of the relevant variables in the situation to be modelled; (c) Development of a conceptual model; (d) Data collection through simulation and forms of mathematization; (e) Production of a prototype and associated mathematical model; (f) Validation of model and prototype through stability and credibility; and (g) Presentation of final product and final mathematical model. This is a cyclical process that can be fully or partially repeated, depending on the validation of the prototype or the mathematical model.



**Figure 1: The mathematical modelling cycle coupled with prototype development**

### Methodological approach

In our study we used physical models in tasks that involved problems driven by the scenario of a request from a client. All the tasks involved simulation and practical work. According to studies that share similar elements, design-based research has been adopted (Cobb, 2000; Kelly et al., 2008). Two classes (46 students aged between 13 and 15) participated in this study. Students were organized in the class into 4 or 5 groups and received identical sets of physical materials and instruments for the development of simulations. The empirical data here addressed were collected from the third out of totally five modelling tasks implemented during the school year, through video and audio recording of group work, as well as detail photographs.

Each modelling task was designed according to a structure that involves four parts: (1) Introduction of the task (when the situation and the various elements are presented and clarified); (2) Practical work with physical models, involving data collection; (3) Representation, analysis of collected data and construction of a prototype; and (4) Preparation of a written report and sharing and discussion of the products obtained.

The task refers to the creation of a biometric database and to the development of a recognition system through biometry/hand geometry. The students had at their disposal a set of paper images of their own hands (obtained through photocopies) and the purpose was to create a recognition system

that validated the biometric data of the elements of each group and rejected those of a person who did not belong. After a prototype of such a recognition system was created by each of the groups, a new image of an unidentified hand was given for testing and validation of the respective prototype.


The categories created for the analysis of the data from the implementation of the biometric recognition task refer to the general purpose of perceiving how the simulation and the construction of a prototype play a role in the mathematical modelling activity of the students and, in particular, how they integrate it in the mathematical modelling cycle. Thus, the categories are defined in terms of the phases of this cycle, as described theoretically (Table 2).

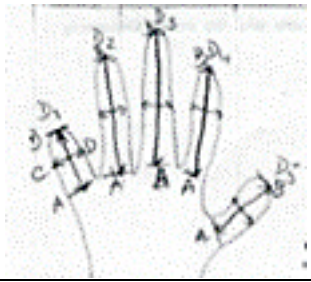
Categories	Prototype development	Modelling cycle phases
<b>understanding</b>	→	(a) Understanding the problem and the goal to be achieved
<b>exploration</b>	→	(b) Identification of the relevant variables in the situation
<b>representation</b>	→	(c) Development of a conceptual model
<b>experimentation</b>	Simulation →	(d) Collection of data and mathematization of the situation
<b>construction</b>	Prototype construction →	(e) Development of a prototype and associated mathematical model
<b>validation</b>	Prototype testing →	(f) Model and prototype validation
<b>communication</b>	→	(g) End product and mathematical model

**Table 2: Categories of data analysis**

### Summary of the results

The selected data were organized to represent the path of the students' work during the task and according to the categories above (Table 2). This path depicts the main features of the students' ways of thinking throughout the task (Table 3).

<b>Understanding</b>	<ul style="list-style-type: none"> <li>• Watching a video about security systems that use biometric data to recognize individuals.</li> <li>• Reading of the task and clarifying questions.</li> <li>• Understanding the problem and the goal.</li> </ul>
<b>Exploration</b>	<ul style="list-style-type: none"> <li>• Naming the biometric database of the group and defining a code for each group element.</li> <li>• Deciding the measures to collect. (Lengths of the fingers, widths of the phalanges; palm area were different choices suggested).</li> </ul> 
<b>Representation and experimentation</b>	<ul style="list-style-type: none"> <li>• Collecting information and data from the image of each hand.</li> <li>• Establishing the main variables to use in the mathematical model.</li> <li>• Recording the numerical data in tables.</li> </ul>
<b>Construction</b>	<ul style="list-style-type: none"> <li>• Defining a rule for the process of comparing and checking a given hand with the database elements (conceptual prototype). An example:</li> </ul>

	Finger 1 (Little finger)		Finger 2 (Ring finger)		Finger 3 (Middle finger)		Finger 4 (Index finger)		Finger 5 (Thumb)	
	width	length	width	length	width	length	width	length	width	length
	$\overline{C_1D_1}$	$\overline{A_1B_1}$	$\overline{C_2D_2}$	$\overline{A_2B_2}$	$\overline{C_3D_3}$	$\overline{A_3B_3}$	$\overline{C_4D_4}$	$\overline{A_4B_4}$	$\overline{C_5D_5}$	$\overline{A_5B_5}$
	<ul style="list-style-type: none"> <li>• Assuming an approximate match.</li> <li>• Establishing an acceptable “margin of error” of 0,2 cm to be taken in the comparison between some given data and the stored data.</li> </ul>									
<b>Validation</b>	<ul style="list-style-type: none"> <li>• Delivering one unidentified hand image for each group.</li> <li>• Obtaining the necessary data from the unidentified hand according to the biometric variables of the identification model.</li> <li>• Checking whether the “scanned” or “standardized” hand belonged to the group or not, according to the recognition model.</li> <li>• Validating the model and the biometric recognition system developed.</li> </ul>									
<b>Communication</b>	<ul style="list-style-type: none"> <li>• Preparing and delivering a report describing the steps taken and presenting the data collected, the mathematical model considered for the recognition, and its validation.</li> </ul>									

**Table 3: A synthesis of the students’ main processes and actions on the task**

## Discussion and conclusions

Much of the students’ thinking and struggling was around the idea of reliability of the process for comparing experimental data with unknown data (of an unidentified hand). The fact that students were simulating a process of capturing biometric data, using full size photos of hands, and realizing the many details and variations of that part of the human body, led them to reflect at length on how to integrate this knowledge in a recognition system performed by a machine, which aims to accept or reject biometric data. Many of the students were quite cautious about creating a model that did not generate false positives, revealing the understanding that the control to be performed by the model would have to be “strict enough” not to allow “a false hand to be accepted”. This resembles what in reality the biometric security systems are concerned with. In fact, a general result from the activity of the students in creating a biometric recognition conceptual prototype was the strong interconnection between ideas and concepts from mathematics, human body biology, and engineering. The idea of creating a prototype engages students on a real attempt to find the most credible model for a real problem in STEM context. The students’ mathematical model for the biometric recognition system was based on the idea of tolerable margin of error. For example, one of the groups assumed that acceptance of an individual would be confirmed in the case where the deviation found for every variable was less than 0.2 cm. Otherwise the system would reject the individual as unrecognized. They therefore based their model on a logical rule of the type if-then-else. The value of 0.2 cm was chosen through the scaling of the hand. The students decided to spread their hands against the table to get an indication of a tolerable dilation of the hand.

The work with physical models in a simulation, such as the use of the actual images of the palms of the students, was a facilitating element for the need to collect real data and for the development of a conceptual model of the situation. The identification of a set of variables to be used was, from the beginning of the activity, an important source of exploration of the situation in the various groups. Different groups considered different ways of defining the measures they assumed as hand identifiers. Thus, the simulation based on physical models has one clear effect over the emergence of different conceptualizations and ways of approaching the real problem; the task allowed students to create different identification criteria for the biometric database.

The ideas of variability, and error or deviation were the focal points of the mathematical approach to the construction of a recognition prototype. This prototype, in turn, elicited a recognition model, which was founded on mathematical ideas and concepts.

Being in the position of obtaining a prototype, that is, of imagining a data capture system and a model aimed to accept or reject new data by comparison, helped students to understand that their model would have to be reliable, that is, it would have to be tested with a new simulation. This gave added meaning to the process of acquiring new data from an unidentified hand, consistent with the system of variables previously defined for data capture.

The mathematical modelling cycle phases are not modified by using simulation and prototype. The students go through all the phases realizing micro modelling cycles.

In general, we conclude that the use of physical models and simulations in experimental tasks, involving the construction of a prototype, promotes the activity of mathematical modelling under conditions that reveal an evident parallelism with real settings where solutions for real problems are searched. Students are faced with genuine obstacles that do not always arise in other types of problems and use their own everyday knowledge and concepts to find ways of establishing the relationship between mathematics and reality. Thus, the integration of simulations and prototype construction, in a STEM educational context, is an important factor to make sense of and also to perform the various steps of the mathematical modelling cycle, so that they become part of a coherent entire process and not just conventional steps to accomplish.

## References

- Birta, L. G., & Arbez, G. (2007). *Modelling and simulation – Exploring dynamic system behaviour*. London, United Kingdom: Springer.
- Blum, W., & Leiß, D. (2007). How do teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modeling – Education, engineering and economics* (pp. 222–231). Burlington, United States: Elsevier Science.
- Carreira, S., & Baioa, A. M. (2017). Mathematical modelling with hands-on experimental tasks: on the student's sense of credibility. *ZDM – Mathematics Education*, 50(1–2), 201–215.
- Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307–330). Mahwah, NJ: Lawrence Erlbaum.
- Diefes-Dux, H. A., & Imbrie, P. K. (2008). Model-eliciting activities for engineering education. In J. S. Zawojewski, H. A. Diefes-Dux, & K. J. Bowman (Eds.), *Models and modeling in*



- engineering education: Designing experiences for all students* (pp. 17–35). Rotterdam, The Netherlands: Sense Publishers.
- Furner, J., & Kumar, D. (2007). The mathematics and science integration argument: a stand for teacher education. *Eurasia Journal of Mathematics, Science & Technology*, 3(3), 185–189.
- Gallegos, R. (2018). Building bridges between the math education and the engineering education communities: a dialogue through modelling and simulation. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited lectures from the 13<sup>th</sup> International Congress on Mathematical Education, ICME 13 Monographs* (pp. 501–519). Cham, Switzerland: Springer International Publishing.
- Gilbert, J., & Boulter, C. (2000). *Developing models in science education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Kelly, A., Baek, J., Lesh, R., & Bannan-Ritland, B. (2008). Enabling innovations in education and systematizing their impact. In A. Kelly, R. Lesh, & J. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 3–18). New York, United States: Routledge.
- Landriscina, F. (2013). *Simulation and learning – A model-centered approach*. New York, United States: Springer.
- Lesh, R., & Doerr, H. (Eds.). (2003). *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, United States: Lawrence Erlbaum.
- Michelsen, C. (2006). Functions: A modelling tool in mathematics and science. *ZDM – Mathematics Education*, 38(3), 269–280.
- Moore, T. (2008). Model-eliciting activities: a case-based approach for getting students interested in material science and engineering. *Journal of Materials Education*, 30(5/6), 295–310.
- Mousoulides, N., & English, L. (2011). Engineering model eliciting activities for elementary school students. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and learning of mathematical modelling, International perspectives on the teaching and learning of mathematical modelling – ICTMA 14* (pp. 221–230). New York, United States: Springer.
- OECD (2016), *Ten questions for mathematics teachers ... and how PISA can help answer them*. Paris: PISA, OECD Publishing. <http://dx.doi.org/10.1787/9789264265387-en>.
- Pollak, H. O. (2003). A history of the teaching of modelling. In G. M. A. Stanic, & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 647–672). Reston, United States: NCTM.
- Stohlmann, M., Moore, T., & Roehring, G. H. (2012). Considerations for teaching integrated STEM education. *Journal of Pre-College Engineering Education Research*, 2(1), 28–34.
- Zawojewski, J. (2010). Problem solving vs modelling. In R. Lesh, P. L. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modelling competencies* (pp. 237–243). New York, United States: Springer.