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An extension of the MAD framework and its possible implication for research

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In this paper we investigate the potential in extending Model Activity Diagrams (MADs) as an analytical tool for studying the activities students engage in when working on modelling problems. By including a sub-division of the task students work on in the MAD framework, we are able to make a more nuanced analysis that in a more explicit way reveals the structure of the students' modelling process. The framework is applied to groups of students engaged in solving a Fermi problem using a two-level analysis, and the result is contrasted and discussed by comparing the outcome of the analysis with the corresponding analysis made using the modelling cycle. In addition, possible applications of the extended MAD framework in research and the training of teachers are discussed.

Keywords: Mathematical modelling, Fermi problems, modelling activities diagrams.

Introduction

In this paper we present a qualitative study focusing on developing a tool to aid researchers in the analysis of mathematical modelling processes students engage in. Existing literature features several theoretical developments that attempt to describe students' modelling processes. Most of these theoretical perspectives are based on the division of the students' work into two domains by separating the real world from the mathematical realm. It is mostly accepted that students go through different stages in a cyclical manner when engaged in modelling as exemplified for example by the so-called modelling cycle of Blum and Leiss (2007). Although this cyclic representation of modelling provides a useful tool and heuristics for teaching and thinking about modelling, there are some concerns and limitations when it comes to its applicability as a research tool.

Firstly, the modelling routes that students are seen to follow when engaged in modelling are complex and typically deviate from the depicted idealized view of the modelling cycle (Borromeo Ferri, 2007). Secondly, some studies (e.g., Ärlebäck, 2009; Czocher, 2016; Aymerich & Albarracín, 2016) highlight the difficulties in identifying the different stages of the modelling process of students' work in a reliable way. In an attempt to provide a complementary perspective on these issues and to partly overcome these difficulties, we propose a construct that extends Modelling Activity Diagrams (MADs) to characterize the activities of students engaged in a mathematical modelling process as an alternative analytical tool to the modelling cycle. To do this, we sought to extend the information provided by the original MAD framework by including a representation of the internal structure of the sub-problems the students engaged in when solving a problem and

applied this extended MAD framework on video-data of students aged 15 to 16 solving a Fermi problem (see Ärlebäck, 2009).

Thus, the aim of this paper is to explore the possibilities of Modelling Activity Diagrams, extended to also explicitly include and represent the structure of the modelling problem in question as well as the complexity of modelling processes. The goal of this paper is to determine whether this extended tool actually provides additional information on the structures underlying the activities the students engage in compared to analyses based on the modelling cycle or the original MADs.

Representations of mathematical modelling: The Modelling Cycle and Modelling Activity Diagrams

In the general discussion, mathematical modelling is often considered to be cyclical in nature, involving going back and forth between the real world and the world of mathematics. Even though there are several ways of conceiving and viewing the structure of a modelling process, it is generally understood that it can be divided into different stages and that the students go through these sequentially, moving from one stage to the next once they consider the work in the current stage has been concluded satisfactorily (Perrenet & Zwaneveld, 2012). The conceptualization of Blum and Leiss (2007) has been widely used to describe the modelling process but has shown to be difficult to use as a tool for analysis when it comes to describe and explain the students' work (Borromeo Ferri, 2007; Ärlebäck, 2009; Czocher, 2016; Aymerich & Albarracín, 2016).

When faced with the need to describe the modelling processes implemented by students, Borromeo Ferri (2007) proposed so-called individual modelling routes with the aim of broadening the modelling cycle's analytical reach. These routes consist of arrow diagrams that show students' work within the theoretical diagram of the modelling cycle, illustrating the non-conformity of students' work relative to the modelling cycle as well as students' seemingly stochastic modelling behaviour. Acknowledging the work by Borromeo Ferri, Ärlebäck (2009) introduced Modelling Activity Diagrams (MADs) as a bi-dimensional graph that depicts the types of modelling activities the students engage in when solving novel modelling problems. The activities proposed by Ärlebäck (2009) and used to characterize the modelling processes in terms of MADs can be found in Table 1.

R: Reading	Reading the statement of the task and understanding it
M: Modelling	Simplifying and structuring the task mathematically
E: Estimating	Making quantitative estimates
C: Calculating	Performing mathematical calculations, such as arithmetic calculations, working with equations, drawing sketches or diagrams
V: Validating	Interpreting, verifying and validating the results, calculations and the model itself related to the real-life context of the problem
W: Writing	Summarizing the findings and results in a report, drafting the solving process as well as the solution

Table 1: Activities that make up MADs

Mathematical modelling and Fermi problems

The use of so-called Fermi problems in our work allows us to propose simple situations that require students to engage in mathematical analysis that promotes the construction of models. Following Ärlebäck (2009), we consider Fermi problems to be open, non-standard problems that require students to make assumptions about the problem situation and estimates of certain quantities before they engage in, often, a series of simple calculations. Efthimiou and Llewellyn (2007) characterised Fermi problems based on their formulation since these always appear to be open questions offering little or no specific information to the solver directing them in the solving process. The key aspect to Fermi problems from the perspective of mathematical modelling is the need to conduct a detailed analysis of the situation presented in the statement of the problem with the objective of decomposing the original problem into simpler and connected problems – addressed as sub-problems in the present work – to reach a solution to the original problem by means of reasonable estimates and reasoning.

Some research studies conducted to date on the use of Fermi problems in the teaching and learning of mathematics have been shown to facilitate the introduction of mathematical modelling in primary and secondary school classrooms as well as at college level. While engaging in solving Fermi problems primary students develop new mathematical knowledge in arriving at their solutions (Peter-Koop, 2009). Social relations and extra-mathematical knowledge in the problem-solving situation are also relevant to the problem-solving process (Ärlebäck, 2009). Czocher (2016), who used Fermi problems to analyse university engineering students' performance in terms of MADs, confirms that the MADs reveal the students' mathematical thinking about variables and constraints related to the problem contexts. Czocher adds an interesting dimension to the interpretation of the MAD when she writes that when a task has become routine for an individual, the modelling route displayed by that individual for solving that particular task resembles the idealized working process of the modelling cycle (e.g., Blum & Leiss, 2007).

Methodology

The aim of our research presented in this paper is to develop an extension of Modelling Activity Diagrams and to investigate the potential and possibilities of this extended framework. For this purpose, we explicitly included a representation of the sub-problems elaborated by the students to achieve their objective within the activity (e.g., solving a Fermi problem). The data analysed in the paper was collected in a secondary school located in the metropolitan area of Barcelona and consists of video-recordings of the problem-solving processes of groups consisting of three regular Grade 4 secondary school students (ages 15 to 16) engaged in solving a Fermi problem. The groups were constructed by choosing students that suggested well elaborated and different solution strategies to previous Fermi problem. The problem the students worked on deals with the estimation of the number of objects needed to fill a large volume presented in a specific context. The problem statement provided to the students was the following:

Water is a scarce resource and it is necessary to be aware of the use we make of it. We have organised a debate to address the amount of water used for different purposes, and, in order to

provide concrete data, we need to answer the following question: How many bathtubs can we fill with the water of a public swimming pool?

The video-recordings were transcribed in terms of the utterances the students made, with each students' single uninterrupted verbal contribution considered as one utterance. We analysed the data in two levels: first both in terms of the modelling cycle and the MAD framework respectively. In the second level of analysis we re-analysed the data and additionally identified the sub-problems the students engaged in to solve the main problem and incorporated this information graphically in the MAD. Notably, the generally most common model used for this type of problem and this particular age-group of students is the iteration of a unit (Albarracín & Gorgorió, 2014), in which the students determine the volume of a (larger) object and compare it to that of its containing (smaller) base units, decomposing the problem into three sub-problems – the calculation of each volume and a final comparison – as illustrated in the diagram shown in Figure 1.

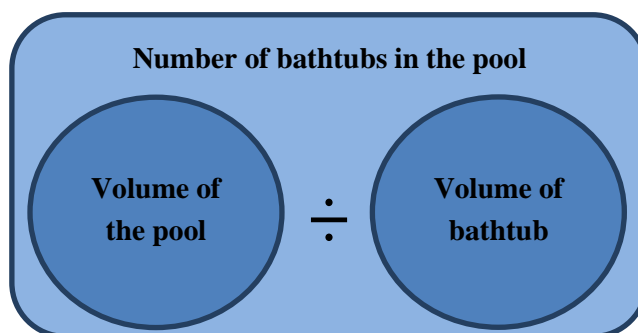


Figure 1: Structure of the Fermi problem used in this study

With this second level of analysis based on identifying the modelling activities manifested through each sub-problem, we intend to describe and investigate the relationship between the MAD and the underlying structure of the modelling processes induced by the sub-problems.

Results

The students in group A worked on the assumption of a swimming pool being 25 metres in length, arriving at an estimated result that 480 bathtubs would fit inside the swimming pool. The participation in the discussion was unevenly distributed, and the problem was mainly modelled and discussed by Ada and Arnau, while Aina drafted the report and participated in the validation process. The total number of utterances for the members of the team were: Ada, 33; Aina, 26; and Arnau, 37. The result of the analysis of group A's performance using the modelling cycle of Blum and Leiss (2007), as was also done in Aymerich and Albarracín (2016) and by focusing on identifying the different stages in the modelling process the students engaged in, can be seen in Figure 2a. This figure shows a complex problem-solving process involving many of the stages in the modelling cycle. One can note however, that none of the students' iterations in terms of modelling routes are closed in the sense of completing (the) stages in the modelling cycle in a clockwise manner, and that the students seem to be struggling in the initial phase of solving the problem. Using the MAD framework Figure 2b illustrates the switching between the different activities the students engaged in, the complexity of how the groups' modelling process unfolded,

and hence conveys, analogous to the representation in Figure 2a, a picture of a somewhat disorganised process.

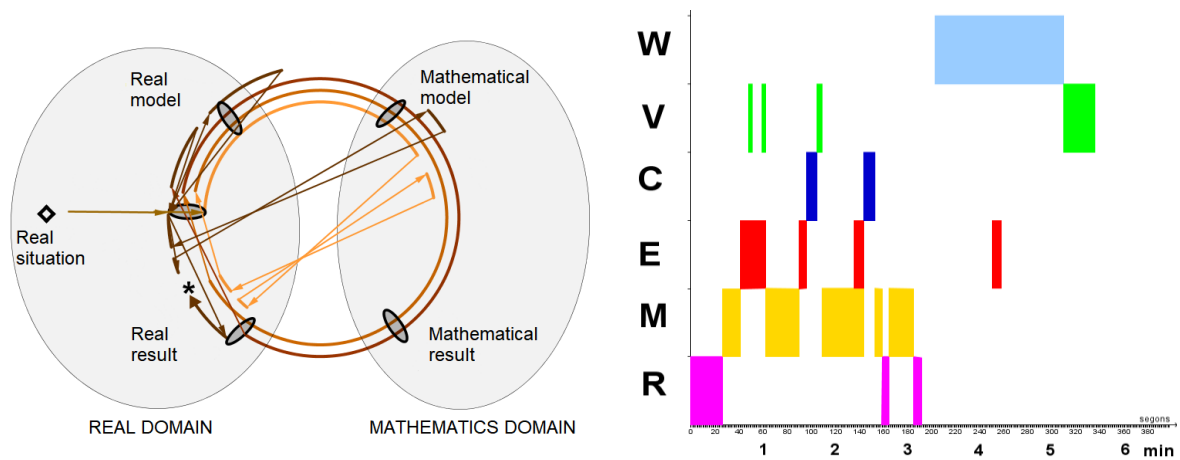


Figure 2: a. Analysis of group A's modelling route in terms of the modelling cycle (left); b. Analysis of group A's modelling route in terms of the MAD framework (right)

If we turn to the analysis including the division into the sub-problems that the students engaged in and how the coded activities are distributed in relation to these, a more nuanced and clear structure is revealed. The MADs show the activities the students engaged in along a timeline, and departing from this, the extension of the MAD is generated by introducing a graduation of intensity of colour to differentiate each of the sub-problems in which the students work. In the lower part of the graph, the specific sub-problems are labelled. In this way, the extended MAD provides a representation merging both the identified modelling activities and the sub-problems the students addressed.

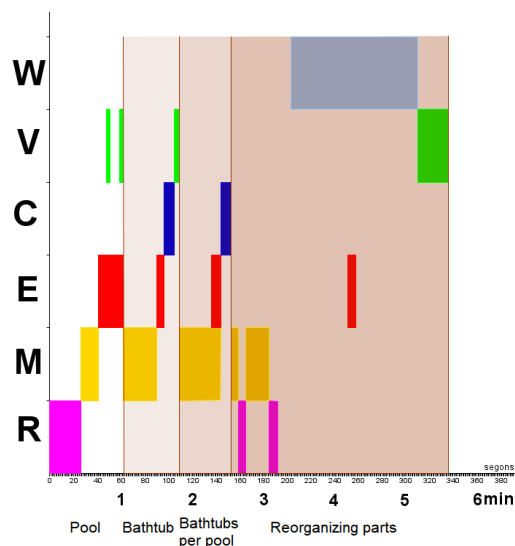


Figure 3: A MAD with the sub-problems group A engaged in during their problem-solving process

Figure 3 shows the activities the students engaged in to solve each of the sub-problem they tackled. The students started working on deciding the size of their swimming pool of choice and estimating its volume (sub-problem one), and then worked on estimating the volume of a bathtub (sub-problem two). The third sub-problem consisted of calculating the number of bathtubs that would fit in the

swimming pool, before finally re-organising the information obtained in order to present a solution and write the report.

Coming back to the point made by Czocher (2016), that students' modelling processes of routine problems tend to better line up with the stages in the modelling cycle, we conclude that this is by no means obvious in terms of students' modelling routes as depicted in Figure 2.a or the activities in the MAD in Figure 2.b. Rather, the impression conveyed by these representations is that the problem solving process the students engaged in is ill-structured and complex. However, the resulting diagram from the extended MAD analysis (Figure 3) provides a more accurate and nuanced representation of the students' engagement within the problem. The MAD in Figure 3 clearly reveals the structure of the activities the student engaged in when solving the problem and shows that the model needed to solve the problem was accessible to them and that they controlled all the details needed to reach a solution. To contrast this straight forward problem-solving process, we illustrate the activities engaged in by a different group of students in solving the same problem. Figure 4 shows that the extended MAD for a second group B and reveals some difficulties in their solution process. These students also calculated the volume of a bathtub and a pool, but they initially used different units in the respective calculations. Hence, which can be seen between 5:39 and 7:41 in Figure 4, they had to decide how to go about to combine their calculated values in order to obtain a result.

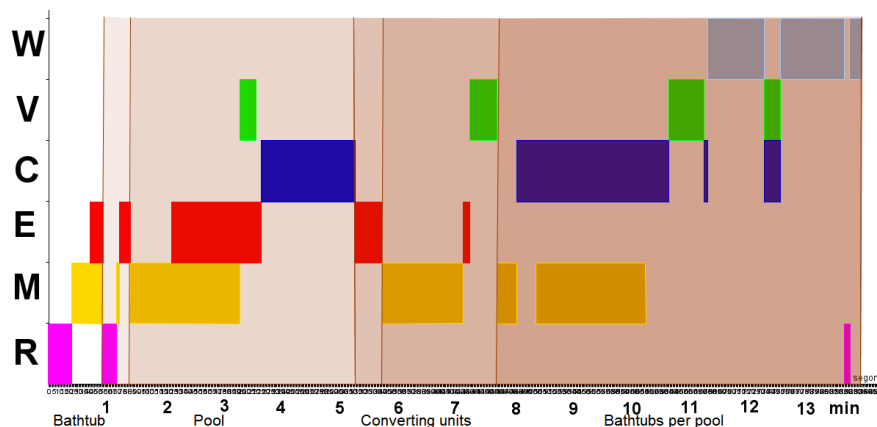


Figure 4: MAD with separation into sub-problems for group B's modelling process

The comparison between the different approaches to analyse the data leads us to conclude that the extended MAD do facilitate an analytical narrative that is closer to the actual activities the students engage in. It does this in the sense that the extended MAD provides more localised information that allows for a clearer and more detailed interpretation of the students' actions and decisions.

Discussion and conclusions

From a methodological point of view, we reaffirm the idea that characterizing the different activities students engage in when solving a modelling problem is straighter forward and clearer in terms of MAD activities compared to that in terms of stages of the modelling cycle. However, we must emphasize that analysing the tasks carried out by a group is a complex task since we can only rely on the observable elements the collected data provide. Sometimes the students in a group split their focuses and interests when working towards the solution, resulting in the students not working

together. Due to their inherent structure, the MAD diagrams make it possible to show this fact explicitly by mapping out different bars for the different activities for the time period in question. In this way, we consider MAD diagrams to be both a richer and more robust tool from a methodological point of view when it comes to make and visualize the connection between the activities the students actually engage in during the problem-solving process and the codes aiming to capture these activities explicitly. From the point of view of the development of an analysis tools, Fermi problems appear suitable as a type of modelling activity for validating the utility of extended MAD. To investigate limitations, the framework needs to be applied to more complex situations where students are engaged in other types of activities (statistical data collection, measurement, prototyping, ...) as well as to the different parts of larger and more complex modelling problems and projects.

Another aspect to note is that the results show that even in the case of students engaged in solving a Fermi problem that is accessible to them, the modelling process involved in arriving to a solution is complex. This complexity is manifested in the large number of the instances that students engage in different activities needed to characterize the problem-solving process, but also in the significant differences between the problem-solving processes of the different groups. This study allows us to affirm that although this complexity exists, the extended MAD framework facilitate us to reveal the internal structure of the proposed Fermi problem, and thus more clearly showing the work pattern of the students. Group A's initial MAD (Figure 2.a) shows a complex problem-solving process that is, however, clearly possible to divide into the solving of several sub-problems as displayed using the extended MAD (Figure 3). For this group the problem given did not pose any great difficulties, neither from a decision-making point of view regarding the modelling process, nor in the workings of the mathematical tools used. This results in an extended MAD of the students' problem-solving process that in a trustworthy way reflects the activities the students engaged in. In part, the modelling behaviour displayed by the students in this group is concurrent with the findings of Czocher (2016), that when students experience the modelling problem at hand as routine, the problem-solving process is more or less straightforward.

This latter fact suggests the possibility of using the extended MAD as a way of representing the activities students engage in during modelling in teacher training programmes. In this context, the extended MAD can be used as the point of departure for discussions on students' performance in different stages of the modelling process and to differentiate between those instances where students have difficulties compared to those in which students make progress and are constructing (sub-) solutions easily. In this sense, and in line with the benefits reported from the use of modelling cycles in teacher preparation courses (Sevinc & Lesh, 2018), we understand that extended MADs can act as an access point or as an element to foster other types of discussions, allowing students' procedures to be connected to the content used. An example of this extra explanatory information the extended MAD makes it easier to identify, is the activities the students in the second group engage in when they detected the inconsistencies in their calculated values due to the units chosen when considering the volumes of the pool and bathtub respectively. The lack of specificity in the formulation of the given problem is part of the nature of Fermi problems and promotes discussion between students, as well as connecting mathematical and extra-mathematical knowledge. The

analytical approach developed in this study shows that extended MAD diagrams can be a helpful tool to visualize some of these hidden elements, and that previously seemingly messy and haphazard problem-solving processes can be understood as highly structured. In fact, we are hopeful that extended MAD diagrams can be further developed and readily applied to study also mathematical modelling processes beyond mere Fermi problems. Given that the MADs were designed to specifically study these types of problems, one of the activities included in the original analysis is estimation, but we suggest that this activity is broadened, or complemented with other types of activities, so that activities such as making measurement, statistical data collection, experimentation or simulation are included in the next version of the extended MAD framework.

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