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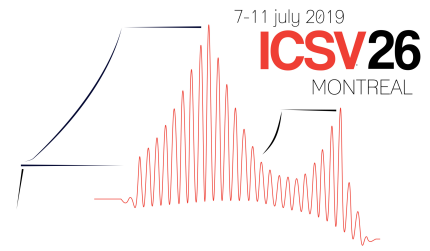
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STATISTICAL ENERGY ANALYSIS AND DIFFUSE FIELD

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Statistical energy analysis (SEA) is a statistical theory of sound and vibration based on an analogy with thermodynamics. The main relationship of statistical energy analysis, the so-called coupling power proportionality, is intimately linked with the establishment of a diffuse vibration field in subsystems. In this study, we explore the conditions under which a diffuse field is enforced. We show that when the subsystem is excited by a point force, a low damping, a high frequency but also a ergodic billiard geometry are required conditions. Then, the energy exchange between two weakly coupled subsystems is proportional to the difference of vibrational energies. But when the field is not diffuse, the exchange of energy does not generally follow this proportionality. Numerical simulations are provided to support the discussion.

Keywords: sound, vibration, statistical energy analysis, diffuse field, ergodic billiard

1. Introduction

Statistical energy analysis [1, 2, 3, 4, 5, 6] is a statistical theory of sound and vibration in the same way it exists a statistical mechanics, statistical physics, statistical electromagnetism. The basic concept is that of diffuse field. A vibrational field, either structural or acoustical, is diffuse if it is random, homogeneous and isotropic [7]. Nowadays, statistical energy analysis represents the most popular method in high frequency vibroacoustics although extensions and/or alternative methods have been developed [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

Identifying the exact assumptions of statistical energy analysis is of primary importance for both, theoretical and practical points of view. The success in applications is conditioned by the knowledge of

this list. But if the main assumptions are well identified, it seems that the exact list of assumptions may slightly differ for the authors [20, 21, 22, 23, 24].

In this study, we explore the status of the assumption of ergodicity of subsystems in statistical energy analysis [25]. For this purpose, we shall systematically compare the prediction of statistical energy analysis with a reference calculation obtained by a direct numerical simulation. For each test case, we shall verify whether or not the exchanged power is proportional to the difference of modal energies, as claimed in statistical energy analysis.

2. Statistical energy analysis

To test whether or not, the geometry of subsystems must be ergodic to ensure the validity of statistical energy analysis, we consider a simple system composed of two coupled subsystems. The subsystems are thin plates in flexural vibrations coupled by a spring. The plates are excited by a stationary random moment whose spectrum is flat and confined within the bandwidth $\Delta\omega$.

The modal density of plate i defined as the number of modes per unit circular frequency ω (rad/s) is

$$n_i = \frac{A_i}{4\pi} \sqrt{\frac{m_i}{D_i}} \quad (1)$$

where m_i is the mass per unit area, D_i the bending stiffness, and A_i the plate area. The number of resonant modes in the frequency bandwidth $\Delta\omega$ is $N_i = n_i \Delta\omega$.

We denote by E_i the expectation of vibrational energy in subsystem i . Expectation must be understood in the meaning of probability theory since the excitation is random. This is the mean value for several realizations of the random process associated to excitation moment. By virtue of stationarity, E_i does not depend on time. We also denote by P_{ij} the expectation of vibrational power exchanged between subsystems i and j . Then, the main result of statistical energy analysis states the power being exchanged between subsystems i and j is proportional to the difference of modal energies

$$P_{ij} = \beta_{ij} \left(\frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \quad (2)$$

This is the coupling power proportionality. The conductivity factor β_{ij} verifies reciprocity $\beta_{ij} = \beta_{ji}$. For two plates coupled by a spring, its value is given by [22]

$$\beta_{ij} = \frac{K^2}{32\pi\omega^2 \sqrt{m_i D_i m_j D_j}} \quad (3)$$

where K is the stiffness of the coupling spring.

3. Ergodic billiard

We selected two different geometries for plates whose properties of ray dynamics are well-known. The stadium geometry is a typical example of ergodic billiard [26, 27]. This imposes that any ray explores during its travel the entire phase space *i.e.* passes through the vicinity of all points and all directions. An example of ray path is given in Fig. 1, left. It can be seen that a single ray spread its energy uniformly and isotropically over the subsystem. This is exactly what is required to get a diffuse field. This illustrates the fact that an ergodic billiard ensures a diffuse field state even if the excitation is a single point and has a high directivity.

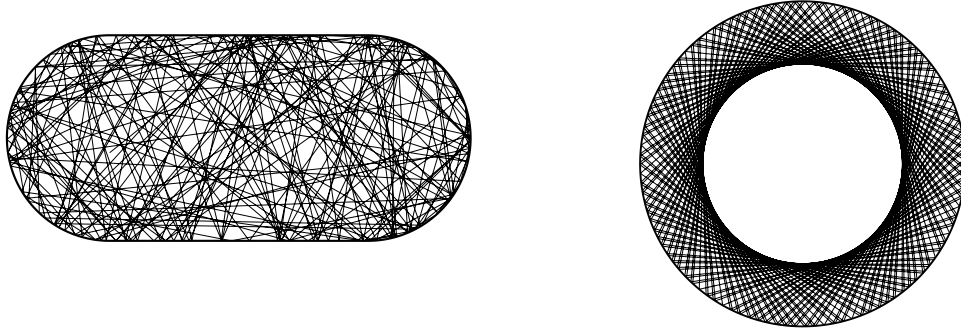


Figure 1: Ray dynamics in various billiards. Left, ray tracing in a stadium. Right, ray tracing in a circle.

The second example is a circular geometry. The conservation of incident angle on boundary is a feature particular to circle. An example of ray path is shown in Fig. 1, right. It can be seen that the ray turns inside the circle but never explores the central zone of the circle. This explains why a circle is not an ergodic billiard and therefore never gives a diffuse field.

4. Parameters of simulation

The parameters of the simulations are the following. The plates are all made of steel with Young's modulus $E_0 = 210$ GPa, density $\rho = 7800$ kg.m⁻³ and Poisson's ratio $\nu = 0.3$. The plates have a damping loss factor $\eta = 0.001$. Plate 1 has thickness 2 mm and mass per unit area $m_1 = 15.6$ kg/m² while plate 2 has thickness 2.5 mm and mass per unit area $m_2 = 19.5$ kg/m². Plate 1 is excited by a moment, or torque, simulated by two closed forces. The forces are out-of-phase but random. The frequency band of excitation is an octave centred on $\omega = 2\pi \times 4000$ rad.s⁻¹.

The size of plates is the following. The width of stadium is 0.7 m and the length is 1.6 m so that its area is $A_1 = A_2 = 1$ m². The circle has diameter 1.13 m and has area $A_1 = A_2 = 1$ m². These dimensions have been chosen to ensure a same number of modes for stadium and circle in the octave band 4000 Hz (see Tab. 1). The damping η is light so that the attenuation of wave per mean-free path, $\eta\omega l/c$ where c is the group speed and l the mean-free path, is small. A small attenuation ensures that rays reflect a large number of times before to vanish [28]. The coupling spring stiffness is $K = 1 \times 10^5$ N/m. The connection strength $\kappa = K/(m_1 A_1 m_2 A_2 \omega^4)^{1/2}$ is small so that the coupling is weak [29, 30, 31].

Table 1: Parameters of plates.

Plate	# Modes	Attenuation	Modal overlap	Connection strength
Stadium 1	450	0.036	0.64	10^{-5}
Stadium 2	360	0.032	0.51	-
Circle 1	450	0.040	0.64	10^{-5}
Circle 2	360	0.035	0.51	-

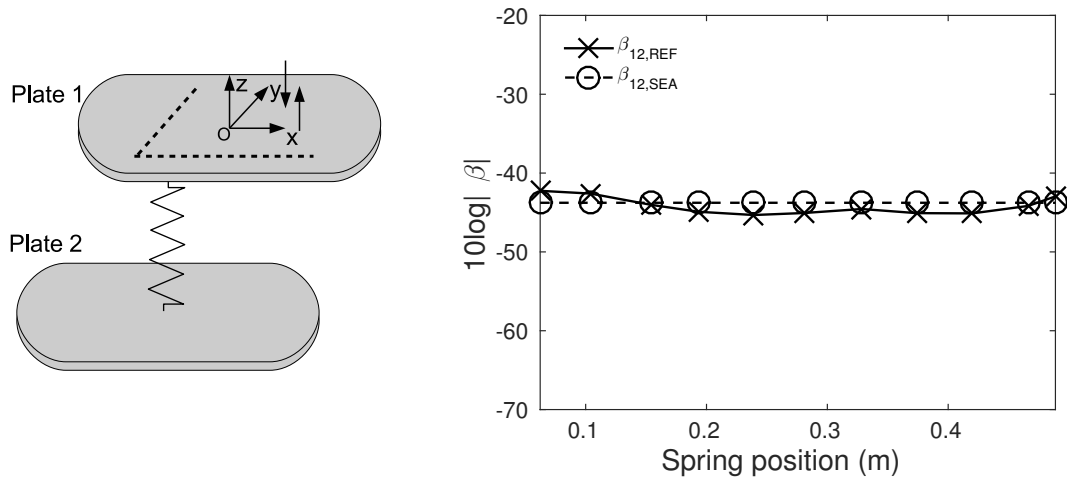


Figure 2: Comparison of energy transfer predicted by statistical energy analysis and reference calculation for two coupled stadium plates excited by a torque on plate 1. Factor β given by Eq. (3) and reference calculation versus position of coupling spring (dashed line in left).

5. Results and comments

In Fig. 2 is shown the ratio β of exchanged power and difference of modal energies calculated respectively from Eq. (3) and by the reference calculation in the case of two coupled stadiums. It is observed that a fine agreement is found between statistical energy analysis and reference for any position of the coupling spring. This agreement is remarkable since the excitation is a unique torque applied. This type of excitation is severe for statistical energy analysis since it is highly localized and highly directive. We are far from the traditional rain-on-the-roof excitation. In this case, homogeneity and isotropy of vibrational field are guaranteed by ergodicity of stadium billiard, while a torque excitation provides neither homogeneity nor isotropy of source.

In Fig. 3 is shown the ratio β in same conditions but for two coupled circular plates. We observe a large disagreement between statistical energy analysis prediction and that of the reference calculation. The reason is that since the excitation is highly localized and directive, and that the subsystems are not ergodic, the vibrational field is never diffuse. The exchanged of energy between the two plates do not follow the thermal law of Eq. (2) since the thermal equilibrium is not reached in subsystems.

6. Conclusion

Diffuse field is an imperative assumption of statistical energy analysis. The requirement of a rain-on-the-roof excitation is a condition which ensures diffuseness of the field. But when the excitation is random but localized and/or highly directive, ergodicity of subsystem is an important condition to get a diffuse field. The counter-example of circular plates show that even if the frequency is high, the attenuation is low, and the coupling is weak, the coupling power proportionality may be violated when subsystems do not have ergodic geometries.

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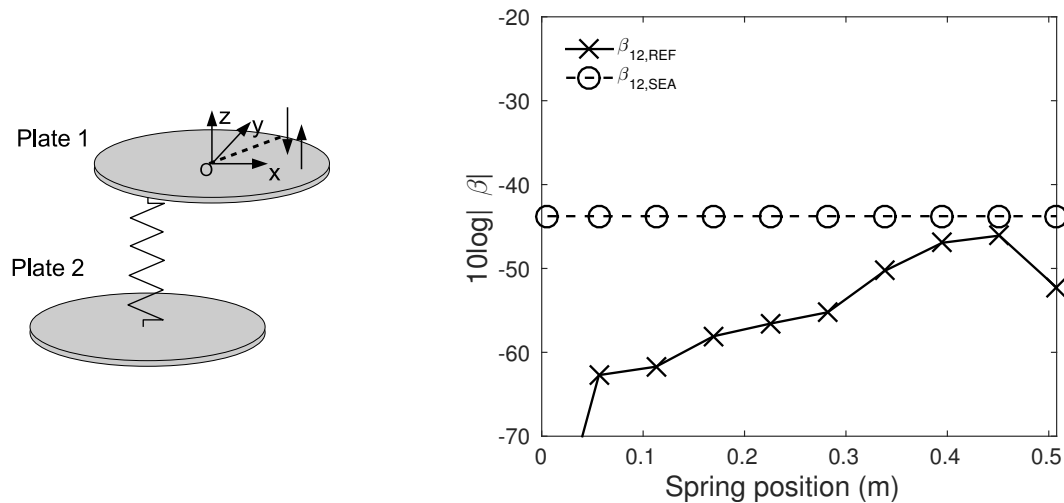


Figure 3: Comparison of energy transfer predicted by statistical energy analysis and reference calculation for two coupled circular plates excited by a torque on plate 1. Factor β given by Eq. (3) and reference calculation versus position of coupling spring (dashed line in left).

REFERENCES

1. A. Le Bot, Foundation of statistical energy analysis in vibroacoustics, Oxford University Press, Oxford, 2015.
2. R.H. Lyon, G. Maidanik, Power flow between linearly coupled oscillators, *J. Acoust. Soc. Am.* **34** (1962) 623-639.
3. P.W. Smith, Response and radiation of structural modes excited by sound. *J. Acoust. Soc. Am.*, **34** (1962) 640-647.
4. D.E. Newland, Calculation of power flow between coupled oscillators, *J. Sound Vib.* **3** (1966) 262-276.
5. D.E. Newland, Power flow between a class of coupled oscillators, *J. Acoust. Soc. Am.* **43** (1968) 553-559.
6. T.D. Scharton, R.H. Lyon, Power flow and energy sharing in random vibration, *J. Acoust. Soc. Am.* **43** (1968) 1332-1343.
7. R.L. Weaver, On diffuse waves in solid media, *J. Acoust. Soc. Am.* **71** (1982) 1608-1609.
8. A. Le Bot, A vibroacoustic model for high frequency analysis, *J. Sound Vib.* **211**(4) (1998) 537-554.
9. A. Le Bot and A. Bocquillet, Comparison of an integral equation on energy and the ray-tracing technique for room acoustics, *J. Acoust. Soc. Am.* **108** (2000) 1732-1740.
10. A. Le Bot, A functional equation for the specular reflection of rays, *J. Acoust. Soc. Am.* **112** (2002) 1276-1287.
11. A. Le Bot, Energy transfer for high frequencies in built-up structures, *J. Sound Vib.* **250**(2) (2002) 247-275.
12. A. Le Bot, Energy exchange in uncorrelated ray fields of vibroacoustics, *J. Acoust. Soc. Am.* **120**(3) (2006) 1194-1208.
13. E. Sadoulet-Reboul, A. Le Bot, J. Perret-Liaudet, A hybrid method for vibroacoustics based on the radiative energy transfer method, *J. Sound Vib.* **303** (2007) 675-690.

14. A. Le Bot, E. Sadoulet-Reboul, High frequency vibroacoustics: A radiative transfer equation and radiosity based approach, *Wave Motion* **51** (2014) 598-605.
15. L. Maxit and J.L. Guyader, Extension of SEA model to subsystems with non-uniform modal energy distribution, *J. Sound Vib.* **265**(2) (2003) 337-358.
16. N. Totaro, J.L. Guyader, Extension of the statistical modal energy distribution analysis for estimating energy density in coupled subsystems, *J. Sound Vib.* **331**(13) (2012) 3114-3129.
17. G. Tanner, Dynamical energy analysis - Determining wave energy distributions in vibro-acoustical structures in the high-frequency regime, *J. Sound Vib.* **320** (2009) 1023-1038.
18. D. Chappell, S. Giani, Tanner G., Dynamical energy analysis for built-up acoustic systems at high frequencies, *J. Acoust. Soc. Am.* **130**(3) (2011) 1420-1429.
19. D. Chappell, Tanner G., S. Giani, Boundary element dynamical energy analysis: A versatile method for solving two or three dimensional wave problems in the high frequency limit, *Journal of Computational Physics* **231**(18) (2012) 6181-6191.
20. F.J. Fahy, Statistical energy analysis: A critical overview. In *Statistical energy analysis an overview with applications in structural dynamics* Edited by A.J. Keane and W.G. Price, Cambridge University Press, Cambridge, 1994.
21. C.B. Burroughs, R.W. Fisher, F.R. Kern, An introduction to statistical energy analysis, *J. Acoust. Soc. Am.*, **101**, (1997) 1779–1789.
22. B.R. Mace, L. Ji, The statistical energy analysis of coupled sets of oscillators, *Proceedings of the royal society A* **463** (2007) 1359-1377.
23. T. Lafont, N. Totaro, A. Le Bot, Review of statistical energy analysis hypotheses in vibroacoustics, *Proc. R. Soc. A* **470** (2013) 20130515.
24. T. Lafont, N. Totaro, A. Le Bot, Coupling strength assumption in statistical energy analysis, *Proc. R. Soc. A* **473** (2017) 20160927.
25. H. Li, N. Totaro, L. Maxit, A. Le Bot, Ergodic billiards in statistical energy analysis, *Wavemotion* **87** (2019) 166-178.
26. G. Tanner, N. Søndergaard, Waves chaos in acoustics and elasticity, *J. Phys. A* **40** (2007) 443-509.
27. J.D. Polack, Playing billiards in the concert hall: the mathematical foundations of geometrical room acoustics, *Applied Acoustics* **38** (1993) 235-244.
28. A. Le Bot, V. Cotoni, Validity diagrams of statistical energy analysis, *J. Sound Vib.* **329** (2010) 221-235.
29. P.W. Smith, Statistical models of coupled dynamical systems and the transition from weak to strong coupling, *J. Acoust. Soc. Am.* **65** (1979) 695-698.
30. B.R. Mace, The statistical energy analysis of two continuous one-dimensional subsystems, *J. Sound Vib.* **166** (1993) 429-461.
31. S. Finnveden, A quantitative criterion validating coupling power proportionality in statistical energy analysis, *J. Sound Vib.*, **330** (2011) 87-109.