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Operation modes and workspace of a 4-rRUU Parallel Manipulator

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Abstract. A lower-mobility parallel manipulator with multiple operation modes can be considered as inherently reconfigurable. This is the case for 4-RUU parallel manipulators that have three different operation modes. Allowing the first revolute joint axis to have any horizontal orientation leads to a dual reconfigurable 4-rRUU mechanism. This paper presents an interpolation approach to determine its operation modes as functions of the orientation of its base revolute joint axes. Furthermore, its limb singularities are characterized by differentiating the constraint equations with respect to the actuated joint variables which are further used to visualize its translational workspace for three mutually perpendicular orientations of its base revolute joint axes.

Keywords: parallel manipulator, dual reconfigurable, 4-RUU, operation modes, workspace

1 Introduction

Different configuration space regions of a parallel manipulator (PM) with possibly different type or number of degrees of freedom are known as the operation modes. They were first observed for the 3-URU DYMO robot by Zlatanov et al. [1] showing that it exhibits five different types of platform motion. A complete characterization of operation modes for various PMs using Study's kinematic mapping and algebraic geometry tools has been accomplished in [2–4]. Moreover, the operation modes of a much more complicated 4-RUU PM with vertical R-joint axes were determined by splitting it into two 2-RUU PMs [5]. Additionally, a novel dual reconfigurable 4-rRUU PM was proposed, constituting a double-Hooke's joint in each limb to reconfigure the R-joint axes to have any horizontal orientations. Its operation modes were determined for some specific configurations [6]. This paper proposes an interpolation approach to determine the operation modes as a function of the orientation of base R-joint axes. Furthermore, its workspace is plotted, thanks to the algebraic expression of its limb singularities.

The organization of the paper is as follows: Section 2 presents the architecture of the dual reconfigurable 4-rRUU PM. Section 3 deals with the derivation of constraint equations and the determination of operation modes of the 4-rRUU

PM in terms of orientations of the base revolute joint axes. Section 4 shows the translational workspace when all the base R-joints are oriented in the same direction.

2 Manipulator architecture

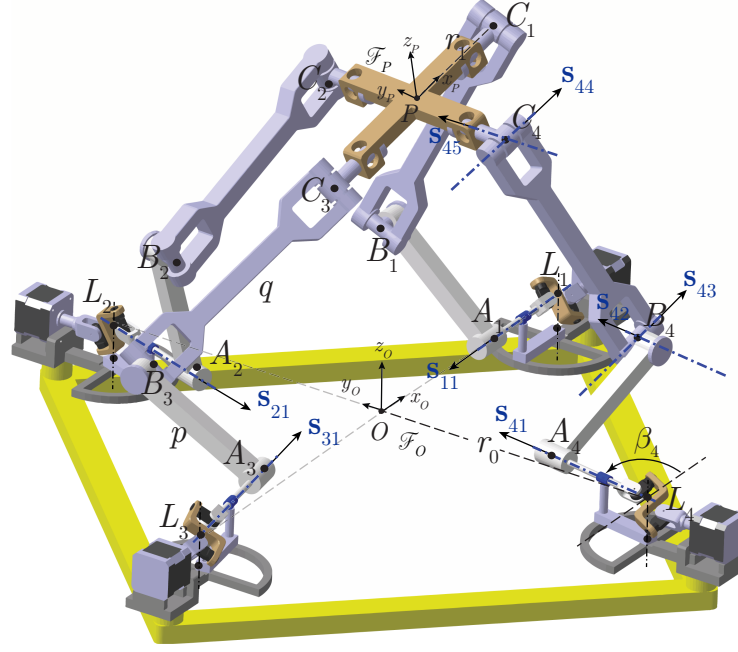


Fig. 1: A 4-rRUU parallel manipulator

The dual reconfigurable 4-rRUUPM with a square base and a platform is shown in Fig. 1. Each limb consists of a double Hooke's joint (rR) followed by two universal joints (UU). Point L_i , $i = 1, 2, 3, 4$ is the intersection point of the pivotal axis of the double-Hooke's joint linkage and the first revolute joint axis. Point A_i lies on the first revolute joint axis of the PM. Points B_i and C_i are the geometric centers of the first and the second universal joints, respectively. Points L_i and C_i are the vertices of the square base and the platform, respectively. \mathcal{F}_O and \mathcal{F}_P are the coordinate frames attached to the fixed base and the moving platform such that their origins O and P lie at the centers of the respective squares. The circum-radii of the base and platform squares are denoted r_0 and r_1 , respectively. p and q are the link lengths. The revolute-joint axes vectors in i -th limb are marked \mathbf{s}_{ij} , $i = 1, 2, 3, 4$; $j = 1, \dots, 5$. Vectors \mathbf{s}_{i1} and \mathbf{s}_{i2} are always parallel, so are vectors \mathbf{s}_{i3} and \mathbf{s}_{i4} . For simplicity, it is assumed that the

orientation of vector \mathbf{s}_{i1} expressed in coordinate frame \mathcal{F}_O is the same as that of \mathbf{s}_{i5} expressed in coordinate frame \mathcal{F}_P .

3 Operation Mode Analysis

3.1 Constraint Equations

Since the reconfigurable revolute joint is actuated, a RUU limb must satisfy the following two constraints:

1. The second revolute joint axis, the fifth revolute joint axis and link BC must lie in the same plane. In other words, the scalar triple product of the corresponding vectors must be null:

$$g_i : (\mathbf{b}_i - \mathbf{c}_i)^T (\mathbf{s}_{i2} \times \mathbf{s}_{i5}) = 0, \quad i = 1, 2, 3, 4 \quad (1)$$

2. The length of link BC must be q :

$$g_{i+4} : \|\mathbf{b}_i - \mathbf{c}_i\| - q = 0, \quad i = 1, 2, 3, 4 \quad (2)$$

Since the length of link BC does not affect the operation modes of the 4-rRUU PM, only the principal geometric constraint from Eq. (1) is considered. To express it algebraically, the homogeneous coordinates of the necessary vectors expressed in frame \mathcal{F}_O are listed below:

$${}^0\mathbf{l}_i = \mathbf{R}_z(\lambda_i) [1, r_0, 0, 0]^T \quad (3a)$$

$${}^0\mathbf{a}_i = {}^0\mathbf{l}_i + \mathbf{R}_z(\lambda_i + \beta_i) [0, 0, l_i, 0]^T \quad (3b)$$

$${}^0\mathbf{b}_i = {}^0\mathbf{a}_i + \mathbf{R}_z(\lambda_i + \beta_i) [0, p \cos(\theta_i), 0, p \sin(\theta_i)]^T \quad (3c)$$

$${}^0\mathbf{c}_i = \mathbf{F} \mathbf{R}_z(\lambda_i) [1, r_1, 0, 0]^T, \quad (3d)$$

$${}^0\mathbf{s}_{i2} = \mathbf{R}_z(\lambda_i + \beta_i) [0, 0, 1, 0]^T, \quad (3e)$$

$${}^0\mathbf{s}_{i5} = \mathbf{F} \mathbf{R}_z(\lambda_i + \beta_i) [0, 0, 1, 0]^T, \quad i = 1, 2, 3, 4. \quad (3f)$$

where $l_i = |A_i L_i|$, $\mathbf{R}_z(\cdot)$ is the homogeneous rotation matrix about the z -axis, λ_i for the i -th limb is given by $\lambda_1 = 0, \lambda_2 = \frac{\pi}{2}, \lambda_3 = \pi, \lambda_4 = \frac{3\pi}{2}$ and θ_i is the actuated joint angle. \mathbf{F} is the following transformation matrix:

$$\mathbf{F} = \frac{1}{\Delta} \begin{bmatrix} \Delta & 0 & 0 & 0 \\ d_1 x_0^2 + x_1^2 - x_2^2 - x_3^2 & -2x_0x_3 + 2x_1x_2 & 2x_0x_2 + 2x_1x_3 \\ d_2 & 2x_0x_3 + 2x_1x_2 & x_0^2 - x_1^2 + x_2^2 - x_3^2 & -2x_0x_1 + 2x_2x_3 \\ d_3 & -2x_0x_2 + 2x_1x_3 & 2x_0x_1 + 2x_2x_3 & x_0^2 - x_1^2 - x_2^2 + x_3^2 \end{bmatrix} \quad (4)$$

with $\Delta = x_0^2 + x_1^2 + x_2^2 + x_3^2 \neq 0$
and $d_1 = -2x_0y_1 + 2x_1y_0 - 2x_2y_3 + 2x_3y_2$,

$$\begin{aligned} d_2 &= -2x_0y_2 + 2x_1y_3 + 2x_2y_0 - 2x_3y_1, \\ d_3 &= -2x_0y_3 - 2x_1y_2 + 2x_2y_1 + 2x_3y_0, \end{aligned}$$

where $x_j, y_j, j = 0, 1, 2, 3$ are called *Study parameters* of the transformation \mathbf{F} . A point $P = (x_0 : x_1 : x_2 : x_3 : y_0 : y_1 : y_2 : y_3) \in \mathbb{P}^7$ represents an Euclidean transformation, if and only if P lies in a 6-dimensional quadric, $S_6^2 \in \mathbb{P}^7$ called the *Study quadric*:

$$S_6^2 : x_0y_0 + x_1y_1 + x_2y_2 + x_3y_3 = 0 \quad (5)$$

Thus, Eq. (1) is derived for each limb algebraically by substituting $t_i = \tan\left(\frac{\theta_i}{2}\right)$ and $v_i = \tan\left(\frac{\beta_i}{2}\right)$, $i = 1, 2, 3, 4$. The equations are not displayed here due to space limits.

The constraint polynomials $g_i, i = 1, 2, 3, 4$ along with Study's quadric form the following ideal³:

$$\mathcal{I} = \langle g_1, g_2, g_3, g_4, S_6^2 \rangle \subseteq k[x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3] \quad (6)$$

To simplify the determination of the operation modes, the 4-rRUU PM is split into two 2-rRUU PMs [5] by considering two ideals:

$$\mathcal{I}_{(I)} = \langle g_1, g_3, S_6^2 \rangle \quad (7a)$$

$$\mathcal{I}_{(II)} = \langle g_2, g_4, S_6^2 \rangle \quad (7b)$$

Furthermore, $\mathcal{I}_{(I)}$ and $\mathcal{I}_{(II)}$ can be decomposed into simpler ideals using primary decomposition to understand the operation modes of the 2-rRUU PMs. Thus, the union of the corresponding prime ideals characterize the operation modes of the whole 4-rRUU PM. Two cases can be considered:

Case 1: When the revolute joint axes are arbitrarily oriented, the primary decomposition of $\mathcal{I}_{(I)}$ and $\mathcal{I}_{(II)}$ leads to one sub-ideal each. These sub-ideals depend on the design parameters and are mixed motion modes which are not of interest in the context of this paper.

Case 2: When the opposite revolute joint axes have the same orientation i.e. $v_1 = v_3$ and $v_2 = v_4$, the operation modes can be determined as follows: All planar orientations of the rR-joint axes are covered by varying $\beta_i \in [-90^\circ, 90^\circ]$ and hence $v_i \in [-1, 1]$. Design parameters were substituted as $r_0 = 2, r_1 = 3, p = 5, q = 7$ to simplify the primary decomposition of ideals $\mathcal{I}_{(I)}$ and $\mathcal{I}_{(II)}$ in Eq. (7). The operation modes could be determined only when arbitrary rational values are substituted for $v_1 = v_3 = v_{13}$ and $v_2 = v_4 = v_{24}$. The primary decomposition is performed in a computer algebra system SINGULAR and it leads to three sub-ideals each. The first two are independent of the design parameters and actuated

³ The ideal generated by the given polynomials is the set of all combinations of these polynomials using coefficients from the polynomial ring $k[x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3]$ [7].

variables. They are of the following form:

$$\begin{aligned}\mathcal{I}_{(I)} &= \mathcal{I}_{1(I)} \cap \mathcal{I}_{2(I)} \cap \mathcal{I}_{3(I)}, \\ \text{where } \mathcal{I}_{1(I)} &= \langle x_0, h_1 x_1 + h_2 x_2, x_1 y_1 + x_2 y_2 + x_3 y_3 \rangle \\ \text{and } \mathcal{I}_{2(I)} &= \langle x_3, -h_2 x_1 + h_1 x_2, x_0 y_0 + x_1 y_1 + x_2 y_2 \rangle\end{aligned}\quad (8a)$$

$$\begin{aligned}\mathcal{I}_{(II)} &= \mathcal{I}_{1(II)} \cap \mathcal{I}_{2(II)} \cap \mathcal{I}_{3(II)}, \\ \text{where } \mathcal{I}_{1(II)} &= \langle x_0, -h_2 x_1 + h_1 x_2, x_1 y_1 + x_2 y_2 + x_3 y_3 \rangle \\ \text{and } \mathcal{I}_{2(II)} &= \langle x_3, h_1 x_1 + h_2 x_2, x_0 y_0 + x_1 y_1 + x_2 y_2 \rangle\end{aligned}\quad (8b)$$

where h_1 and h_2 are functions of v_{13} and v_{24} . For instance, $\mathcal{I}_{1(I)}$ consists of

v_{13}	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	w
h_1	-1	-4	-8	0	8	4	1	$\frac{2}{w}, w \neq 0$
h_2	0	-3	-15	0	-15	-3	0	$1 - \frac{1}{w^2}, w \neq 0$

Table 1: h_1 and h_2 as functions of v_{13} such that $\langle h_1 x_1 + h_2 x_2 \rangle \in \mathcal{I}_{1(I)}$

x_0 and $S_6^2|_{x_0=0}$ irrespective of the value of v_{13} . The remaining polynomial has coefficients h_1 and h_2 , whose values are listed in Table 1 for arbitrarily chosen v_{13} along with their interpolated values for a general $v_{13} = w$. Thus, Eq. (8) can be further simplified as follows:

$$\begin{aligned}\mathcal{I}_{1(I)} &= \langle x_0, 2v_{13}x_1 + (v_{13}^2 - 1)x_2, x_1 y_1 + x_2 y_2 + x_3 y_3 \rangle \\ \mathcal{I}_{2(I)} &= \langle x_3, (1 - v_{13}^2)x_1 + 2v_{13}x_2, x_0 y_0 + x_1 y_1 + x_2 y_2 \rangle\end{aligned}\quad (9a)$$

$$\begin{aligned}\mathcal{I}_{1(II)} &= \langle x_0, (1 - v_{24}^2)x_1 + 2v_{24}x_2, x_1 y_1 + x_2 y_2 + x_3 y_3 \rangle \\ \mathcal{I}_{2(II)} &= \langle x_3, 2v_{24}x_1 + (v_{24}^2 - 1)x_2, x_0 y_0 + x_1 y_1 + x_2 y_2 \rangle\end{aligned}\quad (9b)$$

As a result, the first two operation modes of the 4-rRUU PM are:

$$\begin{aligned}\mathcal{I}_1 &= \mathcal{I}_{1(I)} \cup \mathcal{I}_{1(II)} \\ &= \langle x_0, 2v_{13}x_1 + (v_{13}^2 - 1)x_2, (1 - v_{24}^2)x_1 + 2v_{24}x_2, x_1 y_1 + x_2 y_2 + x_3 y_3 \rangle\end{aligned}\quad (10a)$$

$$\begin{aligned}\mathcal{I}_2 &= \mathcal{I}_{2(I)} \cup \mathcal{I}_{2(II)} \\ &= \langle x_3, (1 - v_{13}^2)x_1 + 2v_{13}x_2, 2v_{24}x_1 + (v_{24}^2 - 1)x_2, x_0 y_0 + x_1 y_1 + x_2 y_2 \rangle\end{aligned}\quad (10b)$$

In general, $\mathcal{I}_1 = \langle x_0, x_1, x_2, y_3 \rangle$ and $\mathcal{I}_2 = \langle x_3, x_1, x_2, y_0 \rangle$. The former corresponds to a 3-*dof* pure translational mode, where the platform is upside down with the z_P -axis pointing downwards. The latter is also a 3-*dof* translational mode, but

the platform is in upright position with z_P -axis pointing upwards. An example would be the configuration (2) in [6] with $\beta_i = 90^\circ, i = 1, 2, 3, 4$.

However, the set of binomial equations $2v_{13}x_1 + (v_{13}^2 - 1)x_2 = 0$ and $(1 - v_{24}^2)x_1 + 2v_{24}x_2 = 0$ can have non-trivial values for x_1 and x_2 if the following conditions are satisfied:

$$\begin{aligned} \frac{2v_{13}}{v_{13}^2 - 1} = \frac{1 - v_{24}^2}{2v_{24}} &\implies \tan(\beta_1) = \tan(\beta_3) = -\cot(\beta_2) = -\cot(\beta_4) \\ &\implies \beta_1 = \beta_3 = \beta_2 - 90^\circ = \beta_4 - 90^\circ \end{aligned} \quad (11)$$

In that case, the first two operation modes are

$$\begin{aligned} \mathcal{I}_1 &= \mathcal{I}_{1(I)} \cup \mathcal{I}_{1(II)} \\ &= \langle x_0, 2v_{13}x_1 + (v_{13}^2 - 1)x_2, x_1y_1 + x_2y_2 + x_3y_3 \rangle \end{aligned} \quad (12a)$$

$$\begin{aligned} \mathcal{I}_2 &= \mathcal{I}_{2(I)} \cup \mathcal{I}_{2(II)} \\ &= \langle x_3, (1 - v_{13}^2)x_1 + 2v_{13}x_2, x_0y_0 + x_1y_1 + x_2y_2 \rangle. \end{aligned} \quad (12b)$$

With $x_3 = 1$, the Study parameters corresponding to the first operation mode, \mathcal{I}_1 are $[0, x_1, x_2, 1, y_0, y_1, y_2, -x_1y_1 - x_2y_2]$. Thus, only four independent parameters are sufficient to characterize this operation mode and it corresponds to a 4-*dof* Schönflies mode in which the translational motions are parametrized by y_0, y_1 and y_2 and the rotational motion is parametrized by x_1, x_2 along with $2v_{13}x_1 + (v_{13}^2 - 1)x_2 = (1 - v_{24}^2)x_1 + 2v_{24}x_2 = 0$. In this operation mode, the platform is upside down with the z_P -axis pointing in a direction opposite to the z_O -axis. The rotational motion is about an axis located at an angle of $\beta_1 - 90^\circ = \beta_3 - 90^\circ = \beta_2 = \beta_4$ from the x_O -axis.

Similarly, with $x_0 = 1$, the Study parameters corresponding to the first operation mode, \mathcal{I}_2 are $[1, x_1, x_2, 0, -x_1y_1 - x_2y_2, y_1, y_2, y_3]$. Thus, only four independent parameters are sufficient to characterize this operation mode and it is a 4-*dof* Schönflies mode in which the translational motions are parametrized by y_1, y_2 and y_3 and the rotational motion is parametrized by x_1, x_2 along with $(1 - v_{13}^2)x_1 + 2v_{13}x_2 = 2v_{24}x_1 + (v_{24}^2 - 1)x_2 = 0$. In this operation mode, the platform is in upright position with rotational motion about an axis located at an angle of $\beta_1 - 90^\circ = \beta_3 - 90^\circ = \beta_2 = \beta_4$ from the x_O -axis.

Hence, it can be concluded from Eqs. (11) and (12) that when all the base R-joint axes of the 4-rRUU PM have the same horizontal orientations, it exhibits a Schönflies motion mode with the rotational *dof* about a horizontal axis with the same orientation as those R-joint axes. An example is the configuration (1) in [6], where $\beta_1 = \beta_3 = 90^\circ, \beta_2 = \beta_4 = 0^\circ$.

4 Workspace for the same orientations of base R-joint axes

The moving platform center of the 4-rRUU PM lies on the boundary of the translational workspace only when at least one of its limbs is in a fully extended

or a folded configuration. In this case, the limb is said to be in a limb or serial or input singularity [8]. It can happen when the 8×4 input Jacobian matrix $\mathbf{J}_I = \frac{\partial \mathbf{g}}{\partial \mathbf{t}_i}$ with $i = 1, 2, 3, 4$ and $\mathbf{g} = [g_1, g_2, \dots, g_8]$ is not full-rank. One approach to deal with the non-square matrix is to search for conditions such that all its 4×4 minors vanish as shown for a 3-RUU PM in [9]. However, the first three equations of the 3-RUU PM do not depend on the variable t_i which is the half tangent of the actuated joint variable θ_i . Likewise, here, g_i does not depend on t_i , $i = 1, 2, 3, 4$. Therefore, the input singularities corresponding to limb i can be simply calculated as $f_i : \frac{\partial g_{i+4}}{\partial t_i} = 0, i = 1, 2, 3, 4$. Eliminating t_i from f_i and g_{i+4} leads to four polynomials, S_i solely in terms of Study parameters. The singularity surfaces (tori shaped) are nothing but the varieties of these polynomials and the workspace boundary is given by their intersection. By considering the 4-rUU PMs in Schönflies motion mode, it is possible to visualize the translational workspace boundaries for different fixed orientations of the moving platform. For instance, the singularity surfaces of a 4-rUU PM with base R-joint axes parallel to x_O -axis and with design parameters $r_0 = 2, r_1 = 1, p = 2, q = 3$ have the following implicit representations:

$$S_1 : x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 - 4x^3 - 4xy^2 - 4xz^2 - 4x^2 - 24y^2 - 24z^2 + 16x + 16 = 0 \quad (13a)$$

$$S_2 : x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 - 4x^2y - 4y^3 - 4yz^2 - 8x^2 - 20y^2 - 24z^2 + 48y = 0 \quad (13b)$$

$$S_3 : x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 + 4x^3 + 4xy^2 + 4xz^2 - 4x^2 - 24y^2 - 24z^2 - 16x + 16 = 0 \quad (13c)$$

$$S_4 : x^4 + 2x^2y^2 + 2x^2z^2 + y^4 + 2y^2z^2 + z^4 + 4x^2y + 4y^3 + 4yz^2 - 8x^2 - 20y^2 - 24z^2 - 48y = 0 \quad (13d)$$

where the orientation of moving platform, $\phi = 0^\circ$ and (x, y, z) are the coordinates of point P . The workspace boundary for this PM is shown in Fig. 2c. Additionally, Fig. 2 shows the workspaces for three configurations of the PM with their base R-joint axes parallel to x_O, y_O and z_O -axes, henceforth named as 4-R_xUU, 4-R_yUU and 4-R_zUU PMs, respectively. The workspaces are plotted for different orientations of the moving platform including their cross-sections about a symmetric axis \mathbf{a} for the 4-R_aUU PM, where \mathbf{a} can be x, y or z .

5 Conclusions and future work

In this paper, a dual reconfigurable 4-rUU parallel manipulator was considered to determine its operation modes as a function of the orientation of its base revolute joint axes. The constraint equations were derived using Study's kinematic mapping for some specific base R-joint orientations such that the coefficients are

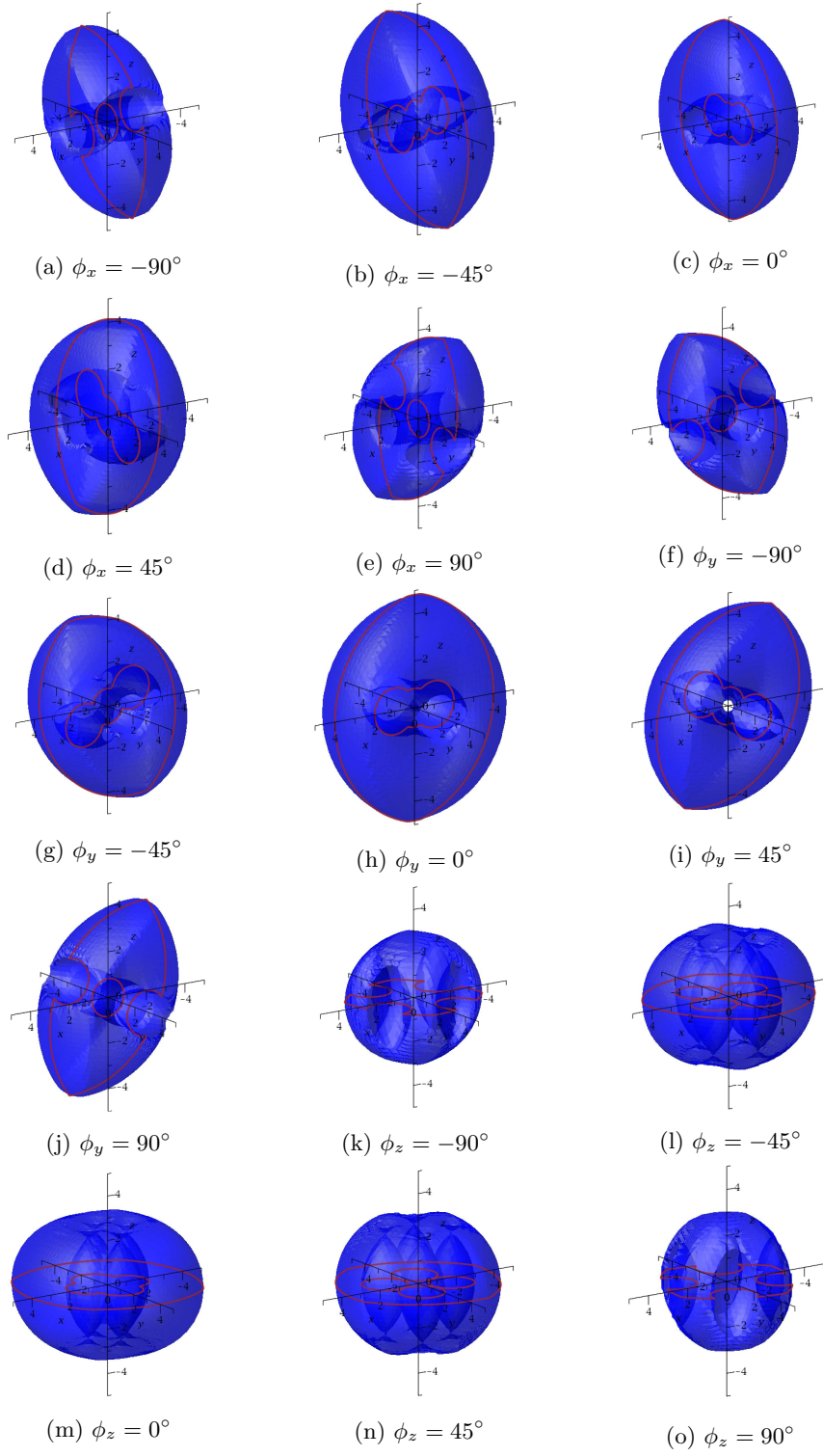


Fig. 2: Workspaces of 4-R_xUU(a-e), 4-R_yUU(f-j) and 4-R_zUU(k-o) PMs

always rational. It simplified the primary decomposition of the constraint equations to determine the operation modes. Then, the polynomials characterizing those operation modes were interpolated to obtain the operation modes for any base R-joint orientations. Furthermore, the translational workspace was plotted for three mutually perpendicular orientations of the base R-joints starting from the algebraic equations describing limb singularities.

As a part of the future work, design optimization of the PM under study will be performed to obtain optimal design parameters so as to realize a functional prototype.

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