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# Optimal Design of Inductors Based on Homothetic Shape and using Geometric Programming

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## Abstract

The efficiency and power density of a converter are two major characteristics but they are antagonists so the designer has to find a trade-off between them. This is generally formulated as a multi-objective optimization problem giving a pareto front, and it can be encountered at the system level, at subsystem level or even at the component level. In general the formulation needs to be different at each level because optimization algorithms are not robust enough and cannot guarantee convergence on large scale problems. This paper proposes a design method for inductors, using a mathematical formalism known as Geometric Programming (GP) that gives strong guarantees of convergence whatever the size of the problem. The design model describes objective and constraint functions using monomials and posynomials to comply with GP rules and works with homothetic shapes. A first formulation using continuous parameters allows selecting the region of operation and provides a good estimate of attainable size and losses, then a second formulation shows how to truncate parameters that need to be discrete values to obtain feasible objects (integer number of turns, discrete core size,) and account for non linear permeability. An evaluation case is presented to test the performance of the proposed algorithm. Last, a finite element analysis is included to account for some non linearities such as fringing and proximity effects, and improve the design accuracy. Because of the GP formulation, it should be possible in future work to use such an inductor model for sub-system level optimization (for example 2nd or 4th order filter) and still guarantee convergence towards a global optimum.

## I. INTRODUCTION

The inherent trade-off, between efficiency and power density, represents a challenge for the design of power electronics systems due to the multi-objective optimization problem [1]. Several works found in the literature focus the optimization mostly on analytic models of the power converter, then following with experimental or simulation verification [2], [3]. The performance of the optimization algorithm, the most computationally efficient way to tackle the problem and cost or the sub-problems to be optimized, are not deeply considered.

The magnetic components, such as inductors and transformers, are very significant part of the volume and weight of power electronic converters, and can also contribute to reduce the system efficiency [1]. Therefore, their design must be carefully addressed as a sub-optimization problem. Among the challenges to consider during the modeling of inductors are the accurate calculation of the core and winding losses, and of the value of the inductance (air-gap fringing effect and non linear permeability). In terms of design, added difficulty comes from the fact that some parameters cannot vary continuously (number of turns, core and conductor size, air-gap) so that once the mathematical optimum is found it is not so obvious to determine the best feasible design.

In the literature it is possible to find several methods for the optimization of the inductors, most of them focusing on analytic models, measurements and finite elements models [1], [4], with fixed or variable geometry [5], [6]. Our aim here is to develop a method that is sufficiently fast to be later used at subsystem or system level with up to hundreds of variables, and still accurate enough to be a very good starting point of a component optimization process. As we will show here, the Geometric Programming (GP) [7], has been identified as the appropriate optimization method to achieve both these goals because of the following properties: • **Assembleability:** Alike most optimization methods, several design models can be easily assembled (e.g. by adding the individual objective functions and grouping the individual constraint functions) to formulate a wider optimization problem. • **Buildability:** Unlike other optimization methods, higher-level problems resulting of the assembly of several lower-level GP-valid formulations inherit from the guarantees of convergence. It is therefore possible to develop and test design models at component-level, to include these models in subsystem or even system-level efficient design tools.

- **Clear-sightedness:** Unlike most optimization methods, GP gives guarantees finding the absolute minima, if any; therefore, it does not need initial guess, nor scaling. That indirectly guarantees reproducibility of results, requires less user inputs, and might find an optimum in an unexpected design region which is especially needed in large scale problems where human intuition might fail.

These properties are obtained at the expense of strict rules to be obeyed when describing the problem, but the methods and tools to solve efficiently these problems are available in [8]:

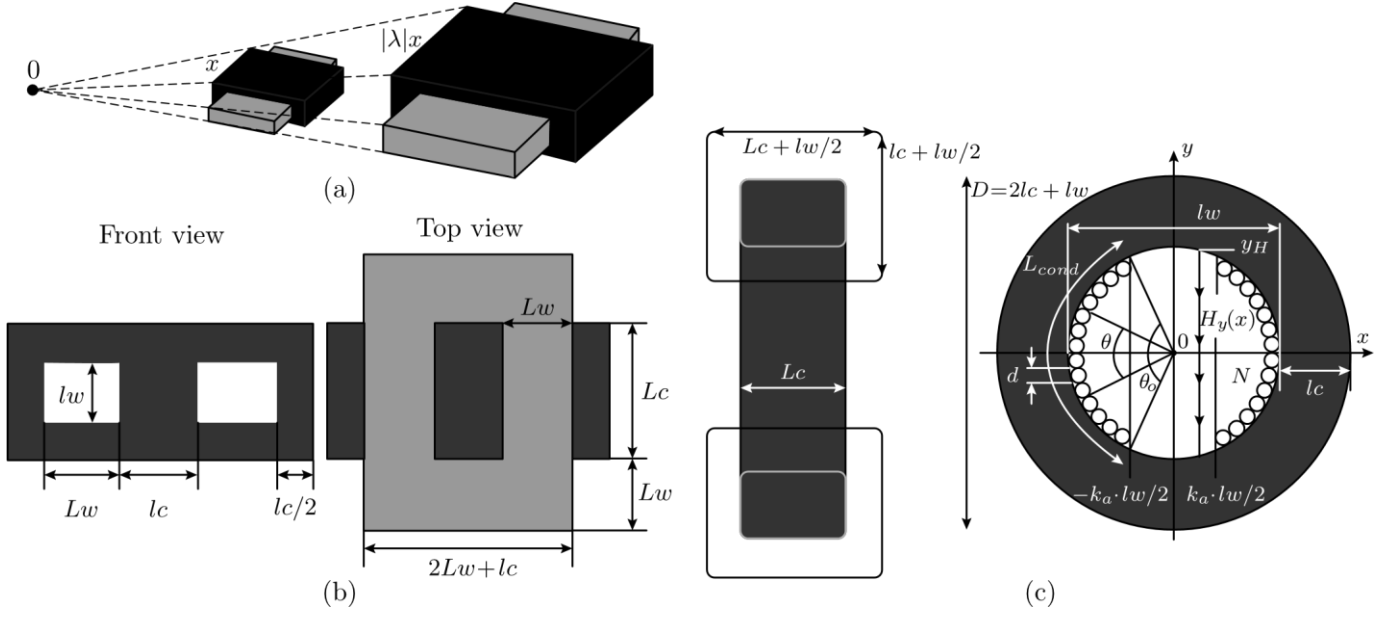


Fig. 1. (a) Homothetic Shape. (b) Characteristics of a rectangular shape. (c) Characteristics of a toroidal shape.

- The validity of the formulation is checked before launching the optimization itself. If a rule is not obeyed, it is pointed out right from the start.
- Solvers for valid GP-form problems are fast and efficient, little knowledge is needed to use them.

## II. GEOMETRIC PROGRAMMING

Geometric Programming (GP) is a mathematical optimization problem [7]. It has the following form:

minimize:

$$f_0(x) = \sum_{l=1}^n (a_{0l} x_1^{k_{0l1}} x_2^{k_{0l2}} \dots x_n^{k_{0ln}}) \quad (1)$$

subject to:

$$f_i(x) = \sum_{l=1}^n (a_{il} x_1^{k_{il1}} \dots x_n^{k_{iln}}) \leq 1, i = 1, \dots, m$$

$$h_j(x) = a_j x_1^{k_{j1}} x_2^{k_{j2}} \dots x_n^{k_{jn}} = 1, j = 1, \dots, p \quad (2)$$

where  $x = (x_1, x_2, \dots, x_n)$  is a vector of the design positive variables,  $h_j(x)$  are the GP-monomials constraints, the coefficients  $a_j$  are positive coefficients, and the exponents  $k_j$  are any real number. However, in this standard formulation the posynomials and monomials are not convex function [9], and can be difficult to solve. However, translating them to the  $\log$  domain makes them convex and easy to solve. Posynomial Functions:

$$\log(h(x)) = \log(ax^{k_1} \dots x^{k_n})$$

$$= \log(a) + k_1 \log(x_1) + \dots + k_n \log(x_n) \quad (3)$$

After such a transformation, the function presents a linear and convex behavior, which guarantees the existence of only one global optimized point. In the next sections we will show that the multi-objective optimization of an inductor aiming at finding the trade-off between power losses and volume, can be quickly solved as long as the formulation is a GP-valid problem [10].

## III. INDUCTOR DESIGN BASED ON HOMOTHETIC SHAPE

An homothetic shape, as shown in Fig. 1 (a) has the advantage of being a linear transformation, which preserves not only the colinearity of points but also the vector addition and scalar multiplication. It means that the distances between points are proportional by a factor  $\lambda$ , areas by  $\lambda^2$ , volumes to  $\lambda^3$ , etc...

By considering fixed shapes of the magnetic cores, and modeling the core and winding losses, the presented approach relates these variable with the area product of the core and the winding ( $A_w A_c$ ) [11]. The area product is proportional to the

TABLE I  
GEOMETRICAL FACTORS

Parameter	Rectangular Shape	Toroidal Shape
$a_{wwc}$	$(a_w / (a_{c1} a_{c2}))^{1/2}$	$\pi / 4 (a_w / (a_{c1} a_{c2}))^{1/2}$
$a_{cwc}$	$1 / a_{wwc}$	$1 / a_{wwc}$
$S$	$\frac{X^*}{(a_w a_{c1} a_{c2})^{1/2}}$	$\frac{2\pi(a_{c1} + a_{c1} a_{c2} + a_{c2} + a_{c2}^2)}{(\pi a_{c1} a_{c2} / 4)^{1/2}}$
$Y$	$\frac{2a_{c1} a_{c2} (a_w + a_{c2} + 1)}{(a_w a_{c1} a_{c2})^{3/4}}$	$\frac{\pi(a_{c2}^2 + a_{c2})a_{c1}}{(\pi a_{c1} a_{c2} / 4)^{3/4}}$
$Z$	$\frac{2a_w (2a_w + a_{c1} + a_{c2})}{(a_w a_{c1} a_{c2})^{3/4}}$	$\frac{\pi(a_{c1} + a_{c2})}{2(\pi a_{c1} a_{c2} / 4)^{3/4}}$

$$*X = 2a_{c1}(2a_w + 3a_{c2} + 1) + 8a_w(a_w + 1) + 2a_{c2}(3a_w + a_{c2} + 3)$$

power  $P$ , which allows to scale the converter volume depending on the operating point. However, it is not easy to express all the core and winding parameters as function of the area product. Therefore, the approach of considering fixed shapes, with dimensions following an homothetic law, simplifies the modeling problem. This homothetic approach is mainly dedicated to standard magnetic cores (E, planar, ETD, toroidal, etc.). Even though the core dimension in a given family are not always homothetic, this approach allows finding the closest possible core.

#### A. Core Shape Specifications

In order to apply the methodology, the geometrical parameters must be expressed in terms of the area product. As a beginning step, three ratios are obtained to describe all the geometric parameters no matter the core geometry. A rectangular shape, as shown in Fig. 1 (b) [11], or a toroidal shape, as shown in Fig. 1 (c) [12].

$$a_w = L_w / l_w$$

$$a_{c1} = L_c / l_w \quad (4) \quad a_{c2} = l_c / l_w$$

where,  $L_w$  and  $l_w$  are related to the winding distances, and  $L_c$  and  $l_c$  with the core distances. These geometrical ratios allow defining the desired inductor characteristic in terms of the area product  $A_w A_c$ , but it is necessary to previously define some geometrical factors, for the rectangular and toroidal shape, as described in the Table I.

With these factors it is possible to express the inductor characteristic as follows:

- Core and winding areas:

$$A_w = L_w l_w = a_{wwc} (A_w A_c)^{1/2} \quad (5)$$

$$A_c = L_c l_c = a_{cwc} (A_w A_c)^{1/2} \cdot$$

Effective inductor volume:

$$Vol_E = (Y + Z/Kw) (A_w A_c)^{3/4} \quad (6)$$

$$Kw > 1$$

where,  $Kw$  represents a winding factor.

- Total exchange surface:  $_{1/2}$  (7)

$$A_{TH} = S(A_w A_c)$$

- Allowed losses:

$$P_{wc} = A_{TH} H \Delta T \quad (8)$$

where,  $\Delta T$  represents the thermal elevation above ambient temperature, and  $H$  is the thermal exchange coefficient.

As can be seen, the volume and thermal losses have been described in terms of the area product  $A_w A_c$ .

#### IV. OPTIMIZATION PROBLEM USING GP FORMULATION

The main rules governing the design of inductors are the following:

- A magnetic component stores energy through the interaction of a magnetic flux and a current ( $\varphi \cdot I > L_{target} \cdot I^2$ ).
- The flux density cannot exceed a certain value ( $\varphi \cdot Ac < B_{sat}$ ).
- The current generates losses in the conductor, and the AC flux generates losses in the core; these losses must be dissipated through an exchange surface.

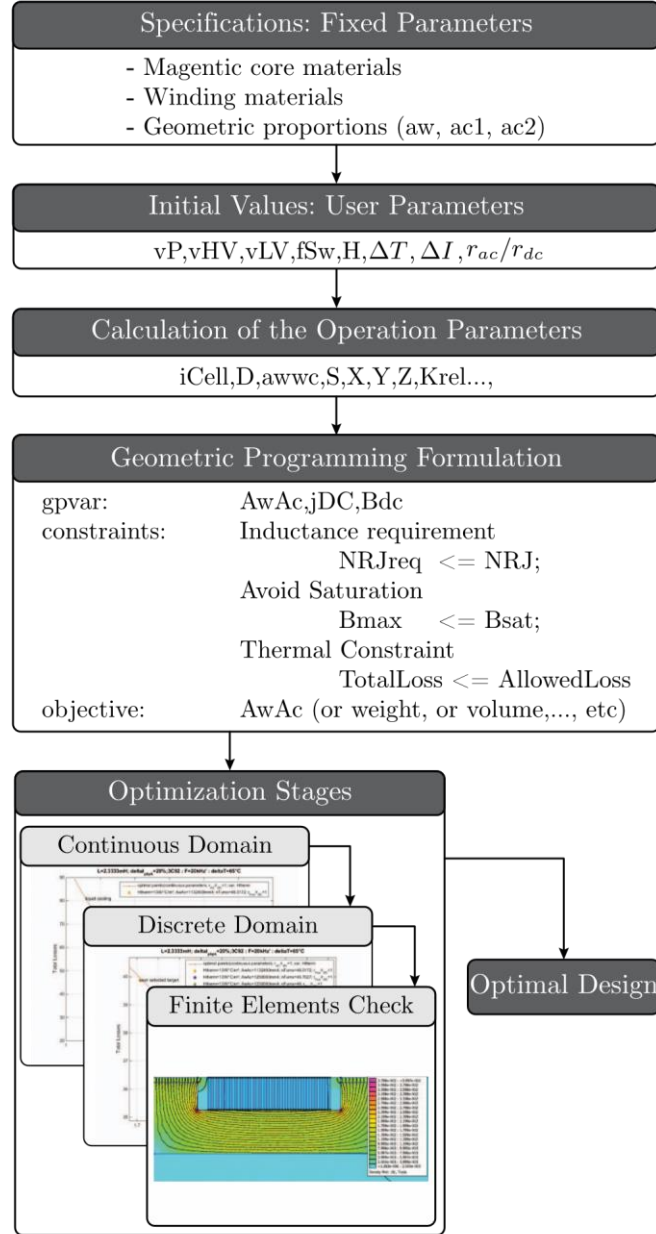


Fig. 2. Optimization procedure of an inductor under user specifications.

In order to address the optimization problem, the proposed procedure is presented in Fig. 2. At the beginning the user selects from a data base set, some standard parameters found in the commercial datasheets (e.g. the material of the core and winding, geometrical proportions based on the type of core, etc.). Then, the user defines the operating points under which the inductor will

work (e.g. nominal power, input/output voltage, switching frequency, etc). After that, the program will calculate some operation parameters based on the user selections, with these parameters all the geometrical and electrical characteristics are defined.

At first instance, the geometric programming formulation is constructed defining the target variables to be optimized  $gpvar$ . The selected variables regard the construction limitations  $A_wA_c$ , inductance requirement  $jDC$  and core saturation  $Bdc$ . The advantage of the GP is that there is not a theoretical limitation on the number of variables to be optimized, as long as the GP formalism is respected. Besides, all the constraints are imposed in this step. Where all the maximum allowed values can be also defined as a function of the variables to optimize  $NRJ$  or, physical limitations of the material  $Bsat$ . For concluding the first GP formulation, the objective is defined. The GP does not limit the number of objectives to be optimized at same time, but in this work the objectives are evaluated independently and consecutively. This will be justified in the following sections.

TABLE II  
EVALUATION PARAMETERS

Parameter	Value
Power $P$	6kW
Voltage at High Side $v_{HV}$	800V
Voltage at Low Side $v_{LV}$	280V
Switching Frequency $f_{SW}$	20kHz
Temperature Difference $\Delta T$	65°C
Thermal Transfer $H$	12°C/W/m <sup>2</sup>
Current Ripple $\Delta I_{cell}$	20%
Magnetic Material	3C92

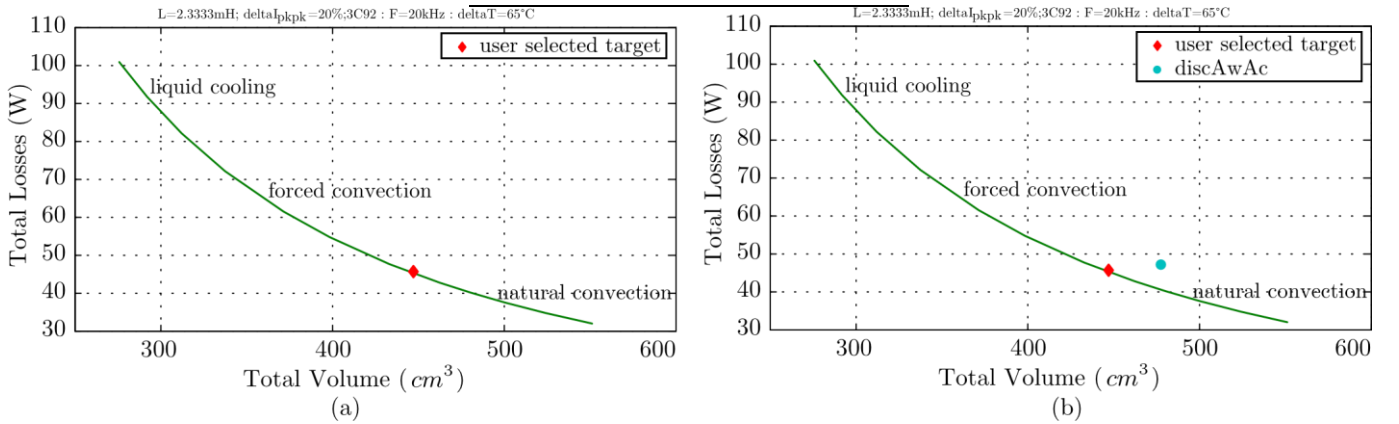


Fig. 3. (a) Volume/Losses Pareto Front, for continuous variables design. (b) Volume/Losses Pareto Front, selecting a feasible core design.

## V. EVALUATION CASE

As an evaluation case the selected core is: a *EI – Planar* type, with a magnetic material *MagneticsMPP300*, and conductor *Copper*, for a Chopper application. Moreover, the design parameters are presented in the Table II.

### A. GP Formulation in a Continuous Domain

At first instance, the GP formulation selects the area product as the optimization objective  $A_wA_c$ . Beside, the first assumption made is that  $r_{ac}/r_{dc} = 1$ . It is made in order to know the volume/loss Pareto front under ideal conditions. The result depicted in Fig. 3 (a), allows the designer to chose a desired operating condition on this design space. This selected target can be made based on the technique for the thermal dissipation (liquid cooling, forced convection or natural convection). In this case, the selected point is:  $H_{therm} = 13W/^{\circ}C/m^2$ ,  $A_wA_c = 1.02623e6mm^4$ ,  $nTurns = 50.4397$ . However, as can be noted, the variables are continuous (e.g.  $nTurns$ ), and it must be an integer value in order to construct the inductor.

### B. GP Formulation for Feasible Variables

1) *Feasible Core Design*: Once the algorithm generates the Pareto front for continuous variables, the designer will be able to find an existing core and an integer number of turns close to the prescribed solution. In the evaluation case, as depicted in Fig. 3

(b), the designer was able to chose an available core. In this case, the selected point is:  $H_{therm} = 13W/^0C/m^2$ ,  $A_wA_c = 1.12e6mm^4$ ,  $nTurns = 52.1402$

It means that the geometry proportion  $A_wA_c$ , is not a variable anymore, but it is an imposed parameter to be set on the GP formulation as follows:

- gpvar: jDC, Bdc (  $A_wA_c$  now imposed to  $A_wA_cDisc$  )
- constraints: *Inductance requirement*  $NRJreq$   
 $\leq NRJ(A_wA_cDisc)$  *Avoid saturation*  
 $B_{max} \leq B_{sat}(A_wA_cDisc)$  *Respect thermal constraint*  
 $totalLosses(A_wA_cDisc) \leq allowedLosses(A_wA_cDisc)$
- objective:  $totalLosses(A_wA_cDisc)$

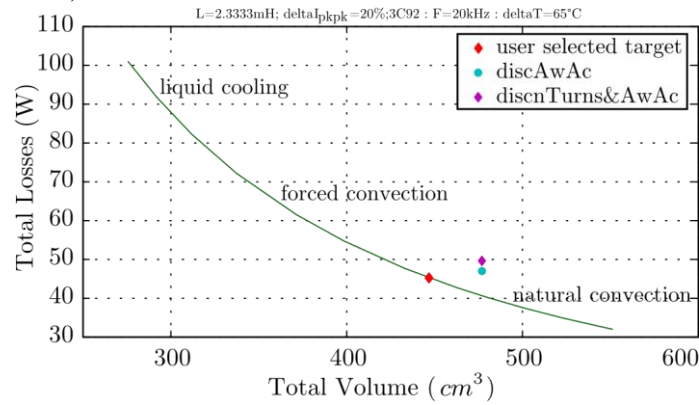


Fig. 4. Volume/Losses Pareto Front, selecting an integer turns ratio.

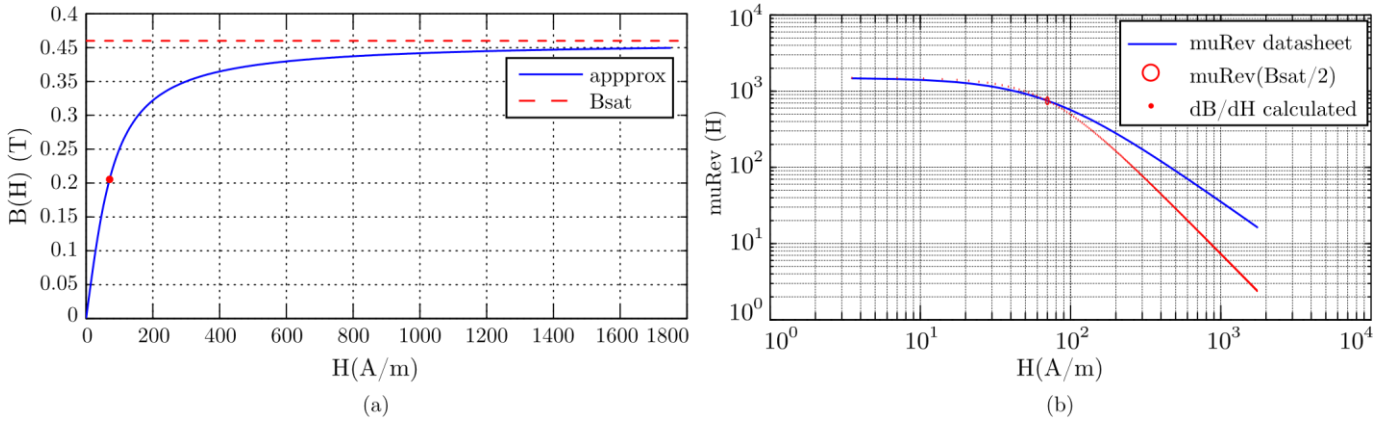


Fig. 5. Optimization procedure of an inductor under user specifications.

2) *Feasible Turns Ratio*: After the optimization, the core is now slightly bigger than strictly required but it still has to truncate the turns ratio  $nTurns$ . There are different ways to do this:

- $nTurnsDisc = \text{floor}(nTurns)$ : It increases  $B_{max}$ , the core losses and wire section. But it decreases dc winding losses.
- $nTurnsDisc = \text{ceil}(nTurns)$ : It decreases  $B_{max}$ , the core losses and wire section. But it increases dc winding losses.

It means that the turns ratio  $nTurns$ , is not a variable anymore, but it is an imposed parameter to be set on the GP formulation as similar to the explained above, with the difference that:

- gpvar: Bdc ( jDC is now imposed)
- constraints: *Inductance requirement*  
 $NRJreq \leq NRJ(A_wA_cDisc, nTurnsDisc)$

*Avoid saturation*

$$B_{max} \leq B_{sat}(A_w A_c D_{isc}, n_{Turns} D_{isc})$$

$$\text{Respect thermal constraint } totalLosses(A_w A_c D_{isc}, n_{Turns} D_{isc}) \leq allowedLosses(A_w A_c D_{isc})$$

• objective:  $totalLosses(A_w A_c D_{isc}, n_{Turns} D_{isc})$

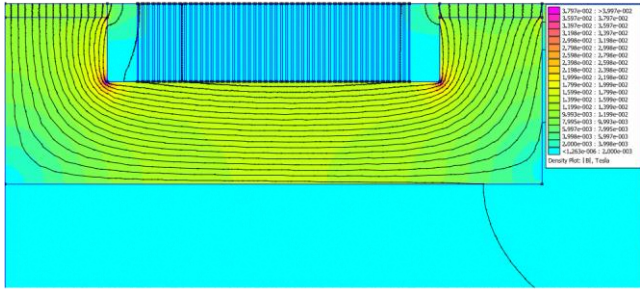
In that evaluation, the selected operating points are depicted in Fig. 4. The values are:  $H_{therm} = 13 W/^{\circ}C/m^2$ ,  $A_w A_c = 1.12e6 mm^4$ ,  $n_{Turns} = 53$ . In case no acceptable design is found, the next bigger core must be selected.

### C. Design using Finite Elements Analysis

The  $jDC/Bdc$  formulation does not involve the  $B(H)$  characteristic of the material; in this approach, the air gap is adjusted at the end of the design to obtain the appropriate induction level and inductance value. Therefore a non-linear  $B(H)$  characteristic can be easily taken into account since  $B$  and  $H$  are already known, as shown in Fig. 5 (a).

The air gap reluctance is found by subtracting the core reluctance (with  $\mu(Bdc)$ ) to the targeted total reluctance, as shown in Fig. 5 (b).

There are some motivations for using Finite Elements:



(a)

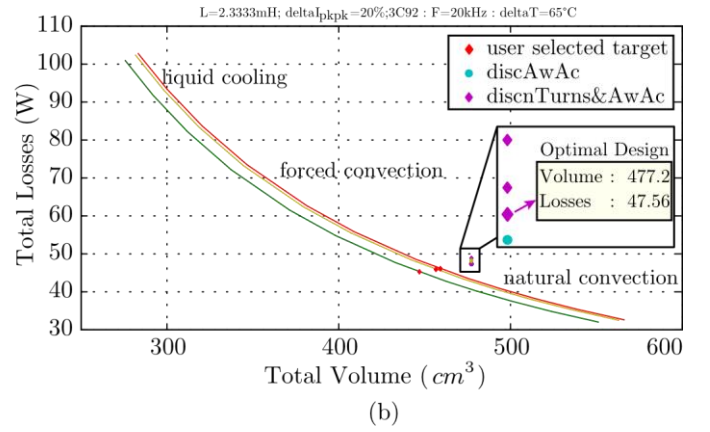


Fig. 6. (a) Flux density results using Finite Elements. (b) Optimization procedure of an inductor under user specifications.

- Relation from airgap to airgap reluctance is complex and involves details of the geometry (e.g. distance to the winding).
- AC losses are difficult to take into account (skin, proximity and air gap induced losses) and each formulation is geometry dependent.

Therefore, FEMM simulations are run to have a good estimate of  $rac/rdc$  and inductance  $L$ . The whole process is repeated until the correction coefficients become as close to 1 as specified. From the results, it was realized that 5% tolerance generally needs less than 4 iterations. Calculation time is around 1 minute, with roughly 99% for FEMM and 1% for GP formulation execution. The results of the FEMM analysis for the selected geometry, is depicted in Fig. 6 (a). In this figure it is possible to see the Flux Density  $B$  on the core.

Moreover, after 3 iterations, the results of the optimization process is depicted in Fig. 6 (b). In this figure, the desired operating points are highlighted, and from the results it is possible to see that for an inductor of  $477.2 cm^3$ , a total power dissipation of  $47.56 W$  is achieved. The result assumes the designer will be able:

- To find an existing core and an integer number of turns close to the prescribed solution.
- To find the air gap that gives targeted  $L$ , accounting for fringing effect and for material non linearity.
- To find a winding topology (round/litz, horizontal/vertical foil,) that gives the assumed  $rac/rDC$ .

## VI. CONCLUSIONS

This work presents a methodology for the optimal design of inductors, based on the homothetic shapes. Due to this criteria of designing, it is possible to represent all the geometrical and electrical characteristics of the inductor in terms of the area product.



Besides, the geometrical programming formalism is used as a powerful tool for the multi-objective convex optimization. Giving as result a volume/loss Pareto front, which allows to the designer to select the available core and turns ratio close to the prescribed solution.

Moreover, other advantage of the geometric programming formalism, is that this design is extremely fast ( $< 20\text{ms}$ ). And it can be used, to explore the design space (weight/losses pareto, iteration on materials or shapes, variation of frequency,), at system level to get good estimates of weight and losses of magnetic components (e.g. in a full conversion chain).

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