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Isabelle Piot-Lepetit, Dominique Vermersch, Robert D. Weaver, . American Agricultural Economics Association. Subvector Technical Efficiency: An Application to French Agriculture. American Agricultural Economics Association Annual Meeting, American Agricultural Economics Association (AAEA). USA., Aug 1995, Indianapolis, United States. 13 p. hal-02403487

**HAL Id: hal-02403487**

**<https://hal.science/hal-02403487v1>**

Submitted on 7 Jun 2020

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# **Subvector Technical Efficiency: An Application to French Agriculture**

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**American Agricultural Economics Association Annual Meeting,  
Indianapolis, U.S.A, August 1995**

## **ABSTRACT**

When quasi-fixed factors exist, only subvector measures of technical efficiency are consistent with economic efficiency. They are shown to be less than or equal to radial measures and to increase as the number of quasi-fixed factors is increased. Nonparametric estimates for French cereal farms indicate these effects can be substantial.

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## Introduction

An important objective of nonparametric estimation of technical inefficiency is the development of recommendations for adjustment of factors of production. Implicitly, these recommendations must be limited to adjustments which are economically efficient. Where technologies are homothetic, expansion paths are rays from the origin. Within this context, recommendations for radial adjustments for technical efficiency are consistent with achieving economic efficiency if adjustment is proposed along the expansion path relevant under the current price structure. In this sense, measures of radial technical efficiency (RTE) would be of interest to managers because they provide recommendations that are consistent with economic efficiency. This correspondence between radial technical adjustment and economic efficiency dissolves when technology is nonhomothetic. A similar situation occurs when the set of technical possibilities is constrained by quasi-fixity of factor services. In this case, even for homothetic technologies, the expansion path will deviate from a radial path. Such fixity might occur as a result of temporal, regulatory, or other constraints (e.g. exogeneity of flows) on adjustment. Within decision horizons where such factor services can not be controlled, recommendations for their radial adjustment would be of little interest to managers. Where the marginal productivity of such services is nonnegative at their constrained level, radial adjustment through disposal of such services would be economically inefficient.

This problem was recognized by Farrell (1957) though he maintained a focus on radial technical adjustment.<sup>1</sup> Hall and Winsten (1959) reconsidered the issue by distinguishing between controllable and noncontrolable variables. More generally, the issue is a case of

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<sup>1</sup> The issue was also considered in a wider context by Färe and Lovell (1978) who introduced input specific measures of input technical inefficiency to generalize the Farrell measure. These « Russell » input efficiency measures are interpretable as nonradial and were shown to provide identification of efficient firms from the primal. In contrast, the Farrell input and output measures provide different rankings of firm efficiency. While Färe and Lovell's Russell measure achieved its objective of providing a unique means of identifying technical efficiency from the primal, Kopp noted that the measure lacks a useful dual interpretation.

measurement of subvector technical efficiency (STE) as studied by Kopp (1981); Färe, Lovell, and Zieschang (1983); and applied by Banker and Morey (1986), Kamakura (1988), Adolphson, et al. (1990). For the case of a single quasi-fixed factor, Banker and Morey (1986) demonstrated that the resulting estimate of subvector technical efficiency will be less than or equal to a corresponding estimate of unrestricted or radial technical efficiency.

In this paper, a formal proof is provided of the relationship between RTE and STE for the case of an arbitrary vector of quasi-fixed factors. Next, the paper establishes that, in general, a type of magnification effect occurs as the vector of quasi-fixed factors is expanded. That is, for a particular firm, as the number of quasi-fixed factors is increased (decreased) the STE estimate decreases (increases). In analogy to the Le Chatelier - Samuelson principle, the proposition states that as the opportunity for adjustment of variable factors is reduced, so the extent of technical efficiency is reduced. Going further, we evaluate the impact of fixity of factors on the efficiency rank of firms.

The presence of efficient markets and opportunities for instantaneous adjustment of factor services in many industries limits the applicability of subvector technical efficiency. However, where ideal economic conditions fail to exist, or where economic conditions are distorted by regulatory constraints, subvector measures of technical efficiency are warranted. To evaluate the empirical implications of use of RTE measures in the presence of quasi-fixed factors, we present estimates of RTE and STE for a sample of French cereal farms drawn from the FADN (Farm Agricultural Data Network) farm record system for 1990. The bias of the RTE measure is evaluated. Given the conditionality of STE estimates on the composition of the subvector of quasi-fixed factors, the robustness of firm efficiency rank is evaluated as the number of quasi-fixed factors is varied.

## Technical Efficiency with Quasi-Fixed Factors

Input based and output based measures of technical efficiency popularized by Farrell indicate an identical proportion by which either all inputs or all outputs may be reduced to place a firm on a particular technical frontier of reference. The usefulness of such aggregative indicators has been criticized because they fail to provide input specific recommendations for adjustment (Fernandez-Cornejo,1994). While the Russell measure of technical efficiency introduced by Färe and Lovell provides input specific estimates, Kopp (1981) noted it lacks a corresponding dual measure. More generally, it may be of interest to access the technical efficiency of subvectors of inputs or outputs. The presence of quasi-fixity of subvectors is an important example. The notion of subvector technical efficiency was considered by Färe, Grosskopf, and Zeischang (1983) for the a single factor case and by Färe, Grosskopf, and Lovell (1994) for the general case. The input based measure of subvector efficiency can be defined following these studies for a set of  $J$  firms indexed  $j= 1, \dots, J$ , each with access to the same technology that transforms a vector of inputs  $x_j \in R_+^N$  into a vector of outputs  $y_j \in R_+^M$ . More generally, for the set of firms define a  $(J \times N)$  input matrix  $X$  and a  $(J \times M)$  output matrix  $Y$ . Suppose the input matrix is partitioned so  $X = (X^v, X^{\bar{v}})$  where  $v \subseteq \{1, 2, \dots, N\}$  with  $v \cap \bar{v} = \emptyset$  and  $v \cup \bar{v} = \{1, 2, \dots, N\}$ . For convenience, the partition is identical for all firms. The elements of  $X^v$  are assumed variable and those of  $X^{\bar{v}}$  are assumed quasi-fixed. Each element of  $X$  can be equivalently written  $x_j = (x_j^v, x_j^{\bar{v}})$ . Suppose the technology satisfies the augmented regularity conditions adopted by Banker, Charnes, and Cooper (1984)<sup>2</sup>. The production possibilities set  $P$  can be written:

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<sup>2</sup> That is, the set of production possibilities is defined by:  $P = \{(x, y) \mid y \geq 0 \text{ may be produced from } x \geq 0\}$  satisfying:

[P1] :  $(x_j, y_j) \in P \quad \forall j = 1, \dots, J$

[P2] :  $P$  is a regular set, i.e. nonempty, closed, and satisfying the origin condition that  $y_j = 0$  if  $x_j = 0$  .

$$P = \left\{ (x^v, x^{\bar{v}}, y) : x^v \geq X^v \lambda, x^{\bar{v}} \geq X^{\bar{v}} \lambda, y \leq Y \lambda, \sum_{j=1}^J \lambda_j = 1, \lambda \in R_+^J \right\} \quad [1]$$

where  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_J)$  is the intensity vector. Each  $\lambda_j$  denotes the intensity at which activity  $j$  is undertaken. By equation (1), the  $j$ th firm's production plan  $(x_j^v, x_j^{\bar{v}}, y_j)$  belongs to the production possibilities set, if, and only if,  $(x_j^v, x_j^{\bar{v}}, y_j) \in P$ . Input based radial technical efficiency (RTE) is defined following Farrell (1957), Charnes, Cooper, and Rhodes (1978), and Färe, Grosskopf, and Lovell (1994):

$$R_j(x_j, y_j) = \min \left\{ R_j : R_j x_j \in I(y_j), R_j \in R_+ \right\} \quad [2]$$

where  $I(y_j)$  is the input requirements set derived from (1). The interpretation of  $R_j$  as a radial measure follows from its scalar dimension which implies each factor will be reduced by the same nonnegative proportion. As an alternative, Färe, Lovell and Zeischang defined asymmetric technical inefficiency where the scalar reduction  $R_j$  is applied to only one element of  $x_j$ . More generally, Färe, Grosskopf, and Lovell define subvector technical efficiency (STE) as follows:

$$S_j^v(x_j^v, x_j^{\bar{v}}, y_j) = \min \left\{ S_j^v : S_j^v x^v \in I^v(x_j^{\bar{v}}, y_j), S_j^v \in R_+ \right\} \quad [3]$$

where  $I^v(x_j^{\bar{v}}, y_j)$  is the input requirements set conditional upon the levels of the quasi-fixed factors. The distinction between  $R_j$  and  $S_j$  rests squarely on the application of a common proportion of reduction (thus, subaggregative) to a subvector of inputs which by maintained

[P3] :  $P$  is consistent with free disposal of inputs and outputs :  $\forall (x, y) \in P$  if  $\bar{x} \geq x$  and  $\bar{y} \leq y$ , then  $(\bar{x}, \bar{y}) \in P$ .

[P4] :  $P$  is convex:  $\forall (x, y) \in P$  and  $(\bar{x}, \bar{y}) \in P, \forall \alpha \in [0, 1]$  ;  $\alpha(x, y) + (1 - \alpha)(\bar{x}, \bar{y}) \in P$

[P5] :  $P$  is the intersection of all sets satisfying P1 to P4.

More formally, P2, P3 and P4 each define linear constraints that define closed subspaces. The intersection of these subspaces satisfies P1 and defines a convex envelop that may be written:

$$P = \left\{ (x, y) : x \geq \sum_{j=1}^J \lambda_j x_j, y \leq \sum_{j=1}^J \lambda_j y_j, \sum_{j=1}^J \lambda_j = 1, \lambda_j \geq 0 \forall j = 1, \dots, J \right\}$$

hypothesis are considered variable and controllable by the manager. Efficient adjustment is defined as conditional upon the levels of quasi-fixed inputs. A direct comparison of these two measures can be made within the context of linear programs that allow their nonparametric estimation following the approach of Data Envelopment Analysis. To allow a convenient comparison, define a base linear program from which the two measures can be drawn as special cases:

$$\min E_j = (E_j^v, E_j^{\bar{v}})$$

$$\text{s.t.} \begin{cases} E_j^v x_j^v \geq X^v \lambda \\ E_j^{\bar{v}} x_j^{\bar{v}} \geq X^{\bar{v}} \lambda \\ y_j \leq Y \lambda \\ \sum_{j=1}^J \lambda_j = 1 \\ \text{where } \lambda \in R_+^J, E_j^v \in R_+, E_j^{\bar{v}} \in R_+ \end{cases} \quad [4]$$

Define  $R_j$  the solution to program "R" defined by the base program (4) augmented by the constraint that  $R_j = E_j^v = E_j^{\bar{v}}$ . Similarly, define program "S" as the base program augmented by the constraints that  $S_j^v = E_j^v$  and  $E_j^{\bar{v}} = 1$ . Solution of program S results in  $S_j^v$  consistent with (3). Banker and Morey graphically compared the solutions  $(R_j, S_j^v)$  for the case where  $x_j^{\bar{v}}$  is a scalar. For the general case where  $x_j^{\bar{v}}$  is a vector, they noted that any optimal solution to program R is a feasible solution to program S. Their proposition is as follows.

**Proposition 1 (Banker and Morey, 1986)**

For any  $j$ ,  $S_j^v \leq R_j$  where  $S_j^v$  is the solution to program S for firm  $j$  and  $R_j$  is the solution to program R.

Formal proof follows directly by reference to the underlying input requirements sets derived from equation (1). In the absence of a constraint on  $x_j^v$ , program  $R$  was derived from equation (2). For  $x_j^v$  fixed, program  $S$  follows from equation (3). To compare the solutions to program  $R$  and program  $S$ , write equation (2) in terms of the constrained level of  $x_j^v$ , say  $\bar{x}_j^v$ :

$$R_j(x_j^v, \bar{x}_j^v, y_j) = \min\{R_j: R_j x_j^v \in I^v(R_j \bar{x}_j^v, y_j), R_j \in R_+\} \quad [5]$$

Given that  $R_j \in R_+$ , free disposal (assumed by P3) implies

$$R_j \bar{x}_j^v \leq \bar{x}_j^v \text{ and } I^v(R_j \bar{x}_j^v, y_j) \subset I^v(\bar{x}_j^v, y_j)$$

Simply stated, the constraint set for program  $R$  is contained within that of program  $S$ . It follows that:

$$R_j(x_j, y_j) \geq S_j^v(x_j^v, \bar{x}_j^v, y_j) \quad [6]$$

By similar logic, define program  $S^{v-1}$  where an arbitrary element of  $x^v$  is fixed, e.g.  $x^k$ . Program  $S^{v-1}$  can be written as:

$$S_j^{v-1}(x_j^{v-1}, \bar{x}_j^k, \bar{x}_j^v, y_j) = \min\{S_j^{v-1}: S_j^{v-1} x_j^{v-1} \in I^{v-1}(\bar{x}_j^k, \bar{x}_j^v, y_j), S_j^{v-1} \in R_+\} \quad [7]$$

However, program  $S^v$  can be rewritten:

$$S_j^v(x_j^v, \bar{x}_j^k, \bar{x}_j^v, y_j) = \min\{S_j^v: S_j^v x_j^{v-1} \in I^{v-1}(S_j^v \bar{x}_j^k, \bar{x}_j^v, y_j), S_j^v \in R_+\} \quad [8]$$

Given that  $S_j^v x_j^k \leq x_j^k$ , it follows that:  $I^{v-1}(S_j^v \bar{x}_j^k, \bar{x}_j^v, y_j) \subset I^{v-1}(\bar{x}_j^k, \bar{x}_j^v, y_j)$  and  $S_j^v \geq S_j^{v-1}$ .

These results provide the basis for the following proposition.

## Proposition 2

The input based subvector efficiency of firm  $j$  varies systematically with the number of quasi-fixed factors which constrain its adjustment of variable factors. As  $v$  decreases from  $M$  to 0,



$$S_j^M = R_j \geq S_j^{M-1} \geq \dots \geq S_j^y \geq \dots \geq S_j^0 = 0 \quad [9]$$

## An Application to French Cereal Production

Propositions 1 and 2 establish that estimates of radial technical efficiency will, in general, over estimate the efficiency of firms which face constraints on adjustment due to quasi-fixity of factor services. In this section, we present results which allow an empirical evaluation of the magnitude of such biases as well as an illustration of the magnification effect presented in Proposition 2. The application is based on a sample of 188 French cereal farms drawn from the Farm Agricultural Data Network (FADN) data set for 1990. On average, cereal production accounted for at least 70 percent of their total gross product. In all cases, the farms were specialized in soft wheat and maize, Vermersh et al. (1992). The sample characterizes 45% of the French cereal farms sampled in the General Agricultural Census of 1988 and in this sense, the sample may be interpreted as representative of population of cereal producers in France.

To proceed, we estimate RTE and STE as alternative hypotheses concerning the fixed of factor services. Under a maintained hypothesis that we label as Hypothesis 1, we represent the product vector as including two outputs (cereal output (soft wheat and corn), and other outputs (mainly, oilseeds); five variable inputs (fertilizer, pesticides, seeds, other intermediate inputs, and equipment); and three quasi-fixed factors (area cultivated under cereal, area cultivated in other crops, and labor (family and hired)). This represents our prior concerning the quasi-fixity of factors. We include labor in the vector of quasi-fixed factors given that family labor accounts for 95% of labor used in the sample. All variables except land and labor are measured in 1990 French francs<sup>3</sup>. Available data necessitated linear aggregation of values<sup>4</sup>. Land is measured in hectares, while labor is measured in annual worker units (AWU<sup>5</sup>). Equipment services were measured based on estimated value, a five year amortization, as well as maintenance expenses, rent, and machinery cooperative fees. Equipment is viewed as variable due to the presence of active rental markets. Descriptive statistics presented in Table 1

<sup>3</sup> Economic data, whether expressed in volume or value, reflects both technical and allocative inefficiency. Past studies of technical efficiency have used both volume and value.

<sup>4</sup> Färe and Primont (1988) present results that indicate that use of value aggregated data will result in negatively biased estimates of technical efficiency. Thomas and Tauer have noted that measurement bias may occur when linear aggregation is used.

<sup>5</sup> One AWU equals 2200 hours of labor.

verify the specialization in cereal production by the sampled farms. Cereal production accounted for over two-thirds of their revenues and occupied almost 75% of their cultivated area. Fertilizers and pesticides dominated expenses.

Estimation of the technical efficiency scores under various maintained hypotheses of quasi-fixity were based on solution of the programs presented in equations (2) and (3). Solutions were implemented using linear programming software available in SAS/OR. Results are reported in Table 2 - 4. Alternatives to Hypothesis 1 for the specification of the quasi-fixed factor vector include Hypothesis V where all factors are variable and Hypotheses 2-4 which maintain alternatives to Hypothesis 1 as defined in Table 3. First, Table 2 indicates that the number of firms identified as technically efficient (based on a technical efficiency score of unity and slack variables estimated as zero) under alternative hypotheses ranged between 88 and 93. This indicates robustness of the classification of efficient firms under the alternative hypotheses. Details on the characteristics of these efficient farms are available from the authors. However, similar robustness is not found across ranges of inefficiency. For example, the number of firms found with scores below .85 varies from 71 under our preferred specification Hypothesis 1, to only 36 under Hypothesis V. This suggests that the distribution of the efficiency estimates is substantially altered by the efficiency measure used (RTE vs. STE) and by the specification of the quasi-fixity of inputs.

Table 3 presents a different view of the distribution of efficiency scores. Results indicate that the mean and standard deviation of scores varies across specification. On average, the estimated technical efficiency score under Hypothesis 1 (land and labor fixed) was .8773 compared to .9319 under Hypothesis V (all factors variable), indicating that a substantial upward bias results from use of  $R_j$  when quasi-fixed exists according to Hypothesis 1. Further, Table 3 indicates the extent of the magnification effect presented in equation (9) can be substantial in quantitative terms, depending on the factors that are fixed. The differences between efficiency estimates based on Hypothesis V vs. Hypothesis 1 is also apparent from 1 to 37.8% compared to only 19.1% under Hypothesis V.

## Conclusions

These results have two important implications. First, where factors are quasi-fixed, both the level of efficiency and the distribution of efficiency among firms may be substantially over estimated when based on Hypothesis V. While econometric study of efficiency has

necessarily recognized the importance of accurate specification of input variability and fixity, these results highlight the importance of this specification decision for nonparametric studies of technical efficiency. Second, the under estimation will provide a misleading basis for recommendations to firms concerning existence of opportunities for input adjustments as well as for more global recommendations concerning the existence of generic opportunities for input adjustment.

Table 1. Summary of data for efficiency analysis  
(188 French cereal producers, 1990)

	Mean	Std-Dev	Min	Max
<b>Outputs</b>				
Cereal (F)	487172	303618	36392	1494657
Other products (F)	199014	141785	393	804672
<b>Quasi-fixed inputs</b>				
Cereal acreage (ha)	59.5	32.4	9.8	214.9
Other acreages (ha)	22.4	18.1	0.5	131.5
Labor (AWV)	1.3	0.5	0.8	5.0
<b>Variable inputs</b>				
Equipement (F)	113463	71827	12402	382560
Chemical fertilizes (F)	85209	47036	10025	304896
Pesticides	72543	51201	2258	278440
Seeds	37398	23731	1184	111519
Others (F)	30274	41321	1535	364914

Table 2. Technical efficiency frequency

	Hv		H1		H2		H3		H4	
	nb	%	nb	%	nb	%	nb	%	nb	%
=1	88	46.8	92	48.9	89	47.3	93	49.5	89	47.3
between 0.95 and 1	29	15.4	12	6.4	26	13.8	22	11.7	17	9.0
between 0.85 and 0.95	35	18.6	13	6.9	33	17.6	31	16.5	18	9.6
<0.85	36	19.1	71	37.8	40	21.3	42	22.3	64	34.0

Table 3. Technical efficiency measurement

Hypotheses	Mean	Std-Dev	Min	Max
Hv	0.9319	0.0969	0.6223	1
H1	0.8773	0.1552	0.5158	1
H2	0.9262	0.1072	0.6006	1
H3	0.9271	0.1047	0.6006	1
H4	0.8915	0.1386	0.5158	1

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