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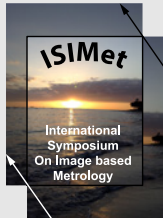
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Discrepancy measure for uniformity qualification of bubble movement in a direct contact heat exchanger

Qingtai Xiao^{1,2}, Jianxin Pan^{3*}, Junwei Huang¹, Jianxin Xu^{1,3}, Hua Wang^{1,2}



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Abstract

This presentation aims to introduce three novel methods for assessing the distribution uniformity of bubbles and determining their mixing time to uniformity. These uniform design based methods are illustrated through image analysis in a direct-contact heat exchanger (DCHE). The novelties include the little constraint of final evaluation value and the efficient algorithm of bubble centroid, leading to fast and accurate characterization of mixing uniformity. To determine the effects of resolution aspect (considering two types of aspect ratios: 16:9 and 4:3) and local region on the measurement, various evolutions of bubbles movement and three mixing measures in the DCHE are investigated experimentally. Real experiments and simulations are conducted, showing that the proposed methods outperform the existing methods such as Betti number. It is also shown that the location effects of bubbles can be measured successfully using these discrepancy measures, which brings a new insight into the comparison of mixing state of different systems.

Keywords

measure of uniformity – bubbles distribution – direct contact heat exchanger

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INTRODUCTION

The purpose of mixing is to obtain a homogeneous mixture or a certain degree of uniformity of mixtures in non-reactive systems [1, 2, 3]. Mixing uniformity has a decisive impact on the overall performance of mixing processes and can sometimes serve as a surrogate for other properties, such as the quality grade of print defects [4], the heat transfer performance of fluids [5, 6], the mechanical behavior of materials [7], the drug content of monolithic devices [8], and so on. There is an increased desire for measuring and comparing mixing uniformity which is required from a practical point of view and for validation of theoretical models as well in various fields [9, 10, 11].

In the literature, the image processing technology has been widely used for feature extraction and multiphase mixing quantification in single phase, gas-liquid, solid-liquid and gas-liquid-solid systems [12, 13, 14]. A large number of experimental studies have been also devoted to demonstrating its efficiency in mixing time estimation for configurable optimization [1, 2, 6, 8]. There is however no universally accepted image-based method for determination of mixing time mainly because each one has its own limitations, such as conductivity method, pH probe method, dual indicator system method [15], tracer concentration method, coloration decoloration method [16] and box-counting method [17], etc. Some limitations have been reported in details, e.g., in [5, 6, 18].

In earlier publications, Betti number was reported in [1] as one of the most efficient methods in determining criti-

cal mixing time of mixing process [19] and acquiring more spatial evolutions information of flow field associated with heat transfer performance of fluids [20]. However, we found that mixing time and uniformity determined by Betti number have an issue of spatio-temporal limitation. Particularly, the mixing uniformity of bubbles in a rectangle area with the same Betti number (β_0 when white pixels are the background or β_1 in other situations) is very different.

The issue of measuring the space-time uniformity of random bubbles in a recorded image was addressed by the uniformity coefficient (UC) method [3] and modified UC method [21] in certain extents. But in the definition of reported UC methods, it is reasonable that the final evaluation value should be between 0 and 1. Otherwise, the definition may be not sensible. Fang et al. pointed out that one should choose a set of experimental points with smallest discrepancy among all possible designs of a given number of factors and experimental runs [22], which satisfies the limit condition. On the other hand, the finding procedure of bubble centroid can be made efficiently using the Image Processing Toolbox (IPT) in Matlab and its in-house function, which performs better in computational complexity than traversal operation of elements by [3] and [21].

Inspired and motivated by the previous work, we aim to investigate the space-time feature by the definition of discrepancy directly and present an analysis of numerical simulations and experiments of the mixing process. Discrepancy concept and thresholding algorithm are applied to quantify the space-time mixing uniformity of random bubbles in a

direct contact heat exchanger (DCHE) in this paper.

1. METHODS

Suppose there are n bubbles of interest (i.e., black or white elements) over a binary image $C^2 = [1, P] \times [1, Q]$ (i.e., a $P \times Q$ matrix mathematically). The goal here is to assess whether or not the bubbles are uniformly scattered on the processed image. Let $F_u(\mathbf{x}) = x_1 x_2$ be the uniform distribution function with a given point $\mathbf{x} = (x_1, x_2)$ where $1 \leq x_1 \leq P$ and $1 \leq x_2 \leq Q$, and $F_{\mathcal{X}}(\mathbf{x})$ be the empirical distribution function of bubbles set $\mathcal{X} = (\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_n^T)^T$, i.e.,

$$F_{\mathcal{X}}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^n I_{[x_i, \infty)}(\mathbf{x}) = \frac{\sharp(\mathcal{X} \cap [\mathbf{1}, \mathbf{x}])}{n} \quad (1)$$

where $I_{[x_i, \infty)}(\mathbf{x})$ is the indicator function and $\sharp(\mathcal{X} \cap [\mathbf{1}, \mathbf{x}])$ denotes the number of bubbles of \mathcal{X} falling into the local region $[\mathbf{1}, \mathbf{x}] = [1, x_1] \times [1, x_2]$. The L_∞ -star discrepancy of \mathcal{X} on C^2 (star discrepancy for short) is defined as

$$D_\infty^*(\mathcal{X}) = \|F_{\mathcal{X}}(\mathbf{x}) - F_u(\mathbf{x})\|_\infty = \sup_{\mathbf{x} \in C^2} |F_{\mathcal{X}}(\mathbf{x}) - F_u(\mathbf{x})| \quad (2)$$

where \mathbf{x} decides the area and controls the number of the given points within the area (i.e., the number of local regions), and sup denotes the superior/maximum of the local discrepancy functions (LDFs).

Statistically, the centered discrepancy (CD) and the wrap-around discrepancy (WD) exhibit some advantages such as permutation invariance, rotation invariance (reflection invariance) and the ability to measure projection uniformity, but star discrepancy does not. Assume that x_{i1}, x_{k1} are the x -axis values and x_{i2}, x_{k2} are the y -axis values of i th, k th bubbles, respectively, the analytical expressions for CD and WD are given respectively as follows:

$$\begin{aligned} \text{CD}(\mathcal{X}) = & \left\{ \left(\frac{13}{12} \right)^2 - \frac{2}{n} \sum_{i=1}^n \prod_{j=1}^2 \left(1 + \frac{1}{2} |x_{ij} - \frac{1}{2}| \right. \right. \\ & - \left. \frac{1}{2} |x_{ij} - \frac{1}{2}|^2 \right) + \frac{1}{n^2} \sum_{i=1}^n \sum_{k=1}^n \prod_{j=1}^2 \left(1 + \frac{1}{2} |x_{ij} - \frac{1}{2}| + \frac{1}{2} |x_{kj} - \frac{1}{2}| \right. \\ & \left. \left. - \frac{1}{2} |x_{ij} - x_{kj}| \right) \right\}^{\frac{1}{2}} \end{aligned} \quad (3)$$

and

$$\begin{aligned} \text{WD}(\mathcal{X}) = & \left\{ - \left(\frac{4}{3} \right)^2 + \frac{1}{n} \left(\frac{3}{2} \right)^2 + \frac{2}{n^2} \sum_{i=1}^{n-1} \sum_{k=i+1}^n \right. \\ & \left. \prod_{j=1}^2 \left(\frac{3}{2} - |x_{ij} - x_{kj}| + |x_{ij} - x_{kj}|^2 \right) \right\}^{\frac{1}{2}} \end{aligned} \quad (4)$$

Obviously, the greater the value of the discrepancy, the more non-uniform the bubbles in a given region. In order to

measure the effects of robustness and assess the performance of three measures, experiments are carried out in a DCHE. As an effective way to use energy, DCHE has been applied and researched extensively in energy recovery from industrial waste. It can transfer the heat of continuous phase (i.e., heat conduction oil) to dispersed phase (i.e., organic fluid), which can be influenced by four factors with three levels. Let E_1 - E_9 denote nine different experimental levels based on orthogonal design table $L_9(3^4)$.

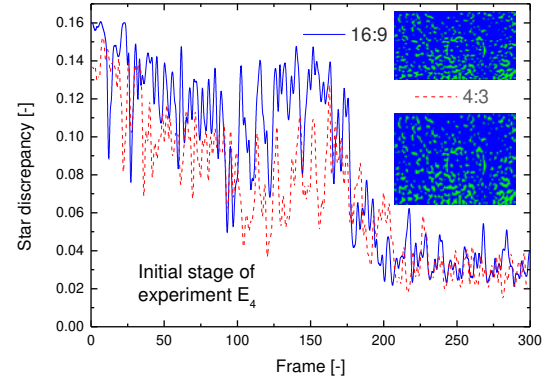


Figure 1. Effect of different resolution on star discrepancy

The images of bubbles are captured using a high-speed video camera with brand PRAKTICA of Germany and resolution 4 million pixels with no LED light. Generally speaking, the objective is bubble (i.e., gas phase) and background represents the liquid phase. For the convenience of engineering calculation and programming, we set $P=1280$ and $Q=720$ while adopting equal interval sampling of 300 which corresponds to 5 min. A processed bubbles image is selected randomly for representing the binary mixture, as shown in the inserted illustration of Fig. 1

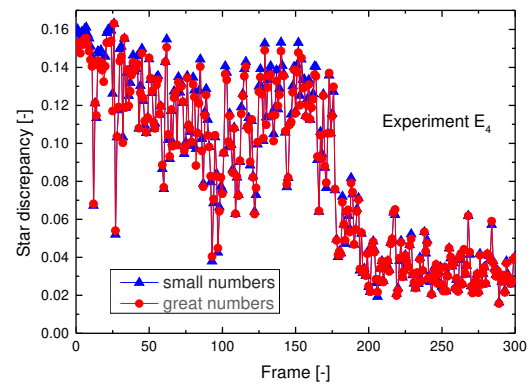


Figure 2. Effect of different local regions on $D_\infty^*(\mathcal{X})$

The novelty of the presented contribution are two-fold: using the mathematical measure of discrepancy and applying

the IPT function `regionprops` directly, leading to low computational cost and high degree of accuracy. Thus, the effects of resolutions aspects and numbers of local regions on the definitions of three discrepancies in this paper are all investigated. To save space, Fig. 1 only gives the $D_{\infty}^*(\mathcal{X})$ trends over time when the pixels size reduces from 16:9 to 4:3. The reason why the results are influenced by the aspect ratio of the image is that the LDFs ratio relies on the area ratio of images with different sizes. In terms of the mixing time determination, it shows that the influence of homogenization curve by our method does not change a lot with the resolution size. Likewise, $CD(\mathcal{X})$ and $WD(\mathcal{X})$ are both dependent on the coordinate axes established for the bubbles image. As magnification factor (px/mm) is still similar, the aspect ratio does not have an influence on the results if adequate equipment is used. As shown in Fig. 2, there is subtle influence of number of local regions on the trend of uniformity curve.

2. RESULTS AND DISCUSSION

2.1 Quantification of mixing state

As shown in Fig. 3, quantitative comparisons of the uniformity curves determined by the proposed techniques are conducted with our previous experimental data. Specially, here there is a clear and distinctive different at the early stage of mixing process of experimental cases. Obviously, any difference in numerical performance has a deep significant role in quantifying the mixing state.

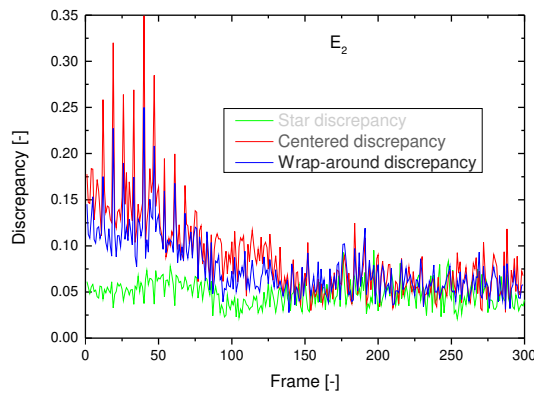


Figure 3. Comparison of three discrepancies curve for E_2 .

The results show that good agreements of the critical point obtained by existing and proposed methods are made, see Table 1. It is observed that the effectiveness of those techniques are valid. However, the Betti number method fails to take into account the location effects of bubbles with respect to mixing transient.

2.2 Account for the location effects

It may not be correct to conclude that the best transients are that present the highest Betti number for special cases,

Table 1. Mixing time obtained by different methods.

methods	β_1	$D_{\infty}^*(\mathcal{X})$	$CD(\mathcal{X})$	$WD(\mathcal{X})$
E_1	156	151	151	151
E_2	93	97	95	96
E_3	168	172	170	170
E_4	225	224	225	224
E_5	122	120	120	120
E_6	84	84	82	83
E_7	262	259	195	198
E_8	117	118	118	118
E_9	128	127	127	127

as the purely numerical approach does not take position details into account. In practical applications, one can more accurately calculate the value of discrepancy for choosing the best experimental condition for space-time feature of bubbles, which is the main advantage of our methods.

In order to verify the feasibility of our methods, two processed images which both present 194 bubbles are taken from an integrated experiment, as shown in Fig. 4 (a) and (b). The right-hand plot in Fig. 4 (c) presents the difference of mixing uniformity in the special transient with the same Betti number $\beta_1=194$. Comparisons show that different experimental cases with the same Betti numbers can be detected by the discrepancy measure. The combined results in Fig. 4 indicate that the proposed evaluations may be enough to obtain a good determination and quantification of mixing state of gas-liquid two-phase flow.

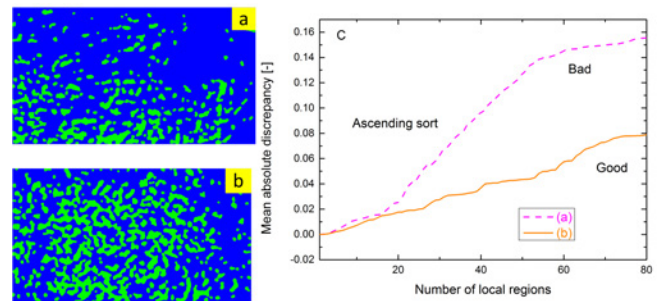


Figure 4. Mean absolute discrepancy evolutions of different experimental images with the same $\beta_1=194$.

In addition, the movement rules of bubbles in DCHE are also discussed by the method of numerical simulation. According to Fig. 5, the green regions in the three inserted images refer to the bubbles in DCHE. In particular, three sets were randomly generated and fixed with $\beta_1 = 234$ by means of the Matlab software. Just like usual, each image has the dimension of 1280×720. Fig. 5 also depicts that the star discrepancy would be influenced by the initial position for calculating local discrepancy function, including top-left (TL), bottom-left (BL), bottom-right (BR) and top-right (TR), but CD and WD do not. It is also interesting to point out that when one wants to assess the mixing uniformity more accurately, the CD may outperform the WD and is more

sensitive to practical engineering application in some sense.

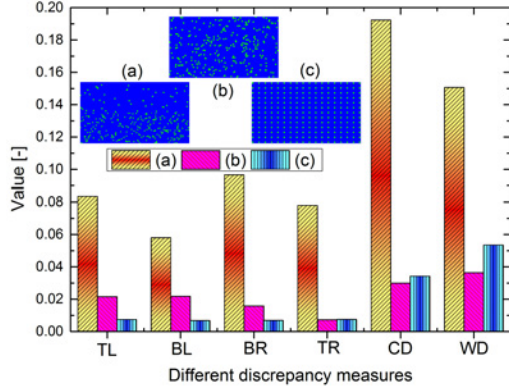


Figure 5. Different discrepancy measures for quantifying the particle mixing state in three different synthetic images with the same $\beta_1=234$.

2.3 Verification of three properties

Assume centroid coordinates of bubbles are written as,

$$\mathcal{X} = \begin{pmatrix} x_{11} & x_{12} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{pmatrix} \quad (5)$$

where n is the number of bubbles in an image processed by classical thresholding algorithm (for example, Otsu method) mentioned earlier. As mentioned early, the first column elements $x_{11}, x_{21}, \dots, x_{n1}$ of \mathcal{X} correspond to the x -axis values and the second column elements $x_{12}, x_{22}, \dots, x_{n2}$ of \mathcal{X} correspond to the y -axis values. Let $CD(\mathcal{X})$ and $WD(\mathcal{X})$ denote the two different measures of uniformity individually.

Theoretically, $CD(\mathcal{X})$ and $WD(\mathcal{X})$ have their own advantages. To begin with, they are both invariant to disrupted order of the experimental points. Second, they are invariant if x_{i1} and x_{i2} are replaced individually by $1-x_{i1}$ and $1-x_{i2}$, $1 \leq i \leq n$. Third, the projection uniformity over all sub dimensions is taken into account, and $CD(\mathcal{X})$ and $WD(\mathcal{X})$ are both invariant on the subspaces. The resulting data verification are depicted in Table 2, 3, and 4, respectively. Hence, we conclude that the different experimental patterns with the same Betti numbers can be identified by the proposed discrepancy measures. Interestingly, those provides a tool to investigate the difference of mixing state quantification between 2-dimensional area and 3-dimensional spaces.

3. CONCLUSIONS

In this paper, the mixing state of gas-liquid two-phase flow in a DCHE for waste-heat utilization is quantified through three novel statistical measures with reference to discrepancy. The proposed methods do not rely on the perceived uniformity (resulting from relaxing the range constraint of

final evaluation values, and the finding algorithm of bubble centroid based on [3] and [21]. Summarily, three assertions for this narrating work are list as follows.

(1) With respect to the star discrepancy, the local discrepancy function of a set of bubbles seems to be a useful concept to measure the irregularity of the distribution of bubbles within a rectangle region. The scheme is based on partitioning of image data obtained from a high-speed video camera, thus leading to a new consideration in the interpretation of flow visualization.

(2) Using the three image analysis techniques (star discrepancy, CD and WD) processed in the ImageJ or Matlab software, the evolution of bubbles movement is tracked experimentally. The influences of expressions, iteration steps and pixels on the mixing efficiency are also discussed. Compared to [3] and [21], the proposed approaches are easy to implement and are computationally low cost.

(3) The CD and WD are more reasonable than the star discrepancy in terms of mixing state quantification in this current study. That is to say, the former two could satisfy the three properties including invariance to permutation, invariance under reflection, and projection uniformity, but the later one does not.

Based on the aforementioned observations, it is believed that the proposed approaches can be applied to measure mixing uniformity with a good accuracy in pharmaceutical, chemical, metallurgical, printing and medical industries, etc.

Table 2. Verification of invariance to permutation

Discrepancy	original order	disordered		
		bubble	coordinates	both
$CD(\mathcal{X})$	0.0249	0.0249	0.0249	0.0249
$WD(\mathcal{X})$	0.0277	0.0277	0.0277	0.0277

Table 3. Verification of invariance under reflection

Discrepancy	no reflected	reflected		
		$x = \frac{1}{2}$	$y = \frac{1}{2}$	both
$CD(\mathcal{X})$	0.0249	0.0249	0.0249	0.0249
$WD(\mathcal{X})$	0.0277	0.0277	0.0277	0.0277

Table 4. Verification of projection uniformity

Discrepancy	no projected	$y=0$	$x=0$	projected
				to origin
$CD(\mathcal{X})$	0.0249	0.6025	0.6009	0.9446
$WD(\mathcal{X})$	0.0277	0.4720	0.4717	0.6812

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