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# A model of the instrumentation process in dynamic geometry

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*Abstract.* The present study investigates students' instrumentation processes while solving open geometry problems in a dynamic geometry environment. It aims at developing an analytical and comprehensive model of students' conceptual schemes and strategies built around the use of the tools available in DGE. The study was conducted in a grade-10 math class in a Lebanese school. Data were collected through whole-class observation with analysis of paper-based data and closer observation of 12 pairs of students. The analysis focused on the instrumentation process that occurred during the construction and manipulation of geometric figures. The results suggest the development of a three-phase model of the instrumentation process.

*Keywords:* Instrumental genesis, instrumentation process, dynamic geometry, grade 10.

## Introduction

For the past few decades, research has investigated students' interactions with digital curriculum materials, analyzing the nature of tasks employed and the ways students act and react in such environments. The Instrumental Approach, elaborated by Verillon and Rabardel (1995) provides an adequate context to analyze such interactions. They made an important distinction between an artifact and an instrument. An instrument does not exist by itself. An artefact, which is the physical object, becomes an instrument when the user appropriates pre-existing schemes, or constructs personal schemes and integrates the artifact within his/her activity. Beguin and Rabardel (2000) and Trouche (2004) defined the concept of instrumental genesis to encompass two dimensions that contribute to the construction and advancement of an instrument: (1) instrumentation: it is subject-oriented and it entails the artifact printing its mark on the subject, i.e. it allows him/her to construct and enhance operative utilization schemes. (2) Instrumentalization, which concerns the development of the artifact component of the instrument.

Many researchers (Alqahtani & Powell, 2016; Artigue & Mariotti, 2014; Baccaglioni-Frank & Mariotti, 2010) have used the Instrumental Approach to analyze the use of DGE tools in learning situations. According to Artigue and Mariotti (2014, pp.338) "representations play a fundamental role in the "generation" of mathematical meanings, and this role is assumed to be crucial in the teaching/learning of mathematics. Digital artifacts can provide representations of mathematical objects with a clear potential of generating mathematical meanings." These shared theories concerning artifacts and mathematical representations highlight the essential role that digital tools play in learning mathematics and hence the importance of understanding the way students instrument these tools in a DGE to create geometrical meanings. Alqahtani and Powell (2016) and Baccaglioni-Frank and Mariotti (2010) studied the instrumentation process of the dragging tool. The analysis showed that the appropriation and application of the dragging tool started with random dragging and then looking at the characteristics of the objects being dragged in order to understand properties of dynamic constructions. The retroactions of the DGE that occurred while dragging different objects helped in identifying and understanding geometrical relationships such as

invariants and dependencies in the figure. While appropriating the use of the dragging tool, the construction of the dynamic figures came last.

Given that in the literature the primary focus in the process of instrumental genesis has been on the role of DGE tools and their impact on students' learning, this study, focuses on the instrumentation process, and aims at developing an analytical and comprehensive model of students' conceptual schemes and strategies built around the use of the tools available in DGE by answering the following question: "How do grade 10 students develop and structure their conceptual schemes and strategies while constructing a geometric figure using DGE tools?" The study consisted of a series of observations of 12 pairs of 10 graders (15-17 years old) who worked on open geometrical proof problems within Geogebra. Data were collected using video-recording and collection of materials, namely any paper trace (sketches, scribbles, conjecture or proof formulations) generated by students, together with their GeoGebra files. This paper presents students' work on two problems previously used by Olivero (2002) and Arzarello et. al. (2002):

*Problem 1 (Olivero, 2002)*

Let ABCD be a quadrilateral. Consider the bisectors of its internal angles and the intersection points H, K, L, and M of pairs of consecutive angle bisectors.

- 1) Drag ABCD, considering different configurations, and explore how HKLM changes in relation to ABCD.
- 2) Write down conjectures and prove them.

*Problem 2 (Arzarello et. al., 2002)*

Given a triangle ABC, consider P the midpoint of [AB] and the two triangles APC and PCB.

- 1) Explore the properties of the triangle ABC which are necessary so that both APC and PCB are isosceles (in this case, the triangle ABC is called "separable").
- 2) Write down conjectures and prove them.

The chosen problems: 1) are in accordance with the definitions of open problems; 2) are problems whose nature is not changed by the DGE, which simply acts as a visual amplifier facilitating the mathematical task; 3) lie in a mathematical conceptual domain familiar to the students, their difficulty does not reside in the understanding of the wording and meaning of the given of the problem; 4) were selected from other studies that analyzed students' work in a DGE.

### **Analysis of the Instrumentation Process**

The observations and analyses showed that, through their interaction with the tool, students gained new geometrical knowledge indigenous to the DGE. These gains were built in a continuous process of exerting intentional actions, processing feedback and adapting techniques. When students employed a certain tool among those offered by the DGE for the first time, or when they employed a tool known to them in a novel context, they processed the feedback it provided and adapted their techniques accordingly. The analysis of the 12 cases helped us identify three phases through which instrumentation and resulting geometric knowledge indigenous to DGE were developed. We

illustrate these three phases using the following examples, chosen among the 12 cases as they are the most representative examples:

### Example 1

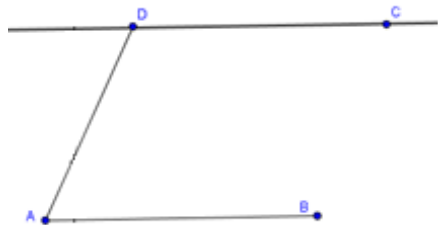
While working on problem 1, a pair of students started their work by constructing two segments  $[AB]$  and  $[AD]$  and selected the *Parallel Line* tool to complete the parallelogram ABCD.

Phase 1: The students clicked on D, the point through which the parallel line to  $(AB)$  should be constructed, and waited, expecting the parallel line to appear. They were not aware of the fact that the construction needed another input, i.e. the line that determines the direction of the parallel line.

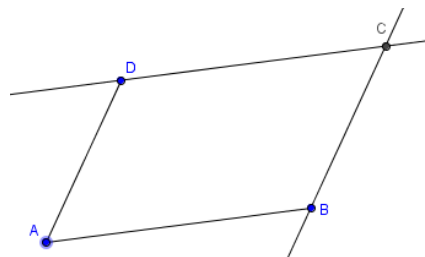
It is observed that the students were not familiar with the type and nature of inputs required by the *Parallel Line* tool. Their choice of input, namely point D, was purely experimental, based on the way they would construct a parallel line through D in the paper-and-pencil environment, i.e. placing the ruler to pass through D in a parallel direction to  $(AB)$ , without explicitly invoking  $(AB)$ .

Phase 2: They refined their input by incorporating the line to which the parallel line should be constructed but they started by clicking on  $(AB)$ , thus the line appeared confounded with  $[AB]$ . They still needed to select the point through which the parallel line should pass. They dragged the line and adjusted it visually till it passed by D and constructed a point C, through which the parallel line passes, by placing it as the fourth vertex of the parallelogram ABCD based on perceptual approximation (Figure 1). Although the parallel line appeared to be constructed in the correct position, but the figure did not hold under dragging since it was based on visual approximation. Therefore, they proceeded by constructing a parallel line to  $[AB]$  through D, a parallel line to  $[AD]$  through B and constructed C as the intersection point of these two lines (Figure 2). In this phase, the students adapted their use of the *Parallel Line* tool to the feedback provided by the tool itself. This adaptation was based on progressively gained knowledge and understanding of the nature of the input requirements for the parallel line to be constructed, rather than on speculations as they did previously.

Phase 3: By the end of the process, the students learned the two defining elements that determine a line parallel to another through a point. They understood the technical functioning of the *Parallel Line* tool. They succeeded in internalizing the utilization scheme of the *Parallel Line* tool since the input was refined based on understanding; their new knowledge was validated for them by the fact that the figure held under dragging.



**Figure 1: Adapting the use of the *Parallel Line* tool based on the tool's feedback**



**Figure 2: Using the *Parallel Line* tool based on conceptual understanding**

## Example 2

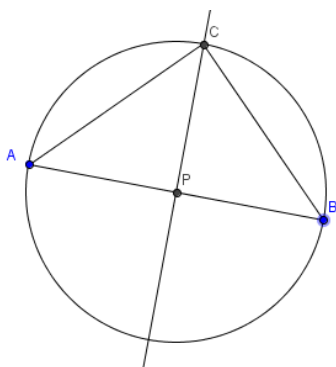
In problem 2, based on a paper sketch, a student hypothesized that ABC is separable when it is right isosceles. They decided to test her hypothesis in DGE.

Phase 1: They constructed a right isosceles triangle ABC by taking C as the intersection point of the perpendicular bisector of [AB] and the circle centered at P, the midpoint of [AB], with radius [PB] (Figure 3). They selected the dragging tool and attempted to drag C on the circle. We note that the choice of the right isosceles condition and the attempt to drag C on the circle were both experimental processes that needed to be refined.

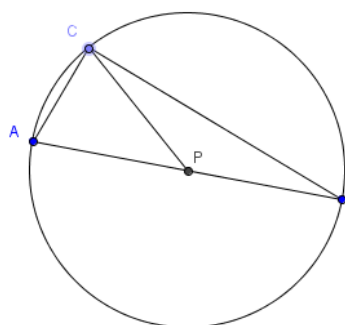
Phase 2: When C could not be dragged on the circle, the students discussed and analyzed what they observed and deduced that, since C is an intersection point, it could not be dragged on the circle. This novel understanding led them to adapting their construction by considering C as a point on the circle, without belonging to the perpendicular bisector of [AB] as well (Figure 4). They dragged C on the circle and observed that ABC is separable when it is a right triangle.

Phase 3: The students were able to develop novel understanding based on the interaction with the dragging tool:

- They understood that intersection points could not be dragged as they are dependent on other objects. This concept is indigenous to the DGE since in the paper-and-pencil environment students are not trained to distinguish dependent from independent points.
- The students were also able to understand that ABC right triangle at C is a sufficient condition for the triangle to be separable; it does not need to be isosceles as well.



**Figure 3: Constructing C based on experimental choices**



**Figure 4: Refining the construction of C based on conceptual understanding**

## Example 3

A pair of students who worked on problem 2, attempted to measure  $\widehat{ABC}$  using the *Angle* tool.

Phase 1: They selected only the vertex of the angle, which does not produce the desired result and keeps the DGE waiting for more input. Then, they selected the three points of the angle but in random order, which, in this case, resulted in displaying the measure of the outer angle.

Phase 2: To solve this problem and find the measure of the inner angle the students read the measure of the outer angle and subtracted it from  $360^\circ$  to find the measure of the inner angle. Here,

the students did not rely totally on the tool, neither did they try to adapt their use of the tool. Instead they solved the problem at hand using a previously acquired geometrical property. They did not attempt to understand the behavior of the *Angle* tool and to adapt their work accordingly.

Phase 3: The students were not able to develop conceptual knowledge indigenous to DGE; although prior geometrical knowledge was used to reach the result that the DGE failed in providing, that knowledge was not novel and was not indigenous to the DGE neither was it invested to build a new knowledge indigenous to the DGE.

### **The three-phase model of the instrumentation process**

From these examples we can identify a common structure for the instrumentation process of DGE tools. Each exploration in each example went through three phases: a phase of *heuristic application* of the tool, followed by a phase of *contextualized adaptation* leading to the third phase of *conceptual understanding*. These phases suggest the development of a three-phase model of the instrumentation process defined as follows:

Phase 1– *Heuristic Application*: The students choose a certain tool, and then select the object(s) they believe to fit the required nature and order of input. The choice of input is experimental based on students' experiences in a paper-and-pencil environment and on assumptions and speculations rather than formal knowledge.

Phase 2 – *Contextualized Adaptation*: After *heuristic application* of the tool, students build on the feedback it provides to adapt their strategies to the situation at hand. This adaptation can be based either on geometrical understanding (i.e. epistemic adaptation) or on practical considerations (i.e. pragmatic adaptation).

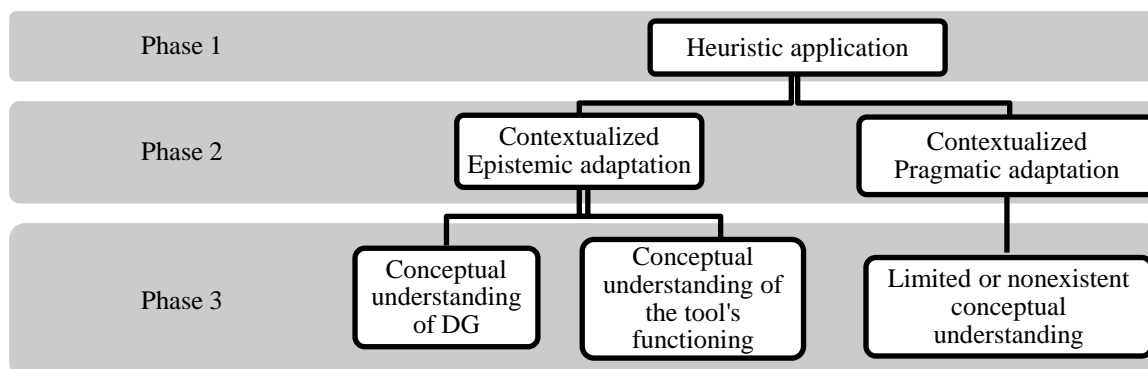
- Epistemic adaptation: Heuristic application of the tool evolves into epistemic adaptation as students benefit from the feedback provided by the tool to improve their strategies and construct new meanings. The utilization scheme of the tool is formalized by the mediation of the heuristic experiments carried out in the first phase.
- Pragmatic adaptation: Students manage to accomplish the given task but in a way that is based on practical considerations; the heuristic experiences do not evolve into mathematically grounded application of the tool. In this case, the construction of geometrical knowledge indigenous to the DGE, i.e. phase 3, is limited or even non-existent.

Phase 3 – *Conceptual Understanding*: The epistemic adaptation generates new understanding imprinted on the student by the tool. This understanding is not totally gained in phase 3; instead it is gradually constructed as of the first phase. The conceptual understanding is relevant to:

- The functioning of the tool: Students construct an understanding of the behavior of the tool, i.e. the type of input required, the context in which the input applies and the outcome that the tool provides.
- Dynamic geometry i.e. geometrical knowledge indigenous to the DGE: Students construct first-hand understanding of the geometry underlying the tool, which may be, in some cases, fundamentally different from the geometry learned in the paper-and-pencil environment.

Through these three phases, students gradually conceptualize a new type of geometry, the dynamic geometry.

The following diagram (Figure 5) illustrates the three-phase model of the instrumentation process.



**Figure 5: The three-phase model of the instrumentation process**

## Discussion and Conclusion

According to Beguin and Rabardel (2000), the instrumentation process, which is learner-oriented, entails the artifact printing its mark on the learner; it thus leads to the construction and development of utilization schemes. Therefore the three-phase model of the instrumentation process can be perceived as a more formalized utilization scheme for DGE tools. Beguin and Rabardel define these utilization schemes as contingent upon the meanings attributed by the individual to a situation; this subjective involvement of the students, while using technology, explains why technology does not have the same power and potential for all students. In fact, in phase 2 of the structure, the contextualized adaptation is contingent upon the meaning that students give to the situation; this is why some students are able to develop a *contextualized epistemic adaptation* and exploit the potentialities of the technological environment, whereas others can only develop a *contextualized pragmatic adaptation* limiting the potential of the DGE. Beguin and Rabardel add that utilization schemes are active structures as they are adapted to an expanding range of situations and their construction necessitates the intelligent use of both conceptual and technical knowledge. This idea is illustrated in phase 3 of the structure presented above, where students adapt their use of the tool to different situations and gain conceptual and technical knowledge of dynamic geometry.

We now compare this structure to a model developed by Leung (2011, 2017). The purpose of this techno-pedagogic task design model is to serve as a heuristic to design mathematical teaching and learning tasks in a technological environment. It is useful to compare these two structures in order to verify whether the way a task should be designed in DGE (i.e. according to Leung's model) is in fact consistent with the way students solve a task in DGE (i.e. according to the common structure of instrumentation processes developed in this study). These two structures are compared and contrasted in Table 0.1.

*A comparative summary of the common structure of instrumentation processes identified in the current study and the techno-pedagogic task design model*

	<b>The common structure of instrumentation processes (identified in this study)</b>	<b>The techno-pedagogic task design model (Leung, 2011, 2017)</b>
Phase 1	<i>Step 1– Heuristic application:</i> The choice of input for a specific tool is based on students’ personal understanding of what is required to define a geometric object and of what is necessary in terms of the nature and order of input.	<i>Establishing Practices Mode (PM)</i> PM1 Construct and manipulate mathematical objects using tools embedded in a technology-rich environment PM2 Interact with the tools in a technology-rich environment to develop (a) skill-based routines; (b) modalities of behavior; (c) modes of situated dialogue.
Phase 2	<i>Step 2 – Contextualized adaptation:</i> Epistemic adaptation or pragmatic adaptation.	<i>Critical Discernment Mode (CDM)</i> Observe, record, re-present (re-construct) patterns of variation and invariant.
Phase 3	<i>Step 3 – Conceptual understanding:</i> The <i>epistemic adaptation</i> generates new understanding of geometrical concepts indigenous to the DGE and/or of the technical functioning of the tool.	<i>Establishing Situated Discourses Mode (SDM)</i> SD1 Develop inductive reasoning leading to making a generalized conjecture. SD2 Develop discourses and modes of reasoning to explain or prove.

In phase 1 of both structures, students practice the use of the tool and develop modes of interaction between them and the tool. Thus the process of instrumentation begins where the student acts on the tool and the tool shapes the student’s knowledge thus creating an artefactual and psychological instrument.

In phase 2 of both structures, namely *contextualized epistemic adaptation* and *CDM*, the process of instrumentation continues; the students shift their attention from establishing routine usage of the tool to constructing mathematical meaning. A process of internalization takes place where technical tools are transformed into psychological tools for the purpose of shaping new meanings. However, the common structure of instrumentation processes includes a step, namely *pragmatic adaptation*, which takes into consideration the case where students fail in transforming the tool into a psychological/cognitive one and do not construct new meaning.

Phase 3 of both structures serves as a connection to the theoretical field. In the techno-pedagogic task design model the theoretical field consists of deductive and inductive reasoning, which leads to developing conjectures and proofs. In the common structure of instrumentation processes, the theoretical field consists of dynamic geometry concepts or technical functioning of DGE tools, which also reflects geometrical understanding relative to DGE.

Based on the results of this study, we make the following recommendations for future research:

- Investigate whether the conceptual understanding of geometrical knowledge indigenous to the DGE developed by the students can be detached from the DGE and become formal geometrical knowledge used in other environments.
- Explore whether the three-phase model of the instrumentation process can be seen as a cycle where the newly gained conceptual understandings in phase 3 can be used later in further explorations at the first phase of heuristic application and pass again through the same phases to reach new conceptual understandings, and so forth; thus suggesting a three-phase instrumentation cycle.

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