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Measuring area on the geoboard focusing on using flexible strategies

Øyvind Jacobsen Bjørkås¹ and Marja van den Heuvel-Panhuizen^{1,2,3}

¹Nord University, Norway; oyvind.j.bjorkas@nord.no

²Freudenthal Institute, FBW, Utrecht University, the Netherlands

³Freudenthal Group, FSW, Utrecht University, the Netherlands; m.vandenheuvel-panhuizen@uu.nl

In this paper we report about a small-scale classroom experiment on teaching area measurement with a digital geoboard. The two-lesson teaching sequence that was developed, was tested in two sixth-grade classes and was aimed at teaching students using measurement strategies in a flexible way. The pre- and posttest data were analysed both on task and student level. The results show that most of the students developed more flexible strategies, but many of them did not use them accurately. Our results indicate that apart from working on the ability of students to find correct answers, also the flexible use of strategies can be realized through education.

Keywords: Measurement, area, digital geoboard, strategies.

Introduction

Students often say that area “is length times width”. This suggest both that the conceptual understanding of area is lacking, and that they only have one strategy available for finding the area of figures, even if they have learned about finding area by counting units earlier: “It [...] is typical of many learning processes, especially in mathematics – that the original sources of insight have been clogged, and the way back to insight is blocked by the processes of algorithmising and automatising.” (Freudenthal, 1983, p. 209). In the new curriculum for Norwegian primary school, in-depth learning is important, and among other things the students are expected do “construct a robust and flexible understanding” (Ministry of Education and Research, 2015). Therefore, it is important to develop teaching that supports the learning of flexible strategies for area measurement.

Theoretical framework and research question

Knowledge about measuring the area of polygons consists of both conceptual and procedural knowledge which are mutually supportive and should both be fostered (Rittle-Johnson, Schneider, & Star, 2015). Having a good understanding of measuring area means that students have several strategies available to determine the area of polygons. For example, they may count square units and/or half square units, make a multiplication to find the area of rectangles, decompose polygons into smaller, simpler polygons and use addition, may embed a polygon into a larger, simpler polygon and use subtraction, and may be able to apply a formula to calculate the area. Essential is that students can apply these strategies in a flexible way fitting to the problems at hand. However, there is evidence (Huang & Witz, 2011) that primary school students fail often in area measurement problems and have difficulties in flexibly handling area measurement problems. According to Huang and Witz (2011) this finding is considered to be linked to the emphasis that in teaching is put on formula memorization rather than on conceptual understanding and explaining your reasoning. Further, students who develop flexibility in problem solving use or adapt existing strategies to unfamiliar problems, and show a greater understanding of concepts (Star, Rittle-Johnson, Lynch, &

Perova, 2009). Therefore, it is important to investigate how students' flexible use of strategies in measuring area and their awareness of using them can be supported.

Most of the research on strategy flexibility or strategy adaptivity has been on mental calculation (e.g., Threlfall, 2009; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) or on solving non-routine word problems (Elia, van den Heuvel-Panhuizen, & Kolovou, 2009). In the study of Verschaffel et al. (2009) a distinction is made between task, subject, and context adaptivity of strategies. In the present study we built on this research and explored how task flexibility of strategies for finding the area of polygons can be taught. We will not use the term "adaptivity" in this paper because although we will discuss appropriateness of strategies with the students, the main goal is to teach multiple strategies and discuss the relationship between them. Further, we will focus on inter-task flexibility in the tests (Elia et al., 2009), investigating whether students change their strategies from pre- to posttest.

One tool that is sometimes used for teaching measurement of area is the geoboard. However, there are only a limited number of empirical studies on this topic. For example, Bair and Cady (2014) observed that by means of a geoboard students could find Pick's theorem and the study of Britton and Stump (2001) showed that students could develop cut-and-paste strategies. To our knowledge, no research has been carried out to investigate whether the geoboard is suited to foster the development of flexible strategies. Therefore in this study our research question was: *Does working with the geoboard help students to use flexible strategies for measuring area?*

Method

To answer the research question, a small-scale classroom experiment comprising a pre-post-retention-test research design was carried in two sixth-grade classes with 46 students during the period of February-July 2017. For this experiment we designed and tested a teaching sequence of two 75-minute lessons on reasoning about area measurement based on working with a digital geoboard.

The digital geoboard ("Geoboard [Computer software]," 2017) is an app for a tablet or computer that resembles the physical geoboard. Students can manipulate polygons and write computations or explanations on the digital geoboard, and they can share their work with the teacher through a sharing platform ("Showbie [Computer software]," 2018) or show it on a projector in the classroom. This makes it possible to select and present students' strategies to emphasize the connections between different strategies and give value to different strategies for the same task.

The lessons were taught by the researcher in the presence of the classroom teacher. In the lessons whole-class discussions led by the researcher were interchanged with students' individual work on the geoboard. In the discussions the use of different measurement strategies was stimulated, both strategies proposed by the students and strategies planned by the researcher, and students were made aware of the relationships between the different strategies. To support these relationships, the strategies were represented visually by drawings on the geoboard. To encourage students to think about their strategies it was emphasized that it is very important to be aware of the used strategy and that good understanding means that one can explain how the answer was found. This

encouragement was given to the students both during their individual work and the whole-class discussions.

Type of task	Description	Task included in		
		Pretest	Posttest	Retention test
Blue task	Rectangle with sides parallel to the axes of the geoboard	✓	✓	✓
Red task	Rectangle with sides 45° to the axes of the geoboard	✓	✓	✓
Brown task	Polygon in which partitioning into triangles and squares can be used	✓	✓	✓

Table 1: Overview of the tasks in the tests which are used in the paper

The pre-, post-, and retention tests were administered in Week 5, 7 and 22 respectively. They contained several types of measurement tasks presented on paper-and-pencil test sheets. Table 1 shows which type of task is included in which test. In every test, for each type of task the to be measured area was varied a bit in order to avoid retest effect.

Figure 1 shows an example of how the tasks in the retention test were presented to the students. The question asked to the students was: «Can you find the area of the shape? Write how you were thinking. You may use the picture to explain.». The yellow square was given as an area unit. On a next page the students were asked to give a justification for their result. During the pretest it turned out that after giving this prompt it was for most students easy to *say* what their thinking was, but writing this down was harder for them. Therefore, the teacher and the researcher made clear to the student that they just had to write down what they said orally.

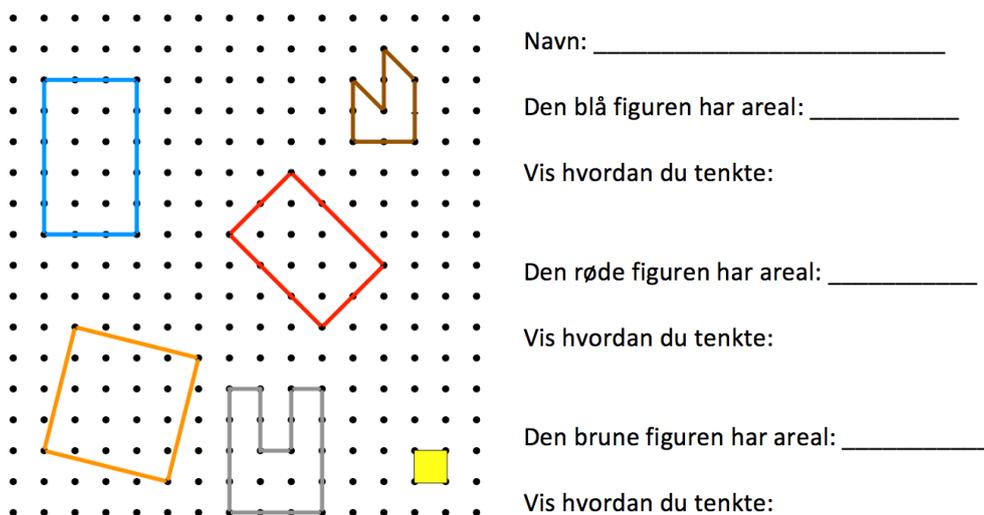


Figure 1: Test sheet with the polygons and the questions the students have to answer

The data from the test sheets were entered into NVivo. Both the students' measurement results and their explanations of the used strategies (see for examples Figure 2 and 3) were coded. When the students found the correct area, the answer was coded as "1" and an incorrect answer was coded as "0". Based on these codes we divided the students per task into four classes: the "00" students (incorrect answer in pretest and posttest), the "01" students (incorrect answer in pretest, correct answer in posttest), the "10" students (correct answer in pretest, incorrect answer in posttest), and the "11" students (correct answer in pretest and posttest). For coding the strategies, we used three main categories, namely "counting", "multiplication", and "restructuring" (decomposing or embedding strategies). Furthermore, we had two additional categories, namely "other" and "no strategy information available".

Results

Table 2 shows for each type of task and for each test the percentage of correct answers. For all the tasks which were both in the pre- and posttest, the students improved their performance of which the most gain was obtained for the Blue task. In the retention test this progress continued only for the Red and the Brown task.

Type of task	% correct		
	Pretest (<i>n</i> = 43)	Posttest (<i>n</i> = 41)	Retention test (<i>n</i> = 43)
Blue task	70%	95%	88%
Red task	44%	51%	58%
Brown task	72%	80%	88%

Table 2: Percentage of students who gave correct answers to the different types of tasks

Even if the overall performance on each task was increasing, individual students showed both progress and deteriorating performance on each individual task. Using the red task as an example: Of the 36 students that participated on both the pre- and posttests, 11 students got the answer wrong

on the pretest but right on the posttest, but 8 students got the answer right on the pretest but wrong on the posttest. Complete results are in table 3.

Type of task	Number of students belonging to a class of students			
	“00” student	“01” student	“10” student	“11” student
Blue task	0	8	2	26
Red task	8	11	8	9
Brown task	3	5	3	25

Table 3: Number of students ($n = 36$) belonging to a particular class of students split out for the three types of tasks included in the pre- and posttest

The next step in our analysis was figuring out what strategies the students used to find the area of the polygons and how flexible they were in using these strategies. We analyse the Red task as an example, because this is in our view the most interesting task that was on both the pre- and the posttest.

The strategies used in the Red task (see Table 4) were counting (in this case the number of squares and triangles), multiplication (length \times width; leading to mistakes because diagonal distances cannot be measured by counting), and restructuring. In the lessons it was made clear that this latter way of measuring the area for this task is very suitable, and we also showed why the multiplication strategy could not work easily.

Used strategy in the Red task in Pretest \rightarrow Posttest	Number of students belonging to a class of students				Total
	“00” student	“01” student	“10” student	“11” student	
Counting \rightarrow Counting	0	3	3	5	11
Counting \rightarrow Restructuring	0	0	4	3	7
Multiplication \rightarrow Counting	1	7	0	1	9
Multiplication \rightarrow Restructuring	2	0	0	0	2
Multiplication \rightarrow Multiplication	1	0	0	0	1
Other	2	1	1	0	4
No strategy in pre- and/or posttest	2	0	0	0	2

Table 4: Students’ ($n = 36$) strategies in the pre- and posttest applied in the Red task

For this Red task, no student used the restructuring strategy before the lessons. Yet, after the lessons, nine students tried to apply this strategy, of which three got the correct answer. The work of one of these students is shown in Figure 2.



Figure 2: Test sheets of students using restructuring as a strategy applied in the Red task in the posttest resulting in a correct answer, and in the Brown task in the pretest making an error

The six students who got a wrong answer made various mistakes. Four of them give a clear explanation of the procedure of embedding the rectangle in a larger square and “subtracting what is in the corners which is not in the red [rectangle]”, but proceeded with computing the wrong area of the square (one student got 7×7 as a result) or computed the wrong area that had to be subtracted (three students). Of the other two students the source of the error is unclear.

The fact that four “10” students go from correctly using a counting strategy to making mistakes with a restructuring strategy, is possibly related to the value we gave to this strategy in the plenary parts of the lessons.

Task type	Strategy	Number of students belonging to a class of students				Total
		“00” student	“01” student	“10” student	“11” student	
Blue	Same	0	6	1	18	25
	Different	0	2	1	5	8
Red	Same	1	3	3	5	12
	Different	3	7	4	4	18
Brown	Same	1	1	0	16	18
	Different	1	2	1	2	6
Total	Same	2	10	4	39	
	Different	4	11	6	11	

Table 5: Number of students using the same or a different strategy in the pre- and posttest split out per task type

Table 5 summarizes the findings of the correctness of the answers per type of task and whether there was a change in strategy between the pre- and posttest. In the Blue and the Brown task more students retained the same strategy than in the other tasks. Moreover, they even did not change the strategy when they got a wrong result in the pretest. In the Red task more changes happened. Here 60% of the students changed their strategy. We see in this table that many students use different

strategies in the pre- and posttest, for instance 18 students changed strategies in the red task, but 7

Student's strategy flexibility for the Blue, Red, and Brown task	Number of students (<i>n</i> = 22)
Using only one strategy	3
Not changing strategy between pre- and posttest but changing it over tasks	2
Changing strategy between pre- and posttest in one task	11
Changed strategy between pre- and posttest in two tasks	6

(3+4) did not come up with the correct answer.

Table 6: Number of students who showed a particular strategy flexibility

Taking again the perspective of the students, we could identify for 22 students their strategy flexibility in the pre- and posttest over the Blue, Red, and Brown task. As it is revealed by Table 6, a few students only used one strategy, but most students changed their strategy from the pretest to the posttest. Note, that in this analysis the students which applied a strategy that could not be classified or was not available in the pre- and/or the posttest are not taken into account.

Student's inter-task flexibility	Pretest	Posttest
Using the same strategy on all tasks	7	5
Using two different strategies	15	14
Using three different strategies	0	3

Table 7: Student's inter-task flexibility on the blue, red and brown tasks (N=22)

We also analysed whether students used the same strategies or different kinds of strategies on the three tasks on the pretest, and the same question on the posttest, to investigate the inter-task flexibility of the students. We see that the inter-task flexibility of the students have increased on the posttest relatively to the pretest (table 7).

Discussion

In this study, we investigated whether working with the geoboard could help students use flexible strategies for measuring area of polygons.

The problems on the pretest and posttest were routine in the sense that the polygons were shapes that the students could have worked with before. However, the problems were given on a grid and without rulers and calculators that the students might be use to have when measuring areas of rectangles. The students showed some inter-task strategy flexibility already at the pretest, choosing

between multiplication and counting strategies. This is in contrast to the situation where students are given unfamiliar, complex problems, where the unfamiliarity and complexity of the problems may hinder the flexible change of strategies (Elia et al., 2009). However, the pretest also showed that some students chose inappropriate strategies, e.g. multiplication on tasks where this could not lead to a correct answer, even though they showed that they knew how to use a counting strategy that would have been appropriate.

Working with the geoboard, the students tried different strategies and shared with each other through whole-class discussion. They also tried the strategies that the researcher emphasized or introduced, e.g. counting or different rearranging strategies. The pre- and posttest results taken together (table 6 and 7) show that many students have developed their inter-task flexibility. Heinze et al. (Heinze, Marschick, & Lipowsky, 2009) raise the question whether the “investigative approach” that was used in the teaching, where students discuss and practice selected strategies, “prevent students from choosing strategies adaptively because they tend to ignore problem characteristics”. Because the posttest items are related to tasks which were discussed in the teaching, we can’t address this question with certainty; however, the results show that many students on the posttest used more appropriate strategies e.g. on the Red task, even if they didn’t use them accurately.

Conclusion

Considering that this was only a small-scale classroom experiment with an intervention consisting of only two lessons, it was hard to draw conclusions. Nevertheless, although the students did not substantially improve their performance in measuring area, we can say that we have experienced that most of the students were active in thinking and trying out strategies for measuring area which were new to them, so working with the geoboard and discussing strategies in whole class helped many students develop their inter-task flexibility for measuring area. This made us more aware that apart from working on the ability of students to find correct answers, also the development of the flexible use of strategies is an important goal in mathematics education to work on.

In further work, we will do a controlled design experiment where the teaching period is longer, to be able to compare working with the geoboard with the ordinary curriculum, and to give the students the opportunity to develop more accuracy and strategy adaptivity.

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