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A computer-based environment for argumenting and proving in geometry

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In this paper, we face the issue of argument and proving in geometry. Overcoming difficulties encountered by the students when moving from argumentation to proof may require suitable didactical interventions. Our contribution to research concerns the design of a specific computer-based didactic environment. We report how 14-15 years old students from high school conjecture and prove within the designed environment and we discuss the preliminary findings.

Keywords: Geometry, argumentation, technology integration, design.

Introduction and theoretical framework

For what geometry concerns, Sinclair, Bartolini Bussi, Villiers, Jones, Kortenkamp, Leung, & Owens (2016), among main strands of contributions to the geometry education they found in reviewing the literature since 2008, identified in particular the understanding of the teaching and learning of definitions and of the proving process. Geometry theorems are a logical consequence derived from an available theory, that is from a set of axioms, theorems previously validated, and definitions, necessary to avoid logical circularity (Fujita, Jones, & Miyazaki, 2018). As reported in the survey by Sinclair et al. (2016), one of the major threads in geometry education concerns understanding of the teaching and learning of definitions, in particular for triangle and quadrilateral.

Definitions comes into play when the students try to construct mathematical proofs, meant as “logical sequence of implications that derive the theoretical validity of a statement” (Mariotti, 2006, p. 182). Moore (1994) highlights that students do not necessarily understand the content of the definitions and how to use them (Moore, 1994).

Boero, Garuti and Mariotti (1996), while not denying the distance between argument and proof, do not consider it an obstacle. Researchers, in fact, highlight a continuity between argument and proof, called cognitive unity. During the problem solving process, argumentative activity usually produces a conjecture. The hypothesis underlying the concept of cognitive unity is that there is a continuity between the argument produced and the proof, which means that the argument can be used by the student in the construction of the proof, reorganizing it appropriately according to a logical scheme. In this sense, open problems (Arsac, Germain, & Mante, 1991), which require conjecture for their resolution, appear to be extremely effective in introducing the concept of proof, as the proofing process is favored by the argumentative one (Boero et al., 1996). However, the analysis of cognitive unity does not cover all aspects of the relationship between argument and proof. Pedemonte (2007) points out that cognitive unity does not take into account the structural continuity between argumentation and proof, when the inferences in argumentation and proof are linked together through the same structure (abduction, induction or deduction) (Pedemonte, 2007).

Difficulties can arise when moving from argumentation to proof, as it requires radical changes in the structure, such as moving from abductive to deductive structures. Facing such difficulties and overcoming them constitute an education goal that may require specific didactical interventions.

Our research study intends to contribute to this issue with the design and the experimentation of a specific didactic environment.

The experimental design

In the following, we are describing the experiment. The aim is to explore the educational potential of a certain working environment with respect to the process of producing conjecture and proving it. The setting of the environment have brought to identify three types of working environment that can be combined. In each of them we tried to create elements that would implement certain hypotheses, based on the theoretical framework.

Hypothesis 1. The use of open problems, i.e. problems where conjecturing is required. This is consistent with various studies (Arsac at al., 1991; Boero, Garuti, & Lemut, 1999; Hadas, Hershkowitz, & Schwarz, 2000; Pedemonte, 2008).

Hypothesis 2. The shift from spontaneous arguments - arisen during and/or after the process of producing a certain conjecture - to the proof may present some difficulties both in terms of reference system and in terms of structure. The formalization environment has been designed to provide support with respect to the expected difficulties. This latter hypothesis is consistent with the results of different studies based on the notion of cognitive unity (Pedemonte, 2002, 2007)

Hypothesis 3. The importance of exploratory talk in collaborative situations is crucial for improving the rise of conjectures and their formalization. This fits the dialogic approach in collective geometrical thinking (Fujita, Doney, & Wegerif, 2017). That is why the design foresees to alternate individual and social tasks where students act “as critical friends”.

The aim of our research study is that of exploring the educational potentialities of a computer-based environment with respect to support students in solving a conjecturing open problem, and subsequently providing a proof of the conjecture. According to the Hypotheses, a computer-based environment was designed offering an organized environment where students work alone and with peers for solving conjecturing problems, producing conjectures and their justifications and formal proofs.

In the following, we describe the main components of the computer-based environment, together with the rationale of their design, and an overview of the methodology; then we present some preliminary findings concerning only one of the three experimental settings.

The computer-based environment

According to Hypothesis 1 we selected a conjecturing problem:

Given a parallelogram ABCD, draw a parallel to the diagonal BD passing through one of the other vertices. Extend one of the sides of the parallelogram that does not contain that vertex until it meets the drawn parallel. Which quadrilaterals can be identified in this figure? What kind are they? Please justify your statements.

The student is required to construct the figure, following the given instructions, then to answer to the posed question concerning the identification of specific quadrilaterals and justify her conjectures. Starting from the parallelogram ABCD and drawing the parallel to BD through A, two different figures can arise: the first one obtained by extending the side BC (Figure 1.a) and the second one by extending the CD side (Figure 1.b).

The mathematical theory related to this problem concerns the definition of parallelogram, trapezoid and rectangle. It is expected that the students recognize the characteristics of a certain figure and use the definition in the direction that allows to derive the name of the figure from its characteristics.

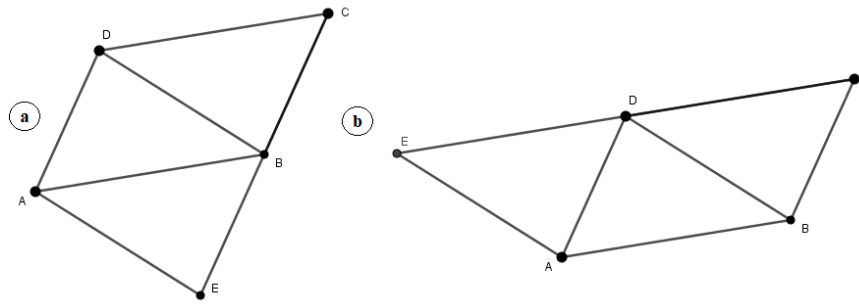


Figure 1: Two possible constructions

The computer-based working environment consists of three working spaces: *Working alone*, *Working with others* and *Assessing the others*, composed of various tasks, some individual and others social. In individual tasks the student does not communicate with her teammates and answers the proposed question herself. In social tasks the student answers individually but discusses with teammates in a forum (Figure 2). The computer-based working environment so designed exploits the potential of the Moodle platform in terms of built-in resources (Questions and Answers Forum, GeoGebra Task, Chat) and of integration of new digital applications (such as ISQ), together with the overcoming of space-time constraints. This allows the design of new activities, inconceivable without the use of the technology - level of Redefinition in the SAMR model (Romrell et al., 2014). Moreover, the automatic reports of the platform allow the teacher to take advantages of an augmented reality for what concerns the students' actions and thinking.

WORKING ALONE	WORKING WITH OTHERS	WORKING ALONE	WORKING WITH OTHERS	ASSESSING THE OTHERS
Problem solving	Conjecturing	Formalizing	Agreeing	Proof contest
Task module	Question and Answer forum	ISQ device	Question and Answer forum	Proof contest forum

Figure 2: the design of the learning activity

We are detailing only the *Working alone* environment, on which we focus in this paper. It consists only of individual tasks, more precisely, the following two individual tasks:

- *Problem solving*: the student is asked to face the above problem. She is equipped with a “blank” GeoGebra page, predefined by the teacher/designer., suitably customized in order to show only the needed GeoGebra commands. The student submits her answer, consisting in: i) a figure constructed according to the given instructions, in particular the student is required to name the vertices fitting such instructions; ii) the identification of the quadrilaterals; iii) the justification of such identification. Using the Moodle *Task* module, the figure related to her GeoGebra construction is attached as the ggb file and the answer is given as a plain text in a blank box.
- *Formalizing*: the student is required to prove her conjectures reorganizing the justifications previously given according to a logical scheme. At beginning, the student is asked to choose one of the two figures shown in Figure 1, if it is the same of the one she constructed, or if it is not the case, she is required to upload her own ggb file. Therefore, starting from her own construction, for each quadrilateral identified, she is expected to prove her statements rearranging her own arguments into a logical chain. Such formalization is supported by a device consisting in a GeoGebra application, called ISQ (Interactive Semi-open Question), integrated into Moodle. It consists of digital language tiles that can be drag and juxtaposed in order to construct a sentence that represents the answer to a given question (Albano & Dello Iacono, 2018). In this case the ISQ has been enriched by some

fixed digital tiles constituting some kind of “Bank of Theory” which contains definitions, properties, theorems that are useful for proving the mathematical concepts at stake. In our case it contains only the definitions of parallelogram, trapezoid and rectangle. We have chosen to write the definitions, distinguishing explicitly between the two directions of the equivalence. This choice aims to make students aware of the existence of the two directions and of the fact that sometimes it is useful to use one direction and sometimes the other direction (Figure 3). For consistency, we always used the formulation “If ... then” and in particular indicated with Def #a the direction "If DEF then PROPERTY" and with Def #b the direction "If PROPERTY then DEF". The construct “If ... then...” has been implemented as “PROPERTY \Rightarrow DEF” and “DEF \Rightarrow PROPERTY”. Concerning the digital tiles, we have chosen to merge the verb “to be” to the next adjective if it has the meaning of “to have the property of” (e.g. “is isosceles” is equivalent to saying “has the property of being isosceles”). Moreover, causal conjunctions constitute digital tiles in themselves to highlight the causal structure.

DEFINITION OF PARALLELOGRAM				
Def. 1a: A quadrilateral is a parallelogram \Rightarrow it has the opposite sides parallel				
Def. 1b: A quadrilateral has the opposite sides parallel \Rightarrow it is a parallelogram				

DEFINITION OF TRAPEZOID				
Def. 2a: A quadrilateral is a trapezoid \Rightarrow it has two parallel sides				
Def. 2b: A quadrilateral has two parallel sides \Rightarrow it is a trapezoid				

DEFINITION OF RECTANGLE				
Def. 3a: A quadrilateral is a rectangle \Rightarrow it is a parallelogram with congruent angles				
Def. 3b: A quadrilateral is a parallelogram with the congruent angles \Rightarrow it is a rectangle				

A	B	C	D	E
by hypothesis		by construction		Def. 3b
Def. 1a		Def. 1b		Def. 2a
is a trapezoid		is a parallelogram		is a rectangle
is parallel to		sides		is on the same line as
opposite		parallel		has
in fact		then		because
				so
				it
				by
				and

Figure 3: ISQ in the Formalizing task

The ISQ contains several overlapped copies of each digital tiles, in order to let using the same tile several times. The available digital tiles allow to build various sentences that can be correct or not correct as well as complete or incomplete answers. The digital tiles, labelled with Def #, allow the production of sentences recalling definitions reported in the Bank of Theory (see on the left of Figure 3).

The methodology

We assume a design based approach with the intention to develop an artifact, i.e. the learning computer-based environment including new developed digital application such as ISQ, which is supposed to be optimized according to the findings of performed empirical studies.

In order to analyze the contribution of the various components of the computer-based environment, we decided to draw three different experimental settings:

- 1) a setting without collaboration but with the support of ISQ, i.e. *Working alone*, to understand the actual contribution of this ISQ device;
- 2) another setting in which students can collaborate with each other, but without the support of ISQ, i.e. *Working with others*, to understand the actual contribution of collaboration;
- 3) the complete setting, with the possibility for students to be supported in the individual phase by the ISQ and, subsequently, to collaborate with each other, so as to be able to observe the contribution of the toolkit and of the collaboration at the same time.

The experimentation involved 72 9th-10th grade students from two different scientific high schools, distributed into the above experimental settings according to the random choice of the teacher. All

the data have been collected by means of the platform's reports and they have been analysed from a qualitative point of view.

In this paper we focus on the setting *Working alone*, for investigating what the ISQ device shows about the students' way of reasoning. To this aim, we have compared the students' transcripts before and after the use of the ISQ device.

Preliminary findings

In this paper we focus on the ISQ device and its functioning within the Working alone setting. We report the analysis of the data and report some findings.

The analysis of the data shows interesting relationships between freely expressed arguments and the corresponding formalized arguments, organized through the use of the predefined digital tiles.

We will discuss in more details the outcomes of three students: Fromix, Antonio and Denisa. For each of them we are considering the figures they freely constructed by GeoGebra (first) and chose among the available (then) and their answer to the question posed by the given problem, in an open-answer mode (first) and by the digital tiles offered by the ISQ (then).

Let us analyze the answer produced by Fromix. He attaches the following GeoGebra figure:

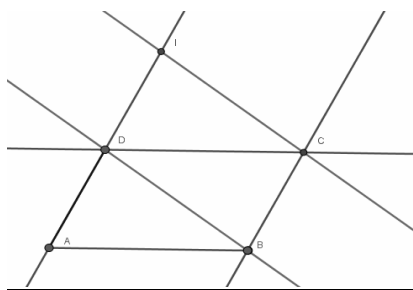


Figure 4: Fromix's GeoGebra construction

and writes the following in the available blank box of the Moodle Task module:

- 1 Fromix: We can highlight as many as 3 figures:
- 2 ABCD : that is the given parallelogram.
- 3 DBCI : another parallelogram, since it has pairs of sides congruent and parallel and we can also notice one of its 2 diagonals (DC).
- 4 AEBC : scalene trapezoid having a longer base parallel to the smaller one and 2 not congruent sides.

Then, Fromix chooses the Figure 1.b and, using the digital tiles (highlighted by grey background) provided by the ISQ, writes the following sentences:

- 5 Fromix: A E B C is a trapezoid by hypothesis because it has two parallel sides
- 6 A B C D is a parallelogram because it has the opposite parallel sides so by Def. 1a
- 7 D B C E is a parallelogram because it has two opposite parallel sides so by Def. 1a

Fromix individuates three quadrilaterals that he classifies as two parallelograms and a trapezoid according to their properties.

We can note that Fromix, moving into the Formalizing task through the use of the digital tiles, does not literally translate the statements freely produced. Some properties that appeared in the first

expressions are neglected, whilst the reference to the definition emerges. According to our hypotheses, the availability of specific digital tiles seems to have determined a shift of attention on the characterizing properties, that is having two parallel sides (row 5), disregarding the presence of two non-congruent sides (row 4). Fromix does not make explicit reference to the digital tiles of the definition of trapezoid. Thus we can interpret this phenomenon as a contribution of the ISQ device that induces the student to analyze the content of the definitions. However, the formalized version is not logically consistent: as matter of fact, it seems that she uses the definition 1a to derive the parallelism of the opposite sides.

Let us analyze the answer produced by Antonio. He attaches the following GeoGebra figure:

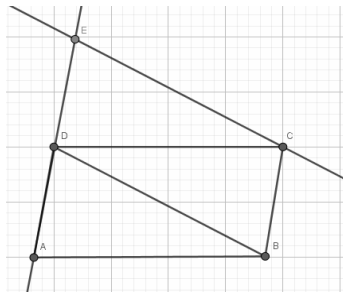


Figure 5: Antonio's GeoGebra construction

and writes the following in the available blank box of the Moodle Task module:

- 8 Antonio: In this figure it is possible to identify the following quadrilaterals: ABCD, BCED, ABCE.
- 9 The quadrilaterals ABCD and BCED are two parallelograms because the opposite sides (AB, CD- BC, AD-DB, CE-CB,DE) are parallel,
- 10 while the quadrilateral ABCE is a trapezoid because it has two parallel sides (AE,CB) and two opposite oblique sides (AB, CE).

Then, Antonio chooses the Figure 1.a and, using the digital tiles provided by the ISQ, writes:

- 11 Antonio: A B C D is a parallelogram by Def. 1.a
- 12 E A D B is a parallelogram by Def. 1.a
- 13 A D C E is a trapezoid because it has parallel opposite sides

We can see that Antonio, using the digital tiles, makes explicit reference to the definition of parallelogram given in the ISQ (rows 11 and 12). Anyway, switching from the free formulation to the one supported by the digital tiles, he loses one step of the argument and it remains unclear how the definition is used, i.e. which are the observed parallel sides needed to apply the definition (row 9), although the digital tiles would have been available to do that. On the contrary, comparing row 10 and 13, we can observe that the characterizing property is expressed but the reference to the definition is missing. Moreover, in any case, moving from the free expression to the formalization, any reference to the specific figure is lost. As in the case of Fromix, we can notice again that the ISQ device seems to divert attention from the non-parallel sides of the trapezium (row 10 and 13).

Let us analyze the answer produced by Denisa. She attaches the following GeoGebra figure:

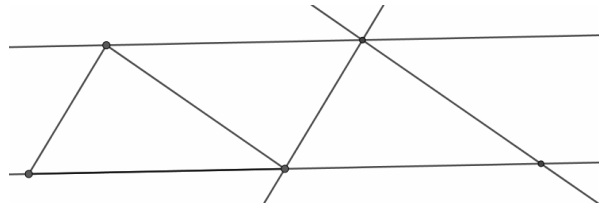


Figure 6: Denisa's GeoGebra construction

and writes the following in the available blank box of the Moodle Task module:

- 14 Denisa: In the figure there are two parallelograms,
 15 because by constructing the parallel to the diagonal BD and extending the side, we have the intersection of the two lines to form another triangle equivalent to half of the initial parallelogram.

Denisa uploads her own figure and, using the ISQ, writes the following sentences:

- 16 Denisa: A B C D is a parallelogram by Def. 1.a it has parallel opposite sides
 17 B E C D is a parallelogram by Def. 1.a it has parallel opposite sides
 18 A B C E is a trapezoid by Def. 2.a it has parallel opposite sides

In the free answer environment Denisa individuates two quadrilaterals, that she identifies as two parallelograms, providing a justification referring to the construction made.

In Denisa's case the change from the free response version and the ISQ-driven version is more evident than in the other cases. First of all, she notices a third figure (a trapezoid) who had not initially identified (rows 18). Furthermore, in the free answer she does not differentiate the two parallelograms and does not provide any real argument to justify the classification, just repeating the construction commands. On the contrary, in the ISQ-driven version, she names the parallelograms and provides arguments referring both to definitions and to the properties (rows 16-17-18). Similarly to Fromix, the use of definition is not logically consistent.

Discussion and conclusions

The previous analysis points out the influence of the ISQ device on the transition from the free expression to the formalized expression, as a matter of fact we can notice a change which highlights interesting elements. On the one hand, the students focus and select the relevant properties, neglecting the others ones, not distinctive. This can be interpreted as due to the availability of the definitions' formulation that shows only the distinctive properties. On the other, formalization is not a simple transcription but it implies a "logical" interpretation of what is expressed in natural language and a transcription of that interpretation into the language using the digital tiles and in particular the connective " \Rightarrow ". Here we can observe how the interpretation of why as explanation is translated by the connective " \Rightarrow ", even if this translation is not consistent with the formal language. What appears is the fact that all students (not only the ones reported above), in the transition to the use of digital tiles, use the wrong direction of the definition. It would seem that they interpret the symbol " \Rightarrow " according to the verbal structure used in the free formulation of the answer. More precisely, comparing the free explanations, it would seem that they reproduce the verbal structure "FIGURE is X because Y" as "FIGURE is X \Rightarrow Y". that is, they replace "because" with " \Rightarrow ", so that in the argument the premise follows the claim.

We want to underline that the logically correct use of the sign “ \Rightarrow ” is not spontaneous, nor the transition from the use of “because” in informal language to the use of “ \Rightarrow ” in formal language. The fact that the ISQ device makes evident such difficulty allows the teachers to identify such kind of obstacle and to intervene opening a discussion on the mathematical meaning of definitions.

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