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# The moment duration scaling relation for slow rupture arises from transient rupture speeds

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## Key Points:

- A Burridge-Knopoff model with only two dimensionless parameters; the homogeneous stress on a fault and a velocity strengthening friction term.
- The simplicity of the model allows for both numerical and analytical calculations of moment versus duration scaling relationships during fault slip.
- Moment versus duration scaling relation for slow events arises from transient rupture speeds.

## Plain language summary

Observations have shown that the duration of earthquakes is related to the seismic moment through a power law. The power law exponent is different for regular earthquakes and slow aseismic rupture, and the origin of this difference is currently debated in the literature. In this letter, we introduce a minimal mechanical friction model that contains both slow and regular earthquakes, and demonstrate that the different power laws emerge naturally within the model because the propagation speed of slow earthquakes decays as a power law in time whereas the propagation speed of regular earthquakes remains fairly constant.

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**Abstract**

28 The relation between seismic moment and earthquake duration for slow rupture fol-  
 29 lows a different power law exponent than sub-shear rupture. The origin of this dif-  
 30 ference in exponents remains unclear. Here, we introduce a minimal one-dimensional  
 31 Burridge-Knopoff model which contains slow, sub-shear and super-shear rupture, and  
 32 demonstrate that different power law exponents occur because the rupture speed of  
 33 slow events contains long-lived transients. Our findings suggest that there exists a  
 34 continuum of slip modes between the slow and fast slip end-members, but that the  
 35 natural selection of stress on faults can cause less frequent events in the intermediate  
 36 range. We find that slow events on one-dimensional faults follow  $\bar{M}_{0,\text{slow},1D} \propto \bar{T}^{0.63}$   
 37 with transition to  $\bar{M}_{0,\text{slow},1D} \propto \bar{T}^{\frac{3}{2}}$  for longer systems or larger prestress, while the  
 38 sub-shear events follow  $\bar{M}_{0,\text{sub-shear},1D} \propto \bar{T}^2$ . The model also predicts a super-shear  
 39 scaling relation  $\bar{M}_{0,\text{super-shear},1D} \propto \bar{T}^3$ . Under the assumption of radial symmetry, the  
 40 generalization to two-dimensional fault planes compares well with observations.  
 41

## 1 Introduction

Over the last decades, an increasing number of slow slip events on faults have been reported (Bürgmann, 2018). A measure that is viewed as a key to unravelling the mechanism of slow and fast rupture is the relation between seismic moment  $M_0$  and slip event duration  $T$ . Regular fast earthquakes have long been known to follow a moment duration scaling relation of  $M_0 \propto T^3$ . Ide et al. suggested that slow events follow a unified scaling relation  $M_0 \propto T$  (Ide et al., 2007). They suggested that the linear relation between moment and duration for slow events can be explained in two ways: (1) the average slip is proportional to the fault length as for fast propagation, and the stress drop is constant for all events, which gives the relation  $M_0 \propto T$ . (2) the slip amount is constant for all events, and the fault area increases linearly with time  $L^2 \propto T$ , which results in  $M_0 \propto T$ . Peng et. al (Peng & Gomberg, 2010) elaborated on the ideas of Ide et. al (Ide et al., 2007) and reached a different conclusion; that rupture should span a continuum between fast and slow velocity end-members. However, almost 10 years after the suggestion of Peng et. al, observations of events between the fast and slow end-members are still sparse. Later studies have reported on a variety of scalings between moment and duration ranging from  $M_0 \propto T$  to  $M_0 \propto T^2$  (Ide et al., 2008; Aguiar et al., 2009; Frank et al., 2018; Gao et al., 2012), although the definition of events can vary somewhat between different studies.

The shape of slip patches can also influence the observed scaling. Ben-Zion (Ben-Zion, 2012) argued that fractal slip patches can result in a scaling relation  $M_0 \propto T^2/\log(T)$  because the average displacement is approximately constant rather than proportional to the rupture dimension. Bounded propagation can also play an important role (Ben-Zion, 2012; Gomberg et al., 2016). Gomberg et. al (Gomberg et al., 2016) suggested that the scaling relation between moment and duration is the same for slow and fast events, but that a transition occurs between a two-dimensional scaling and a one-dimensional scaling when the rupture propagation switches from unbounded to bounded in one direction. Assuming the fault displacement can be approximated using dislocation theory, this results in a transition from  $T^3$  to  $T$ . They suggest that there should be a bimodal but continuous distribution of slip modes, and that a difference in scaling relations alone does not imply a fundamental difference between fast and slow slip. The above mentioned theoretical considerations implicitly assume constant rupture velocity. However, this contradicts observations by Gao et. al (Gao et al., 2012) that show that the average rupture speed for slow events decreases with increasing seismic moment, which is a strong indication of transient rupture speeds.

Slow slip events emerge in numerical models with rate-and-state friction. Colella et. al (Colella et al., 2011) simulated a Cascadia-like subduction zone using rate-and-state friction. They analyzed a large number of slip events, and found that the seismic moment  $M_w$  scales as  $M_w \propto T^{1.5}$  for  $M_w \leq 5.6$  with a transition to  $M_w \propto T^2$  for  $M_w > 5.6$ . Shibazaki et. al (Shibazaki et al., 2012) modeled the subduction zone of southwest Japan with rate-and-state friction. For slow events, they found a scaling  $M_0 \propto T^{1.3}$ . Liu et. al (Liu, 2014) used rate-and-state friction on a 3D subduction fault model and found a scaling  $M_0 \propto T^{1.85}$ . Romanet et. al (Romanet et al., 2018) highlighted the role of interactions between faults. They argue that the scaling relationships of slow slip events and earthquakes emerge from geometrical complexities due to interactions between fault segments. The moment duration scalings have not only been addressed using rate-and-state friction. Ide et. al (Ide, 2008) introduced a Brownian walk model for slow rupture, where the assumption is that there is a random expansion and contraction of the fault area, so that its radius can be described as a Brownian walk with a damping term. This model predicts  $M_0 \propto T$  for large  $T$ .

Here, our goal is to answer the following two questions: (1) Is there a separation of two distinct classes (Ide et al., 2007), or is there a continuum of possible scaling

95 relations between the fast and slow end-members (Peng & Gomberg, 2010)? (2) Can a  
 96 difference in  $M_0 - T$  scaling relations alone be attributed to different physical mecha-  
 97 nisms behind slow and fast rupture? We address both (1) and (2) through a Burridge-  
 98 Knopoff type model with Amontons-Coulomb friction with a velocity strengthening  
 99 friction term that has previously been shown to contain a large variety of rupture  
 100 phenomena, including sub-shear, super-shear and slow propagation (Thøgersen et al.,  
 101 2019). Velocity strengthening friction has been shown to be a generic feature of dry  
 102 friction (Bar-Sinai et al., 2014), and has been reported in Halite shear zones at low  
 103 slip speeds or large confining pressures (Shimamoto, 1986). The friction law can also  
 104 be interpreted as a transition from a dry contact to a lubricated sliding regime with  
 105 increasing velocity (a Stribeck curve) under the additional assumption that the transi-  
 106 tion from dry to contact to lubricated sliding occurs at a small sliding speed (Gelinck  
 107 & Schipper, 2000; Olsson et al., 1998).

108 For homogeneously stressed faults, the model can be reduced to only two dimen-  
 109 sionless parameters  $\bar{\tau}$  and  $\bar{\alpha}$  representing the prestress and a velocity strengthening  
 110 friction term, respectively. The advantage of such approach is that the simplicity of the  
 111 model allows us to calculate moment duration scaling relations both through numer-  
 112 ical simulations and through analytical calculations. Through numerical simulations,  
 113 we demonstrate that there exists a continuum of rupture modes between the slow  
 114 and fast end-members, but that the most likely selection of  $\bar{\tau}$  in nature produces two  
 115 distinct classes separating sub-shear and slow rupture velocities. Through analytical  
 116 calculations, we show that the scaling relation for slow fronts arises due to long-lived  
 117 transients in the rupture velocity. Such transient rupture velocity has been observed  
 118 in nature through a dependence on the average rupture speed on the seismic moment  
 119 for slow fronts (Gao et al., 2012). In addition, the model predicts a separate scaling  
 120 for super-shear rupture not previously reported in the literature.

## 121 **2 A one-dimensional Burridge-Knopoff containing slow and fast rup-** 122 **ture**

123 We solve the one-dimensional Burridge-Knopoff model (Burridge & Knopoff,  
 124 1967) with Amontons-Coulomb friction with a linear velocity strengthening term. The  
 125 dimensionless equation of motion for a chain of  $N$  blocks can be written as (a detailed  
 126 derivation can be found in the supplementary information)

$$\ddot{u}_i - \ddot{u}_{i-1} - \ddot{u}_{i+1} + 2\ddot{u}_i + \bar{\alpha}\dot{u}_i - \bar{\tau}^\pm = 0, \quad \forall i \in [0, N], \quad (1)$$

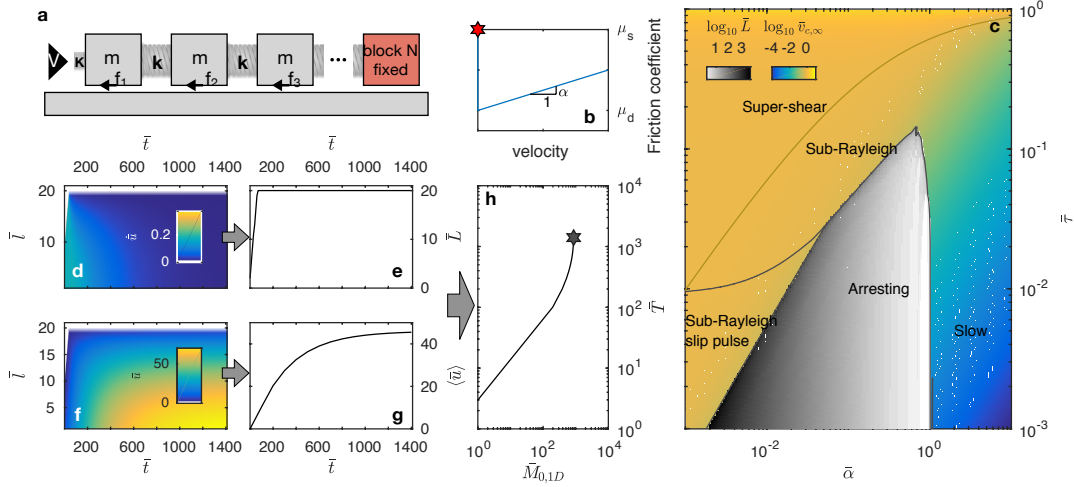
127 which is integrated using the Euler-Cromer method with  $d\bar{t} = 10^{-3}$ .  $\bar{u}$  is the dimen-  
 128 sionless displacement, and

$$\bar{\tau}^\pm = \frac{\tau/\sigma_N \mp \mu_k}{\mu_s - \mu_k} \quad (2)$$

129 is the dimensionless prestress where  $\sigma_N$  is the normal stress,  $\tau$  is the initial shear  
 130 stress, and  $\mu_s$  and  $\mu_k$  are the static and dynamic coefficients of friction, respectively.  
 131  $\pm$  denotes the sign of the block velocity. For positive velocities, we only need to  
 132 consider  $\bar{\tau}^+$ , but negative velocities can occur in a small subset of our simulations. In  
 133 such situations, we need to prescribe the relation between  $\mu_s$  and  $\mu_k$ , which we set to  
 134  $\mu_s = 2\mu_k$ , so that  $\bar{\tau}^- = \bar{\tau}^+ + 2$ . In the rest of the paper we will use  $\bar{\tau}$  as a reference  
 135 to  $\bar{\tau}^+$ . The second dimensionless parameter

$$\bar{\alpha} = \frac{\alpha}{\sqrt{\rho E}} \quad (3)$$

136 is a viscous term, where  $\rho$  is the density,  $E$  is the elastic modulus, and  $\alpha$  is a velocity  
 137 strengthening term with units [Pa s/m].  $\bar{\alpha}$  can range from 0 to infinity, where  $\bar{\alpha} = 0$  re-  
 138 covers the ordinary Amontons-Coulomb friction without viscosity.  $\bar{\tau}$  has an upper limit  
 139 of 1, where the prestress equals the static friction threshold. For  $\bar{\tau} < 0$ , steady state

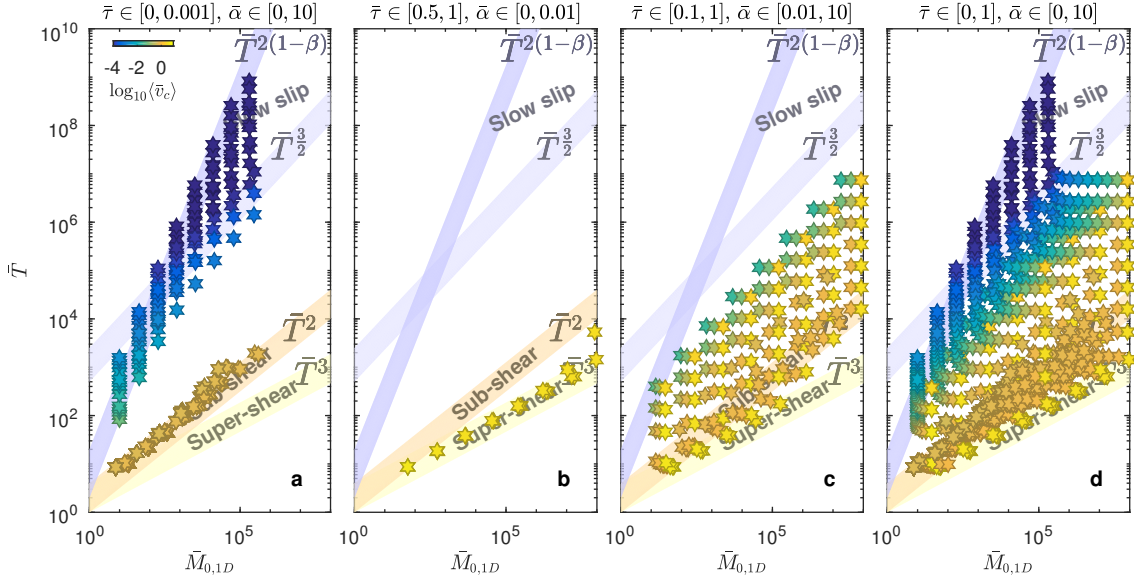


**Figure 1.** (a) We solve the one-dimensional Burridge-Knopoff model with Amontons-Coulomb friction with velocity-strengthening dynamic friction for homogeneously loaded faults.  $V$  is the driving velocity,  $K$  is the driving spring constant,  $m$  is the block mass,  $k$  is the spring constant, and  $f_i$  is the friction force on block  $i$ . The friction law is given by a static friction coefficient  $\mu_s$ , and a dynamic friction coefficient  $\mu_d$  plus a velocity strengthening term  $\alpha v$  (b). To obtain the seismic moment and duration for a given maximum fault length we fix the block at position  $N$ . The model can be written in dimensionless units with only two parameters:  $\bar{\tau}$  representing the prestress on the fault, and  $\bar{\alpha}$  representing the velocity strengthening friction term. This simple model produces a large variety of slip, including, slip pulses, cracks, sub-Rayleigh rupture, super-shear rupture, slow rupture, and arresting fronts (c). The colorbars show the fault length  $\bar{L}$  of arresting fronts, and the steady state rupture speed  $\bar{v}_{c,\infty}$  for given  $\bar{\tau}$  and  $\bar{\alpha}$  (adapted from (Thøgersen et al., 2019)). Each event consists of a single simulation, which gives the block sliding velocity  $\dot{u}$  as a function of position  $\bar{l}$  and time  $\bar{t}$  (d), from which we extract the front position  $L$  (e). We also measure the block displacement  $\bar{u}(\bar{l}, \bar{t})$  (f), from which we extract the average displacement  $\langle \bar{u} \rangle$  (g). Using equation 6 we obtain the seismic moment and the duration of the event marked with a star in (h).

140 propagation does not occur (Amundsen et al., 2015). This corresponds to a prestress  
 141 below the dynamic friction level. We thus simulate  $\bar{\tau} \in [10^{-7}, 1]$  and  $\bar{\alpha} \in [10^{-3}, 10]$ .  
 142 The boundary conditions assuming triggering through soft tangential loading (small  
 143 driving velocity  $V$  and driving spring stiffness  $K$ ) are given by  $\bar{u}_{-1} = \bar{u}_0 + 1 - \bar{\tau}$ .  
 144 The rightmost block is fixed so that  $\bar{u}_N = 0$ . Blocks start to move once the static friction  
 145 threshold is reached, which in dimensionless units can be written as

$$\bar{u}_{i-1} + \bar{u}_{i+1} - 2\bar{u}_i \geq 1 - \bar{\tau} \quad (4)$$

146 Moving blocks restick if the velocity changes sign. The system is sketched in Figure 1a.  
 147 This model has previously been used to determine the steady state rupture velocity  
 148 which includes sub-shear, supershear, and slow rupture, as well as an arresting region  
 149 at low  $\bar{\tau}$  and intermediate  $\bar{\alpha}$  (Thøgersen et al., 2019). The steady state front speed  
 150  $\bar{v}_{c,\infty}$  can be found exactly when  $\bar{\alpha} = 0$  (Amundsen et al., 2015). For  $\bar{\alpha} > 0$  we can use



**Figure 2.** One-dimensional seismic moment  $\bar{M}_{0,1D}$  and event duration  $\bar{T}$  obtained from simulations. The color of the markers show the average front speed  $\langle \bar{v}_c \rangle$ . The origin of the four different scaling exponents  $\{2(1 - \beta), \frac{3}{2}, 2, 3\}$  is discussed in detail in the text. (a) In the limit of small  $\bar{\tau}$ , there is a separation in two distinct scalings for fast and slow events. (b) For large  $\bar{\tau}$  the model predicts super-shear rupture, which has a different scaling exponent than regular sub-shear earthquakes. (c) For intermediate  $\bar{\tau}$ , the central part of the diagram is populated. (d) Results from the entire range of  $\bar{\tau}$  and  $\bar{\alpha}$  show that moment duration can exhibit a continuum of slip modes in between the slow and fast end-members.

151 the empirical expression (Thøgersen et al., 2019)

$$\bar{v}_{c,\infty} \approx \frac{1 - e^{-\frac{\bar{\tau}}{\bar{\alpha}(1-\bar{\tau})}}}{\sqrt{1 - \bar{\tau}^2}}. \quad (5)$$

### 152 3 Moment duration scaling relations

#### 153 3.1 Model results

154 For each simulation we predefine  $\bar{\tau}$  and  $\bar{\alpha}$ , and each simulation consists of a  
 155 single event. From each event we extract the displacement  $\bar{u}$  and the fault length  $\bar{L}$ ,  
 156 which is found from the position of the rightmost block that has ruptured. We run the  
 157 simulations until all blocks are immobile, or until the average velocity reaches 0.1% of  
 158 the steady state slip speed  $\bar{\tau}/\bar{\alpha}$  (Thøgersen et al., 2019). In dimensionless units the  
 159 zeroth order moment for rupture propagation along a line is

$$\bar{M}_{0,1D} = \langle \bar{u} \rangle \bar{L}, \quad (6)$$

160 where  $\langle \bar{u} \rangle$  is the average displacement on a fault of length  $\bar{L}$ . The seismic moment  
 161 and the duration are measured when 99% of the total displacement has been reached.  
 162 Figure 1 shows how  $\bar{M}_{0,1D}$  and the event duration  $\bar{T}$  is measured in the simulations.  
 163 For each simulation, we measure the duration, as well as the fault length  $\bar{L}$  and the  
 164 average block displacement  $\langle \bar{u} \rangle$ . This results in a single point in the  $(\bar{M}_{0,1D}, \bar{T})$  diagram  
 165 for each event.

166 Figure 2 shows the measured  $\bar{M}_{0,1D}$  and event duration  $\bar{T}$  for 1120 simulations  
 167 with  $N \in 5 \times 2^{\{0,7\}}$ . If the stress drop is small compared to the absolute shear stress, as  
 168 is often found for faults (Shearer et al., 2006),  $\tau$  should often lie close to the dynamic  
 169 level, which corresponds to  $\bar{\tau} \simeq 0$ . For low values of  $\bar{\tau}$ , the arresting region in  $(\bar{\tau}, \bar{\alpha})$   
 170 gives rise to a separation of these scaling relations, so that fast and slow rupture  
 171 fall into two distinct lines in the moment duration diagram (Figure 2a). This is in  
 172 line with the ideas of (Ide et al., 2007). This separation occurs because steady state  
 173 propagation at small  $\bar{\tau}$  and intermediate  $\bar{\alpha}$  is forbidden (Figure 1a). If we include also  
 174 larger prestress values we obtain a continuum of slip modes in the moment duration  
 175 diagram (Figure 2d), in line with the suggestions of (Peng & Gomberg, 2010). The  
 176 model also predicts a second scaling relations for super-shear rupture, which is found  
 177 at large  $\bar{\tau}$ , that has not previously been reported (Figure 2b).

### 178 3.2 Origin of scaling relations - analytical calculations

179 The simplicity of the model allows an analytical treatment of several aspects  
 180 which helps explain the various scaling relations between seismic moment and event  
 181 duration. We summarize the analytical predictions for slip, front speed and event  
 182 duration, and explain why the different scaling relations appear. A detailed derivation  
 183 is given in the supplementary information.

184 First, we can determine the average slip on a fault. If the stress is at the dynamic  
 185 level after rupture (the stress drop equals  $\bar{\tau}$ ), we can calculate  $\langle \bar{u} \rangle$  exactly

$$\langle \bar{u} \rangle = \frac{\bar{\tau} \bar{L}^2}{3} + \frac{(1 - \bar{\tau}) \bar{L}}{2}. \quad (7)$$

186 Equation 7 is derived for soft tangential loading, and we stress that a different boundary  
 187 conditions could lead to different dependencies between  $\bar{L}$ ,  $\langle \bar{u} \rangle$  and  $\bar{\tau}$ . Combining  
 188 equation 7 with equation 6 we find that the seismic moment can be written as

$$\bar{M}_{0,1D} = \frac{\bar{\tau} \bar{L}^3}{3} + \frac{(1 - \bar{\tau}) \bar{L}^2}{2}, \quad (8)$$

189 which only depends on the prestress  $\bar{\tau}$  and the length of the fault  $\bar{L}$ . To obtain the  
 190 moment duration scaling relation we need to determine  $\bar{L}(\bar{T})$ , and thus have to combine  
 191 equation 8 with information about the rupture propagation and the afterslip (i.e. the  
 192 amount of slip after the propagation has stopped).

193 A key observation on the rupture propagation is shown in Figure 3. While fast  
 194 fronts exhibit short transients and quickly reach the steady state propagation speed  
 195 given by equation 5, slow rupture contains long transients where the propagation  
 196 speed decays. In the figure, we have illustrated this effect as a decay in the average  
 197 rupture speed  $\langle \bar{v}_c \rangle \equiv \frac{\bar{L}}{\bar{T}}$  with increasing seismic moment  $\bar{M}_0$ . This result is in line with  
 198 observations by (Gao et al., 2012).

#### 199 3.2.1 Fast regime

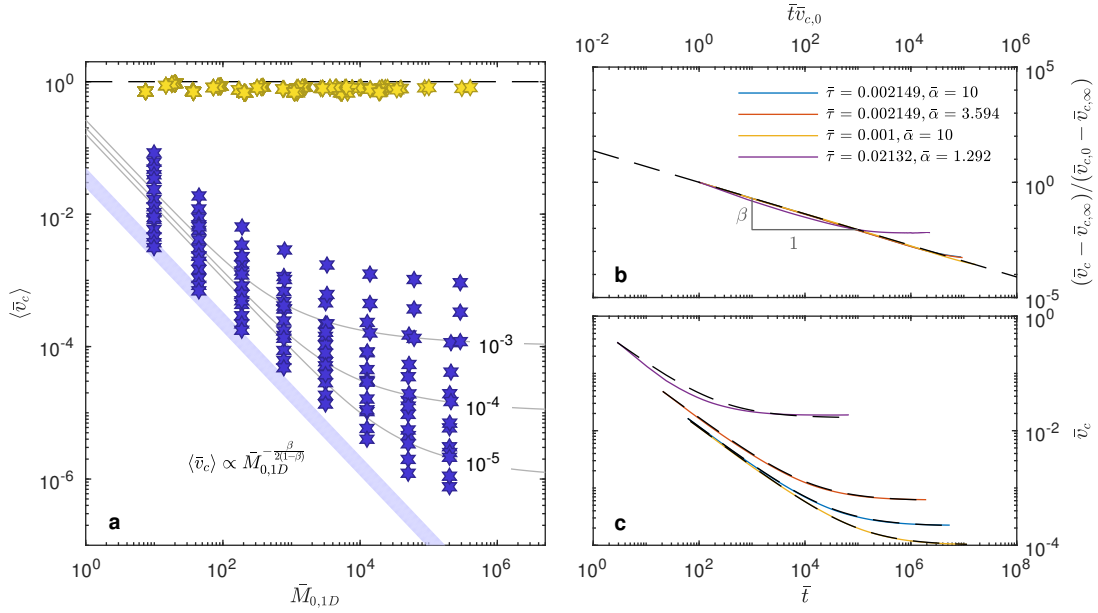
200 The short transients in the fast regime indicate that we can approximate the  
 201 fault length by

$$\bar{L} = \int_0^{\bar{t}_{\text{rupture}}} \bar{v}_c(t') dt' \approx \bar{v}_{c,\infty} \bar{t}_{\text{rupture}}, \quad (9)$$

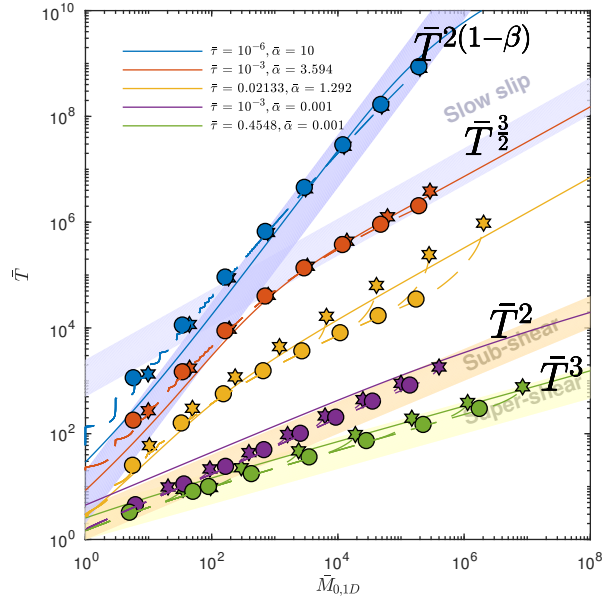
202 where  $t_{\text{rupture}}$  is the time it takes for a rupture front to reach the end of the fault.  
 203 The time it takes to arrest completely depends upon the existence of a backward  
 204 propagating arresting front. If we assume that this backward propagation occurs at  
 205 roughly the same speed as the forward propagation we obtain

$$\bar{T} \approx \frac{2\bar{L}}{\bar{v}_{c,\infty}} \quad (10)$$





**Figure 3.** (a) Average propagation speed  $\langle \bar{v}_c \rangle$  as a function of seismic moment for  $\bar{\tau} \in [10^{-7}, 10^{-3}]$  and  $\bar{\alpha} \in [10^{-3}, 10]$ . Yellow markers show fast fronts while blue show slow fronts. Grey lines show predictions for  $\bar{\alpha} = 10$  and  $\bar{\tau} \in [10^{-5}, 10^{-3}]$  from equation 14. The prediction for  $\bar{\tau} = 0$  follows  $\langle \bar{v}_c \rangle \propto \bar{M}_{0,1D}^{-\frac{\beta}{2(1-\beta)}}$ . (b) This transient velocity can be approximated by subtracting the steady state front velocity  $v_{c,\infty}$  from equation 5 and scaling with the initial rupture velocity  $v_{c,0}$  found from equation S32. The dashed line shows  $(\bar{t}\bar{v}_{c,0})^{-\beta}$  with  $\beta = 0.685$ . (c) The same fit when the steady state is not subtracted.



**Figure 4.** Moment duration scaling examples. Dashed lines show the evolution in time of the seismic moment. (star) shows the final value of the duration and the moment, while (circle) marks the point when the front reaches the end of the fault (i.e without afterslip) for  $N = 5 \times 2^{\{0,7\}}$ . The solid lines show the predictions of moment versus duration discussed in the text. The top three curves use the slow scaling (equation 17 and 18), while the bottom two use the fast scaling relation (equation 11). The colored regions highlight the different scaling exponents discussed in the text.

206 so that

$$\bar{M}_{0,1D} \approx \frac{\bar{\tau}}{3} \left( \frac{\bar{v}_{c,\infty} \bar{T}}{2} \right)^3 + \frac{1-\bar{\tau}}{2} \left( \frac{\bar{v}_{c,\infty} \bar{T}}{2} \right)^2. \quad (11)$$

207 This relation implies that there is a separate scaling for sub-shear ( $\bar{\tau} \rightarrow 0$ ) and super-  
208 shear rupture ( $\bar{\tau} \rightarrow 1$ ) in our simulations:

$$\bar{M}_{0,\text{sub-shear},1D} \propto \bar{T}^2 \quad (12)$$

209 and

$$\bar{M}_{0,\text{super-shear},1D} \propto \bar{T}^3. \quad (13)$$

210 Note that we also predict that  $\bar{M}_{0,\text{sub-shear},1D}$  will transition to a  $\bar{T}^3$  scaling for large  $\bar{L}$   
211 if  $\bar{\tau}$  is small but nonzero. The moment duration, in the fast regime using equation 11, is  
212 shown in the two bottom lines of Figure 4. This figure also shows numerically obtained  
213 values for the moment duration. The agreement between the numerical simulations  
214 and equation 11 is good.

### 215 **3.2.2 Slow regime**

216 For slow fronts,  $\bar{v}_c(\bar{t})$  is transient, and we observe that  $\bar{v}_c(\bar{t})$  is well described by  
217 a power law with exponent  $\beta$  for large  $\bar{\alpha}$  and small  $\bar{\tau}$ . To obtain an approximation  
218 for  $\bar{v}_c(\bar{t}, \bar{\alpha}, \bar{\tau})$ , we plot  $\bar{v}_c(\bar{t})$  for a selection of  $\bar{\tau}$  and  $\bar{\alpha}$  in Figure 3. All curves collapse  
219 when we subtract the steady state front velocity  $v_{c,\infty}$  and scale with the initial rupture  
220 velocity  $v_{c,0}$  given in equation S32. We can then write down

$$\bar{v}_{c,\text{slow}} \approx (\bar{v}_{c,0} - \bar{v}_{c,\infty})(\bar{v}_{c,0} \bar{t})^{-\beta} + \bar{v}_{c,\infty} \quad (14)$$

221 Figure 3 shows that this relation fits well with simulations for small  $\bar{\tau}$  and large  $\bar{\alpha}$ , and  
222 we measure empirically the exponent  $\beta = 0.685 \pm 0.002$  (using  $\bar{\tau} = 10^{-3}$  and  $\bar{\alpha} = 10$ ).  
223 To obtain a parametric equation for  $\bar{M}_0$  and  $\bar{T}$ , we need to find  $\bar{T}(\bar{L})$ .  $\bar{T}$  has two main  
224 components; the time it takes to rupture a fault of length  $\bar{L}$ ,  $\bar{t}_{\text{rupture}}$ , and the time  
225 it takes for all motion to stop. Unlike for fast fronts, the arresting phase in the slow  
226 regime is not governed by a backward propagating arresting front, but rather a slow  
227 exponential decay in velocity. We denote this time  $\bar{t}_{\text{afterslip}}$ , and define

$$\bar{T} = \bar{t}_{\text{rupture}} + \bar{t}_{\text{afterslip}}. \quad (15)$$

228  $\bar{t}_{\text{rupture}}$  can be found from equation 14

$$\bar{L} = \int_0^{\bar{t}_{\text{rupture}}} \bar{v}_c(\bar{t}') d\bar{t}' \quad (16)$$

$$= \frac{(\bar{v}_{c,0} - \bar{v}_{c,\infty})}{(1-\beta)\bar{v}_{c,0}^\beta} \bar{t}_{\text{rupture}}^{1-\beta} + \bar{v}_{c,\infty} \bar{t}_{\text{rupture}}, \quad (17)$$

229 The afterslip time can also be found analytically, and the detailed calculation is given  
230 in the supplementary information. The result is

$$\bar{t}_{\text{afterslip}} = \log(100) \frac{2\bar{L}^2 \bar{\alpha}}{\pi^2} \quad (18)$$

231 where  $\log(100)$  indicates that we take the time when 99% of the slip has been ac-  
232 cumulated (which is necessary because the afterslip is exponentially decaying). The  
233 prediction of seismic moment versus duration can then be found using equation 8 for  
234 the seismic moment, equation 17 for  $\bar{t}_{\text{rupture}}$  (this has to be solved numerically for  
235 nonzero  $\bar{\tau}$ ), and equation 18 for  $\bar{t}_{\text{afterslip}}$ , with  $\bar{T} = \bar{t}_{\text{rupture}} + \bar{t}_{\text{afterslip}}$ . The excellent  
236 agreement between the analytical approach and the numerical simulations is demon-  
237 strated in Figure 4.

238 We can determine the bound on the slow front scaling relation by noting that  
 239 for infinitesimal  $\bar{\tau}$ ,  $\bar{v}_{c,\infty} \approx 0$  and  $\bar{M}_{0,1D} \approx \frac{\bar{L}^2}{2}$ . This yields

$$\bar{T}_{\bar{\tau}=0} = \bar{v}_{c,0}(1 - \beta)^{\frac{1}{1-\beta}} \bar{L}^{\frac{1}{1-\beta}} + \log(100) \frac{2\bar{L}^2 \bar{\alpha}}{\pi^2}, \quad (19)$$

240 where the first term will dominate over the second term (negligible afterslip) for large  
 241  $\bar{L}$  because  $\frac{1}{1-\beta} > 2$  for the measured  $\beta = 0.685 \pm 0.002$ . We can then solve for

$$\bar{L} \approx \frac{\bar{T}_{\bar{\tau}=0}^{1-\beta}}{(1 - \beta) \bar{v}_{c,0}^{1-\beta}} \quad (20)$$

242 which gives us

$$\bar{M}_{0,\text{slow},1D,\bar{\tau}=0} \approx \frac{\bar{L}^2}{2} \propto \bar{T}^{2(1-\beta)} \quad (21)$$

243 with  $2(1 - \beta) \approx 0.63$ . We also observe a transition to a different scaling at large  $\bar{M}_{0,1D}$   
 244 when  $\bar{\tau}$  is nonzero. To obtain the exponent in this regime, we note that in this limit  
 245 the steady state rupture velocity is reached, so that

$$\bar{T} \approx \frac{\bar{L}}{\bar{v}_{c,\infty}} + \log(100) \frac{2\bar{L}^2 \bar{\alpha}}{\pi^2}. \quad (22)$$

246 For large  $\bar{L}$  and nonzero  $\bar{\tau}$ , the afterslip will dominate, so that  $\bar{L} \propto \bar{T}^{\frac{1}{2}}$ . The cubic  
 247 term in equation 8 will eventually dominate, which results in

$$\bar{M}_{0,\text{slow},1D,\bar{L} \gg 1, \bar{\tau} > 0} \propto \bar{T}^{\frac{3}{2}} \quad (23)$$

248 This means that the moment duration scaling relation in the slow regime is expected  
 249 to follow a power law with exponent  $2(1 - \beta)$  with a possible transition to  $\frac{3}{2}$  at large  
 250  $\bar{M}_{0,1D}$

## 251 4 Discussion

252 We have demonstrated that a simple Burridge-Knopoff model with Amontons-  
 253 Coulomb friction is capable of reproducing the range of power law scaling relations  
 254 between seismic moment and duration observed in nature. The simplicity of the model  
 255 means that we can calculate the scaling relations analytically, and we find the one-  
 256 dimensional exponents  $2(1 - \beta)$  with a transition to  $\frac{3}{2}$  for large seismic moments for  
 257 slow rupture, 2 for sub-shear rupture, and 3 for super-shear rupture, where  $\beta$  is the  
 258 power law exponent of the transient slow rupture velocity.

259 In this letter, we aimed to address two questions. First, whether there is a  
 260 separation of two distinct classes, or is there a continuum of possible scaling relations  
 261 between the fast and slow end-members. We argue that the most likely value for  $\bar{\tau}$  is  
 262 close to 0, which corresponds to shear stress at the dynamic level, or to ruptures where  
 263 the stress drop is small compared to the background stress like in faults (Shearer et  
 264 al., 2006). If this is indeed the case, the moment duration scaling should contain a  
 265 continuum of slip modes between the slow and fast end-members. However, because  
 266 large  $\bar{\tau}$  would in this case be unlikely, it would result in a distinction of fast and  
 267 slow scalings simply because this is more likely. This would indicate that both the  
 268 interpretations by (Ide et al., 2007) and by (Peng & Gomberg, 2010) hold in the  
 269 sense that there is a continuum of slip modes, but the natural variation of  $\bar{\tau}$  could  
 270 result in more frequent events along the end-member scalings. In our simulations, the  
 271 separation into the slow and sub-shear scaling relations occurs spontaneously under  
 272 the assumption that  $\bar{\tau} \approx 0$ .

273 The second question we aimed to address was whether a difference in  $M_0 - T$   
 274 scaling relations alone could be attributed to different physical mechanisms behind

275 slow and fast rupture. Our model contains only two dimensionless parameters, which  
 276 highlights that the observed scaling relations do not necessitate complex underlying  
 277 mechanisms. The same friction law with different values for the coefficients and a  
 278 varying prestress can explain the entire range of scaling relations, and the slow scal-  
 279 ing regime arises simply because slow rupture speeds are transient. We have previ-  
 280 ously shown that fast rupture is governed by inertia, while slow rupture is non-inertial  
 281 (Thøgersen et al., 2019), which has consequences for whether the slow and fast scaling  
 282 relations can be attributed to different underlying physical mechanisms. While the  
 283 derivation of the scaling relations presented in this letter does not require specifica-  
 284 tion of the underlying physical mechanism causing transient rupture speeds, transient  
 285 rupture speeds are only observed in the non-inertial regime. This suggests that the  
 286 different scalings observed in the model originate because fast rupture is inertial while  
 287 slow rupture is not.

288 To compare our results to observations on faults, it is instructive to discuss  
 289 relations that would be obtained for rupture on a 2D plane. If we can assume radial  
 290 symmetry, we can use the same expression for  $\langle \bar{u} \rangle$  as in 1D, but  $\bar{M}_{0,2D} = \langle \bar{u} \rangle \bar{L}^2$ , which  
 291 changes the scaling by a term  $\bar{L}$ . This changes the scaling relations to

$$\bar{M}_{0,\text{sub-shear},2D} \propto \bar{T}^3 \quad (24)$$

$$\bar{M}_{0,\text{super-shear},2D} \propto \bar{T}^4 \quad (25)$$

$$\bar{M}_{0,\text{slow},2D} \propto \bar{T}^{\{3(1-\beta),2\}} \quad (26)$$

294 where  $3(1 - \beta) \approx 0.945$  is the exponent that is dominant for  $\bar{\tau} = 0$  at large  $\bar{L}$ .  
 295 This is remarkably close to the hypothesized exponent of 1 from observations (Ide  
 296 et al., 2007). However, it should be noted that the prestress is not expected to be  
 297 radially symmetric, which puts limitations on this extension. We stress that future  
 298 studies should incorporate two-dimensional simulations to address these scaling relations  
 299 without such limitation.

300 The transition from  $3(1 - \beta)$  to 2 also indicates that a simple linear scaling relation  
 301 between seismic moment and duration for slow events is not appropriate, because it is  
 302 only valid at  $\bar{\tau} = 0$ . We find it likely that a scaling in the approximate range  $M_0 \propto T$   
 303 to  $T^2$  should be observed for slow events, depending also on the decaying exponent  
 304  $\beta$ . For a constant  $\bar{\alpha}$ , this variation in the power law exponent occurs due to changes  
 305 in the stress state of the interface. This is in line with observations, where different  
 306 studies have reported on scaling exponents ranging from approximately 1 to 2 (Ide et  
 307 al., 2007, 2008; Aguiar et al., 2009; Frank et al., 2018; Gao et al., 2012).

308 From our results in Figure 3 we are in a position to explain the observed relation  
 309 between average rupture speed and seismic moment (Gao et al., 2012). A transient  
 310 rupture speed with a decaying exponent  $\beta$  would result in a two-dimensional scaling  
 311 relation  $\langle \bar{v}_c \rangle \propto \bar{M}_0^{-\frac{\beta}{3(1-\beta)}}$ . Gao et. al (Gao et al., 2012) observed that slow events fol-  
 312 low the approximate relation  $\langle \bar{v}_c \rangle \propto \bar{M}_0^{-0.5 \pm 0.05}$ , which indicates that  $\beta \approx 0.6 \pm 0.025$ .  
 313 Using equation 26 yields a moment duration scaling relation for slow rupture follow-  
 314 ing  $\bar{M}_0 \propto \bar{T}^{\{1.1,1.3\}}$ , which is fully consistent with their observed linear relationship  
 315 between seismic moment and duration.

316 Here, we have assumed that propagation is not bounded. (Gomberg et al., 2016)  
 317 demonstrated that there will be a change from a two-dimensional scaling to a one-  
 318 dimensional scaling when the rupture propagation goes from unbounded to bounded in  
 319 one of the directions. While we have demonstrated that different scalings can originate  
 320 without such effect, a bounded system would add a number of possible transitions in  
 321 moment duration, and would in principle allow for scaling relations following both the  
 322 two-dimensional and the one-dimensional exponents.

## 5 Conclusion

Linear elasticity and Amontons-Coulomb friction with a viscous term is sufficient to produce a large variety in scaling exponents between seismic moment and duration. This suggests that different scaling relations for fast and slow slip events do not require complex underlying physical mechanisms. However, our findings do suggest that whether rupture is dominated by inertia or not plays an important role because fast inertial rupture fronts propagate at fairly constant speeds while slow non-inertial rupture fronts contain long-lived transients. Our findings also suggest that there exists a continuum of slip modes between the slow and fast slip end-members, but that the natural selection of stress on faults can cause less frequent events in the intermediate range. We find that the sub-shear scaling follows  $M_0 \propto T^2$  (which corresponds to  $T^3$  in 2D), while the slow scaling follows  $T^{2(1-\beta)}$  (which corresponds to  $T^{3(1-\beta)}$  in 2D) with a transition to  $T^{\frac{3}{2}}$  ( $T^2$  in 2D) for larger seismic moments depending on the prestress.  $\beta \approx 0.685$  corresponds to the power law decay in the slow rupture velocity with time. The model also predicts a separate scaling for super-shear rupture with  $M_0 \propto T^3$  ( $T^4$  in 2D).

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