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Learning fractions using a teaching model designed with applets and the number line: The cases of Alvaro and Fernanda

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As part of a general study, a teaching model for fractions based on applets created with GeoGebra and the number line as a conceptual and didactical resource was designed. The model has eight stages; each one is composed of an interaction-exploration part, and questions to favour students' thinking about their interaction with the applets. The purpose of the model is to contribute to the building up of better fraction mental object of students who finished elementary school. The performances of two students are characterized in this paper; both improved their fraction mental object. Before the teaching experiment, they only had ideas related to the use of fractions as fracturer. After working with the applets, those ideas changed. Both students could do tasks related to fractions as number and as measurer, particularly, they could represent fractions on the number line, identify which of two fractions was bigger, and distinguish proper and improper fractions.

Keywords: Aspects of fractions, applets for teaching, fractions on number line, teaching models.

Introduction

Investigating the processes of teaching and learning fractions has been a significant theme in the field of mathematics education. The previous statement can be linked to the fact that fractions are part of the basic education curriculum in the world (NCTM, 2000; SEP, 2011). Nevertheless, fractions have been considered as one of the complex concepts studied in basic education (Siegler, Fazio, Bailey, & Zhou, 2013), and as a consequence, students have low performance in mathematics. Therefore, it is necessary to design tools that promote the improvement of students' fraction mental object, in Freudenthal's (1983) sense; since a good mental object of this concept will allow pupils to have adequate performance in mathematics from elementary to high school (Siegler, Duncan, Davis-Kean, et al., 2012). Due to the inclusion of digital technologies as a cognitive resource for teaching mathematics (Drijvers, 2013), the design of a teaching model based on the aforementioned resources is one of the main purposes of the research described in this paper.

Research objectives

The main objective is to construct a teaching model in order to enrich the current one for Mexican elementary school. In this context, two particular purposes for students who finished elementary school are 1) to characterize their fraction mental object, and 2) to promote the building up of better fraction mental objects. The aim of this paper is to show evidence of the purposes defined above.

Theoretical framework and related literature

The Local Theoretical Model (LTM) (Fillooy, Rojano, & Puig, 2008) is used as a theoretical and methodological framework. The LTM serves to focus on an object of study through four

interrelated components: (1) formal competence, (2) teaching models, (3) models for cognitive processes, and (4) models of communication. These components serve to design a teaching model.

The formal competence component. From the example of the didactic phenomenology of fractions elaborated by Freudenthal (1983), and taking into account ideas of other authors, for example, those of Kieren (1992) and Behr et al. (1983), the formal component of the LTM is constructed. This component is summarized in a scheme by means of which uses and aspects of fractions and their relationships are identified from everyday language to formalization through mathematical constructions. The aspects of fractions that are considered according to Freudenthal (1983) are descriptor, fracturer, comparer, measurer, operator, and number. For more about those uses and aspects of fractions see Valenzuela, Figueras, Arnau and Gutiérrez-Soto (2017a).

The teaching models component. A characterization of the teaching model of the last cycle of the Mexican primary school - 5th and 6th grades, children between 11 and 12 years old- was made. Results allowed the identification of situations in which all aspects of fractions underlie, and different representations and models are used. However, greater emphasis is placed on fractions as fracturer and as number. This led to design a new teaching model based on technology and focusing on fractions as measurer and as number; in particular, the emphasis is placed on notions of equivalence, order, density and the characterization of proper and improper fractions. To obtain the link for using the teaching model write to the authors (an English version will be available soon).

The cognitive processes component. As part of this component, reference is made to errors, difficulties, and students' actions when they use fractions. An important purpose of this study is to have a catalogue that serves as a reference for analysis of students' performances. Among the difficulties students have, informed by other researchers, are the following: 1) difficulties to identify a fraction represented in a figure with more than one fractional unit (Saxe, Taylor, McIntosh, & Gearhart, 2005); 2) difficulties to identify improper fractions (Tzur, 1999); 3) consideration of the whole as the entire segment and not the defined unit segment in segments of a number line with more than one unit (Ni & Zhou, 2005); and 4) lack of association of a fraction with a point or the right end of the segment on the number line (Petitto, 1990).

The communication component. This component is related to the processes of communication between different actors in a teaching and learning situation. In this case, the fundamental process considered is established between the teacher/researcher and the students via their interaction with the applets. Figure 1 shows how technological tools in this research are designed to promote the communication process. Through a series of codes, the teacher/researcher provides information to students; this favours interaction between student and applet. Students' information provided through applet interaction is saved in a database, which is used for the analysis of their behaviour.

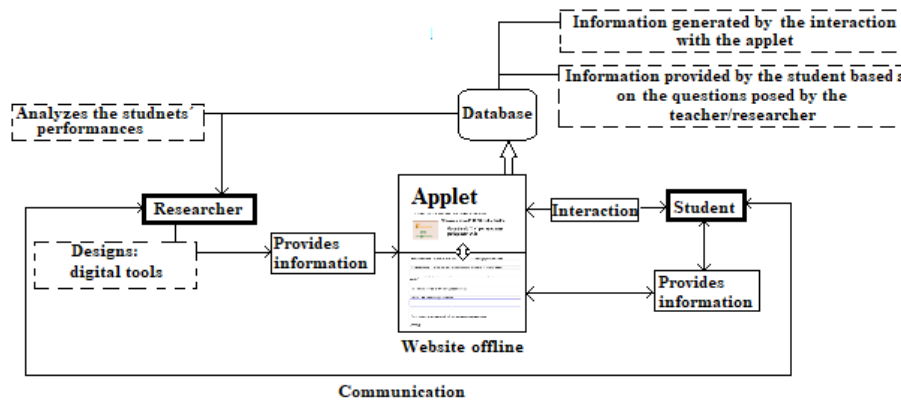


Figure 1: The communication process between Teacher/Researcher-Applet-Student

The teaching model

The final teaching model is formed by eight phases (see Figure 2); in each one, students' interaction is similar as the description in Figure 1, but every phase has a different purpose, related to the study of order, equivalence, density, and characteristics of proper and improper fractions. The design of the first two applets is described in Valenzuela, Figueras, Arnau and Gutiérrez-Soto (2017b).

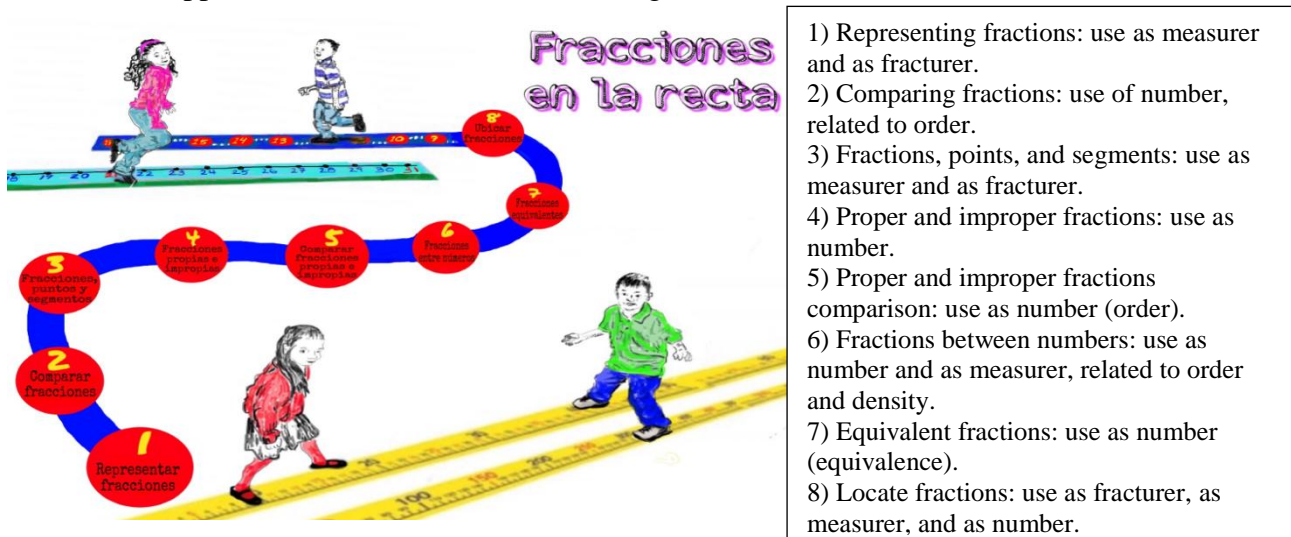


Figure 2. Screenshot of teaching model sequence offline

Population and method

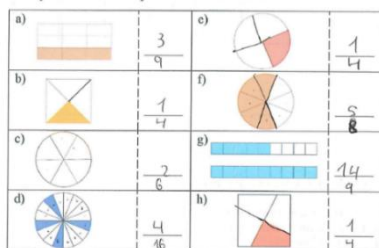
The experimentation component of the study had two phases. A pilot study; 45 students (4 groups) of a public secondary school located in a troubled zone of Valencia, Spain, took place. It helped to validate the teaching sequence, modify questions and verify applets' operation, and was considered necessary to realize an introductory activity to explore GeoGebra. In the main trial, 10 pupils of a public primary school in a marginalized zone of Zacatecas, México participated. In both, students worked individually and no help was given. Teachers only participated when applying the initial and final paper and pencil tests (evaluation and validation instruments). For analysis, the evidence stored in the computer as a product of the student-applet interaction was used. This report concerns Alvaro and Fernanda's performances. Alvaro is a 13-year-old student with low grades who participated in the pilot study, he had already used a computer before the experimentation. Fernanda

is 12 years old, and has medium-high grades, she participated in the main trial and had never used a computer.

The pilot study was carried out in 5 sessions of 45 minutes each, the main trial in 6 sessions from 60 to 90 minutes, including the activity to explore GeoGebra. In both approaches, an initial test was applied, the teaching sequence was carried out, and a final test was used. The tests are employed as evaluation instruments that help to validate the teaching sequence, in which aspects of fraction as measurer, as number and as fracturer are taught. Tests have 6 tasks. The first three assesses aspects related to the use of fraction as fracturer in continuous and discrete models. In task 4, the uses of fraction as measurer and as fracturer in the number line model are evaluated. Task 5 deals with the use of fraction as number, and in task 6, different uses of fractions appear in a context of problem-solving. Following this order, Alvaro's and Fernanda's performances are described below.

Alvaro's performance

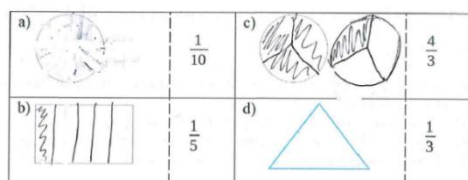
Alvaro's answers in the initial test, confirms that his fraction mental object allows him to establish a fracturing relation in a graphic representation of a fraction, particularly when using discrete and area models. In Figure 3, the processes to establish the fracturing relation is shown, the student extends the linear partitions to identify the minor fractional unit (items b, e, f, and h). The trace of the pen allows seeing that there is a tendency to count the number of parts, especially when the partition has a "large" number of parts (items d and f).



Instructions: In the following figures, what fraction represents the part that is painted? Write your answer on the line in the box.

Figure 3: Alvaro's answers to represent fractions symbolically in an area model, initial test

Alvaro is also successful representing graphically fractions from a symbolic expression (proper and improper), except in the case of a triangle to represent $1/3$, see Figure 4. In the processes of partitioning, Alvaro estimates the equality of the parts by congruence. He has technical difficulties to make a partition, but not conceptual ones as sustained by Streefland (1991).



Instructions: In the following figures, represent the indicated fractions.

Figure 4: Alvaro's answers to represent fractions in an area model, initial test

Alvaro was not able to represent fractions on the number line. Apparently, he fails to transfer his knowledge about partitioning in an area model to a linear model -number line-. He attempts to answer task 5, however, there is a tendency to use decimal numbers to provide fractions between

two integers, but has difficulties to supply fractions. He does not recognize the terminology of proper and improper fractions and faces difficulties in solving problems; he did not answer task 6.

Answers given by students during the interaction with the applets in the teaching sequence are analysed to identify: 1) processes they follow to represent fractions on the number line, 2) recognition and justification of fraction comparison, 3) recognition of equivalent fractions, and 4) characterization of proper and improper fractions.

During the teaching sequence, Alvaro did not achieve the purpose of stage 1. A tendency to dissociate numerator and denominator in his interpretation of symbolic and graphic representations of fractions on the number line was identified. But, this idea changed. In stage 3 he recognized a fracturing relationship between part and whole. In stage 2 the student wrote three proper fractions ($1/2$, $2/3$ and $2/4$) but no improper ones. To justify the order of fractions he focuses on the length of the segment that represents a fraction; he justifies his answer writing: “ $2/4$ is smaller because it occupies less than the others”. In stage 4, Alvaro faces difficulties to express characteristics of proper and improper fractions, but in stage 5 characterizes an improper fraction, comparing the numerator and denominator, he wrote: “the numerator is larger than the denominator”. In the teaching sequence, the student only managed to identify two equivalent fractions: $4/1$ and $40/10$.

As mentioned in the initial test, Alvaro failed to represent fractions on the number line, but after exploring the applets of the teaching sequence, in the final test (solved in a paper), he succeeded locating fractions on the number line, as shown in Figure 5.

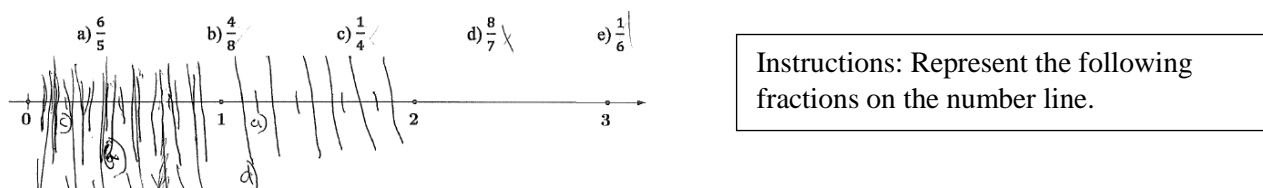


Figure 5: Alvaro's answers in the final test to represent fractions on the number line

As observed, the student makes the complete partition for representing each fraction. One strategy he uses is to make stripes of different sizes to distinguish the different fractional units. In addition, a trace of counting the parts is observed.

Also in Figure 6, there is evidence that Alvaro improved his partitioning processes to represent fractions in tasks related to an area model, although this was not the focus of the teaching sequence.

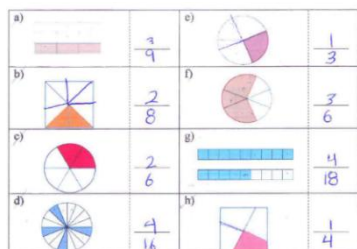


Figure 6: Alvaro's answers to represent fractions in an area model, final test

Fernanda's performance

In the initial test, Fernanda showed solid ideas about fraction as a fracturer, both to establish a fracturing relation in continuous (Figure 7) and discrete models, as well as its use in a fracturing

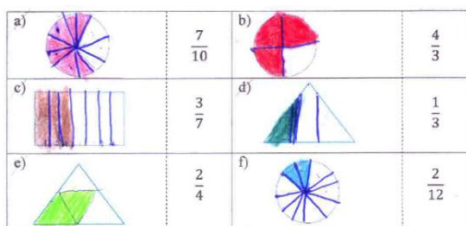
operator aspect (Figure 8). She only faced difficulties in establishing a fracturing relation in graphic models where the difference in the size of the parts is not so evident (items e and f of Figure 7). But after studying the teaching sequence, the student managed to respond even to this type of situations.



Instructions: In the following figures, what fraction represents the painted part? Write your answer on the line in the box.

Figure 7: Fernanda’s answers to represent fractions symbolically from area model, initial test

The processes used by Fernanda to establish the fracturing relation are based on counting the number of parts (items d, f, and g). In fact, in item f she did not distinguish the differences between the sizes of parts. The student also extends the partitions to identify the minor fractional unit (items b, e, and h). In item e, probably she had a counting error. Moreover, Fernandas’ fraction mental object allowed her to use partitioning processes to establish a fracturing relation in a graphic representation from a symbolic expression of a fraction (see Figure 8).

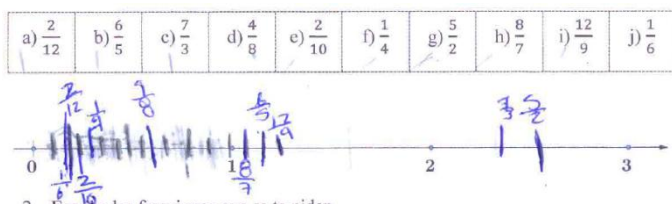


Instructions: In the following figures, represent the indicated fractions.

Figure 8: Fernanda’s answers to represent fractions in an area model, initial test

Fernanda could express the fracturing relationship in continuous models, but did not manage to extend the model to represent improper fractions (item b), she changed numerator by the denominator. She also showed difficulties in fracturing a whole in a large number of parts -12 -. With respect to the number line, Fernanda was not able to transfer her knowledge about partitioning in an area model to a linear model -number line-. In the initial test, she did not answer tasks related to representing fractions on the number line, nor those regarding the use of fractions as a number.

Fernanda’s performance in the teaching sequence was among the best in her group. She only showed less ability in stage 2, the criteria used to establish order between fractions was a comparison of numerical values of a fraction, which led to errors. However, with respect to the other subjects taught, she was more successful in stages 1 and 3, related to fractions as fracturer and as measurer. She also managed to characterize proper and improper fractions comparing numerator with denominator. Fernanda was able to establish several equivalent fractions during her interaction with the applets in the teaching sequence, but also later, since in the final test she showed skills to write equivalent fractions of some given fraction. After studying the teaching sequence, in the final test, Fernanda was able to answer correctly tasks related to fraction’s uses as measurer and as number. Particularly, she managed to represent fractions on the number line (see Figure 9).



Instructions: Represent the following fractions on the number line.

Figure 9: Fernanda's answers in the final test to represent fractions on the number line

In the process to represent fractions, Fernanda made lines of different sizes to distinguish partitions. Apparently, she did not split all the unit segments. For example, to represent fraction $7/3$, only a third part was indicated in the interval $[2, 3]$. Also, she expressed $2/12$ and $1/6$ in the same place, which shows that she recognizes their equivalence, and she puts $2/10$ to the right of $1/6$, an idea about order between fractions. Additionally, the student improved her partitioning processes to establish a fracturing relation in both static and dynamic processes.

Conclusions

Although in the experimentation a small period of time was devoted to interacting with the applets, there is evidence that students learned other aspects of fractions, mainly those related to their location on the number line. In the initial test, it was observed that Alvaro and Fernanda's fraction mental object allowed them to solve mainly tasks related to the use of fracturer in continuous and discrete models. However, this knowledge could not be transferred to represent fractions in a linear model. So the teaching in a single approach is not a guarantee for students to be able to transfer that knowledge to solve tasks where other uses of fractions are required. The proposal to use new technological resources that allow visualization and manipulation of actions that are done on the objects (fracturing, comparing and ordering) helped students to improve their ideas related to fractions as measurer and as number. Moreover, there is evidence that despite the fact that in the teaching model emphasis is not placed on the area model, students were able to improve their partition processes in order to establish a fracturing relation in this type of models. That is, a teaching model based on the number line and the use of applets helps students to improve their ideas about the use of fraction as fracturer in an area model. The use of a computer in a marginalized community did not involve difficulties in the teaching model, in fact, students were able to transfer ideas from the manipulative computer task to the "non-manipulative" paper task.

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