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► To cite this version:

Maria T Sanz Garcia, Olimpia Figueras, Bernardo Gómez. A tool to evaluate students' performance in solving fraction word problems. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02401082

HAL Id: hal-02401082

<https://hal.science/hal-02401082>

Submitted on 9 Dec 2019

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A tool to evaluate students' performance in solving fraction word problems¹

Maria T. Sanz Garcia¹, Olimpia Figueras², and Bernardo Gómez¹

¹ Departamento de Didáctica de la Matemática Universidad de Valencia, Valencia Spain;
m.teresa.sanz@uv.es, bernardo.gomez@uv.es

² Centro de Investigación y de Estudios Avanzados del IPN, Ciudad de México, México;
figuerao@cinvestav.mx

In this paper a tool to evaluate students' performance in solving particular fraction word problems is described. It is a pencil and paper test with seven tasks, which have been designed ad-hoc from a previous historical-epistemological study of this kind of problems. The tasks have in common that all of them involve iterative processes to calculate part of the part complement. The test was validated through reliability and content validity. The difficulty of each task was obtained through the resolution success probability based on the analysis of the responses of 202 Mexican students between 15 and 16 years old. Difficulty of the task is increased when its structure is more complex. Surprisingly, only a small number of students used algebra to solve the problems.

Keywords: Fraction word problems, part of part of a whole, problem solving methods, validation test.

Introduction

Currently, problem solving emerges with renewed interest in curricular proposals because it is considered a basic competence, particularly in the development of arithmetic and algebraic thinking. An example of this assertion is that it appears explicitly in the curriculum of basic education of various countries (i.e., in Mexico (SEP, 2011)). Word problems have played a central role in the school context (Puig & Cerdán, 1988). Textbooks include a wide range of these problems, whose wording includes a setting, which is not expected to give answer to some practical situation. In Polya's typology (cited by Puig & Cerdán, 1988) such problems are refer as "finding problems" –it is requested that, under certain conditions, a quantity be determined from others that are therefore known–.

The fact that they appear in school curriculum does not imply that their teaching or learning is simple. A big number of authors have studied how this type of problems is a source of serious difficulties for many students (Korpershoek, Kuyper, & Van der Werf, 2015; Nesher & Teubal, 1975; Riley, Greeno, & Heller, 1983; Verschaffel, De Corte, & Pauwels, 1992). Daroczy, Wolska, Meurers, and Nuerk (2015) show that difficulties can be caused by: a) the linguistic complexity of the statement, b) the numerical complexity, or c) the relationship between the linguistic and

¹ This research has been supported by Conselleria d'Educació, Investigació, Cultura i Esport through the GVPROMETEO2016-143 project, the Ministerio de Educación through EDU2017-84377-R (AEI/ERDF, EU) project, and the University of Valencia and Cinvestav within the framework of the Iberoamerica-Santander Research Scholarship Program.

numerical complexity. Other authors consider the problem difficulty as the probability to solve them correctly (Ivars Santacreu & Fernández-Verdú, 2015; Riley, Greeno, & Heller, 1983).

More often than not, these studies about word problems are based on statements that involve whole numbers. However, in the case of fraction word problems the literature is scarce, both regarding its classification, and the identification of students' difficulties encountered when solving them. This lack in the literature leads us to the first aim of the study described in this paper: To characterize students' performance when solving particular fraction word problems and to measure their difficulty through the probability of solving them correctly.

The word problems chosen for this research have a common feature: an unknown quantity, which is partitioned into parts expressed by fractions. These problems repeatedly appear in the history of Mathematics, in books from different times and cultures, often solved in different ways (Gomez, Sanz, & Huerta, 2016). In this report, we chose to focus on problems with the common syntagma "of what remains", that is referred to a part of another part, which is the complement to the unit. The following is an example of this type of problems:

I found a stone, but I did not weight it; after taking away a $\frac{1}{7}$ and then a $\frac{1}{13}$ [of what remains], I found that its weight was 1 manna. What was the original weight of the stone? (Mesopotamian origin, Katz 2003, p. 27)

The common syntagma is often repeated, as in the next example:

A pilgrim carried a certain amount of money. He gave away half the amount (to Brahmins) at Prayaga. He spent two-ninths of the remaining amount in Kashi. One-fourth of the remainder was paid as duty. He spent the $\frac{6}{10}$ th part of the remainder in Gaya. Finally, he returned home with 63 *niskas*. If you know the fractional residues, find the amount he carried. (Lilavati, 1150/2006, p. 59)

This situation allows us to raise the second aim of the research: To determine if the increase of steps in the statement entails an increase in the problem difficulty.

Methodology

Students between 15 and 16 years old were considered for the research. In the inquiry carried out in a public school at Mexico City a sample of 202 individuals was formed.

For data collection, a paper and pencil test was designed with seven fraction word problems of the type aforementioned. A previous study was taken into account in which those problems were classified and mathematical elements necessary for their resolution identified (see Gómez, Sanz, & Huerta, 2016).

In a 90-minute session, students completed individually the test. They were allowed to use a pen, but not calculators, or correctors that could interfere with qualitative analysis that was going to be carried out.

The answers to the tasks were categorized following the guidelines of Taylor and Bogdan (1987), and a validation of the test was carried out according to Lacave, Molina, Fernández and Redondo (2016), as well as an analysis of the answers with a descriptive approach using IBM SPSS Statistics 22.

Design of the test

Paper and pencil test design was structured taking into account two aspects: a Historical and Epistemological Analysis (hereinafter HEA) and a Didactic Analysis (hereinafter DA). According to Gómez (2003), the first aspect allows to investigate how the teaching of mathematics has been configured at different moments in history. In particular, regarding the solving of fraction word problems, it provides information about structure, context, quantities involved and solving methods that most relevant authors have included on documentary sources. The second aspect has the purpose to identify analytical readings, that is, the quantities involved in a problem and relationships between them (Puig & Cerdán, 1988).

Test problems

To design the test's tasks, previous work is considered. Gómez, Sanz, and Huerta (2016) and Sanz and Gómez (2018) made a classification of fraction word problems through an HEA study. In Figure 1, part of this classification is shown; in it, the problems under study in this inquiry are included.

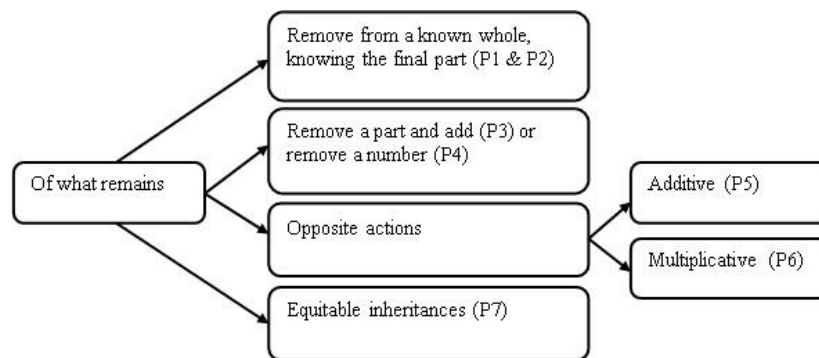


Figure 1: Classification of word problems of the type: “of what remains”

Table 1 shows the problems used in the test, they are an adaptation of the classical problems identified through HEA. Despite the fact that P1 and P2 have the same structure (see Table 1), P1 needs two steps to be solved, and P2 needs three. Given that the easier type was initially considered due to its analysis-synthesis scheme, it was decided to explore a possible change of students' performances when introducing an additional step (aim 2 of this research).

P1. A pole is painted in red, blue and black. The black part is a $\frac{1}{3}$ of the length of the pole, the part in red is $\frac{2}{3}$ of what remains, and the blue part measures 2.70 meters. How long is the pole?

P2. Miguel plays a video game. He has to end it in the time indicated on the screen. He plays one-third of the time scheduled, pauses the game and goes to have dinner. When he comes back, he moves up one-fifth of what remains. After supper, he moves up one-seventh of what remains. A message appears on the screen: ‘32 minutes left’. What time was indicated on the display at the beginning of the game?

P3. A thief went into a palace and found a bag full of coins. When he tried to escape, a palace doorman caught him; the thief offered him half of the coins to let him escape. The doorman gave back 80 coins to the thief and let him go. Another doorman took the thief by surprise, and again the thief offered half of the coins of what remained in the bag to let him escape. The second doorman received the number of coins but he gave 50 coins back. Finally, when the thief goes out, he has 200 coins in the bag. How many coins were there in

the bag at the beginning?

P4. A man who has been picking up some oranges returns to his hometown. He visited two friends and he gave to one of them half of the oranges and half an orange. He gave half of the rest of oranges plus the other half orange to his second friend. After his visit, he left 10 oranges. How many oranges had he picked up?

P5. Carlos has a glass of wine and drinks $\frac{1}{4}$ of it. He fills it with water and drinks $\frac{1}{3}$ of the mixture. He refilled it with water and drank $\frac{1}{2}$ of the liquid obtained. What part of pure wine does he have left to drink?

P6. Juan went on an excursion and took a bottle of water. When he found a faucet, he duplicated the content of the bottle and drank $1 \frac{9}{10}$ dl. Later, he doubled the content that he had left and drank $1 \frac{9}{10}$ dl. Surprise! The bottle was empty. How much water was in the bottle at the beginning of the excursion?

P7. A man made his will and divided the heritage among his sons. One received a thousand pesos and $\frac{1}{7}$ of the rest; other received 2000 thousand pesos and a $\frac{1}{7}$ of the rest; another received 3000 thousand pesos and a $\frac{1}{7}$ of the rest, and so successively for all his sons. One of his sons said the distribution was not fair, but the father said all had the same amount of money. How much did each son receive? How many sons did the man have?

Table 1. Fractions word problems included in the paper and pencil test

Theoretical resolution of problems (analysis-synthesis)

The analysis of students' performances took into account ways to solve problems. In order to obtain all possible strategies that a student can follow in a problem solving procedure, an AD was carried out through an Analysis-Synthesis (A-S) scheme (Puig & Cerdán, 1988) using the graphic symbols employed by Cerdán (2007), see Figure 2.

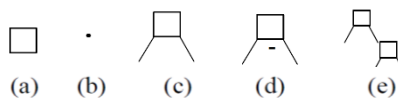


Figure 2: Symbols for A-S: (a) Unknown quantities, (b) Known quantities, (c) Relationships among them, (d) Operation/relationship between them, and (e) Nesting between quantities

In order to show the process followed, an analysis of problem P1 is recorded. Two A-S schemes of the first task (P1 in Table 1) are included in Figure 3 to show that solving procedures are not unique and that each one represents a form of reasoning, a way to relate the quantities involved. For the case of P2, the process starting from Q (see Figure 3) is repeated (due to shortage of space in this paper, the A-S scheme of P2 is not included).

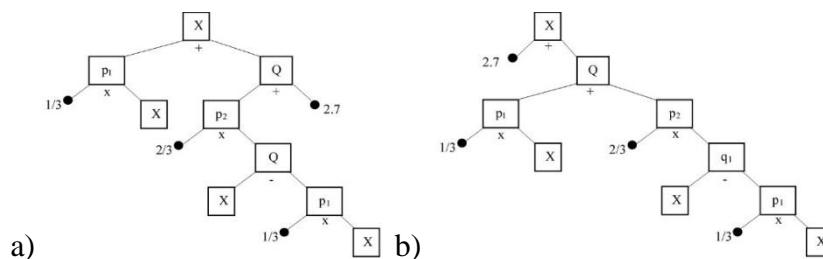


Figure 3: A-S Scheme for P1. a) x =total pole, p_1 =part painted in black, Q =the rest after painting in black, p_2 =part painted in red. b) x =total pole, p_1 =part painted in black, Q =the rest painted in red and black, p_2 =part painted in red, q_1 =the rest after painting in black

Categorization for the analysis of resolutions

The A-S scheme allows us to categorize the task resolutions. These categories will enable to identify profiles of students' performances and to carry out a subsequent qualitative analysis of the answers. In addition, the quantitative analysis, both descriptive and inferential (to be done with a large amount of data), requires this type of categorization in order to obtain more precise analysis.

Table 2 shows an example of the categories and encoded resolutions for a task, in this case P1 (Table 1). There are two different types of categories. On the one hand, the C3, C4, C5 and C6 codes are related with the analysis of students' solving procedures through A-S schemes. For example, C3 is related with p_1 (see Figure 3) or C4 is related with Q (in Figure 3). On the other hand, C1, C2 and C7 codes have the intention to collect other types of information. For example, the method used in C1 could be: graphic, direct arithmetic, algebraic, inverse arithmetic, or trial and error (methods identified by Gómez, Sanz and Huerta (2016) using HEA). Moreover, if a graphic support is used for solving the problem, C2 has been included, since the literature shows how graphic support during solving procedure leads to greater success in finding the solution (Greeno & Hall, 1997). Finally, a category is introduced to value the numerical results obtained, C7, that is, if the solution is correct or not.

It should be pointed out that 'Nothing' is included in all categories; there are students who do not write anything or who wrote calculations, but without any answer highlighted, and this situation must be considered in the analysis.

	C1	C2	C3	C4	C5	C6	C7
Code	Method	Graphic support	Part of a whole	What remains	Part of a part	What remains	Correct Solution
0	Graphic	No	No	No	No	No	No
1	Direct Arithmetic	Yes	Yes	Yes	Yes	Yes	Yes
2	Algebraic	Nothing	Nothing	Nothing	Nothing	Nothing	Nothing
3	Inverse Arithmetic						
4	Trial and Error						
5	Nothing						

Table 2. Categories and encoded resolutions for Problem P1

Test Validation

As mentioned before test validation is done according to Lacave, Molina, Fernández and Redondo (2016). Reliability was tested with the Alpha Cronbach coefficient as a measure of the internal consistency, obtaining 0.9248. The selected coefficient is higher than 0.90 (George & Mallery, 2003, p.231), so reliability is excellent. In addition, the homogeneity index was calculated to decide if any task had to be eliminated, but it was not necessary. Content validity was made through a peer discussion with nine colleagues from different educational levels in México. They modified the

wording to adapt it according to students' use of Spanish in Mexico City, but the content and structure of the problems was not modified.

Results

The measure of the difficulty (d) of the fraction word problems (those in Table 1) is shown in Table 3. It is calculated as the probability of solving them correctly according to $d = \frac{\text{Correct}}{\text{Total}-\text{Nothing}}$, where Correct is the number of students who obtained the correct answer; Total is the sample size; and Nothing is the number of students who have left the problem without an answer.

	P1	P2	P3	P4	P5	P6	P7
Correct	25	8	54	6	35	11	1
Nothing	7	58	24	17	14	51	51
Total	202	202	202	202	202	202	202
d	0.128	0.056	0.303	0.032	0.186	0.073	0.007

Table 3: Summary of correct solutions by problem

From results of Table 3 it can be asserted that for Mexican students between 15 and 16 years old, P1 is easier than P2 ($d_1=0.128>0.056=d_2$), this confirms the second aim of this study: to add a step in the statement of the fraction word problems makes its resolution more difficult.

To expand the study about students' performances, a detailed analysis, related to the categorization, for problem P1 is shown. Table 4 exhibits details of the analysis' categories for such problem. C3, C4, C5, and C6 are subsequent categories, that is, if a student reaches C5 it means that C3 and C4 have been achieved. Beside absolute frequencies, percentages are in brackets. Even though the A-S scheme indicated that P1 was the most straightforward problem, only twenty-five students gave a correct solution.

Code	0	1	2	3	4	5	Total
C1 Method	24 (11.88)	162 (80.20)	8 (3.96)	0	0	8(3.96)	202
C2 Graphic support	77 (38.12)	117 (57.92)	8 (3.96)				202
C3 Part-whole	140 (69.31)	54 (26.73)	8 (3.96)				202
C4 What is left	147 (72.77)	47 (23.27)	8 (3.96)				202
C5 Part-part	150 (74.26)	44 (21.78)	8 (3.96)				202
C6 What is left	163 (80.69)	31 (15.35)	8 (3.96)				202
C7 Pole height	164 (81.19)	25 (12.38)	13 (6.44)				202

Table 4: Students' frequencies in each category of analysis, percentage is shown in parenthesis

Seventy-seven students used graphical representations, but always as support for the resolution; the part of a part can be seen only in twenty-four cases. The rest is as indicated: a rectangle is divided into three parts, and each is painted in one colour without pointing out a relationship between them.

Regarding category C7, incorrect answers were very varied, among them the following are highlighted: a) No answer (8/202); b) Pole height 8.10 m (34/202); in this case students considered all parts as equal, 2.7 m long, and since the pole was divided into three parts, they multiplied 3 times 2.7; c) Pole height 3.7 m (46/202), students added $1/2$ plus $2/3$ plus 2.7 m. They could not

identify that such fractions did not represent a number but an operator; d) Pole height 10.8 m (16/202), in this case students computed the part of the whole and the part of the part but could not accomplish correctly the last step of what is left to reach the solution.

Conclusion and Future Research

This research is a part of a project with a general aim: to include this type of problems in an Intelligent Tutorial System (ITS) to help students accomplish learning, by predicting their performance in a following task. A mathematical model to predict student's performance and to provide a sequence of problems adapted to his capabilities will be used. This model will be build up from data obtained from the ITS, but it is necessary to dispose previous information, for instance, the difficulty of each problem, diverse solving methods, previous fraction knowledge, learning comprehension or student's age. Such information is collected by means of paper and pencil tests, and in this paper, we present some of them.

With the tool described in this paper, information about solving a particular word fraction problem was collected, which have a common characteristic, in its statement the syntagma "what remain" appears. The mathematical meaning of this syntagma refers the part of the part of the complement. The information provided is diverse. On the one hand, the difficulty of each problem increases when the steps of the problem as mentioned before increases. This result will be used in the mathematical model as initial information to start the specific sequence for each student. On the other hand, the different solving methods for the same problem have been obtained through the different A-S scheme. This information will be written in the ITS to help the student when they choose one way to solve the task.

Other kind of results obtained in this study remark the arithmetic solving methods in contrast to a remarkable lack of an algebraic approach, even though students had previous contact with algebra in secondary education (SEP, 2011). Moreover, difficulties with fraction knowledge learned in primary and secondary, such as graphic representation or operations, were observed. This result leads us to believe that basic fraction knowledge should be examined previously and therefore, the relationship between them and the solving of fraction word problems should be investigated further.

Finally, note that this research is done with a sample of Mexican students between 15 to 16 years old. But, the ITS can be used by students with different ages and different nationalities, then other random and statistically significant student samples of different ages should be studied.

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