Flexible mental calculation skills of freshmen and graduates

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Theoretical Framework

Researchers and mathematics educators agree on the importance of flexibility in mental calculation, and in the last two decades a lot of research on students’ development in flexible mental calculation was conducted (brief overview see Rathgeb-Schnierer & Green, 2017). Torbeyns et al. (2011) investigated flexibility and adaptivity in adults and revealed that they frequently and efficiently apply indirect addition on subtraction problems up to 100.

Baumert & Kunter (2011) describe four professional competences: beliefs, motivational orientations, self-regulation and professional knowledge. Professional knowledge is again subdivided in five category groups. The performance of flexible mental calculation can be considered as content knowledge. During their teacher training program students should acquire the competences which are important for their future teaching activity. Therefore, it is necessary to know about students’ knowledge at the beginning and the end of university.

Regarding the described aspects, we summarize that teachers are supposed to be flexible in mental calculation. Furthermore, we expect all students who currently entered university to perform flexibly since this has been a central aim in German primary schools for the last 15 years.

Overview on the Project

Due to the fact that there is hardly any research on teachers’ professional knowledge regarding flexibility in mental calculation, this study focuses on this aspect. Thereby, there is a special focus on the comparison of freshmen and graduate students as well as students with mathematics as major and minor subject. Three questions are posed in this study: Do freshmen and graduates use adaptive expertise? Do students with mathematics as major subject perform better than students with mathematics as minor subject? Can we find differences both between freshmen and graduates and between students with mathematics as subject and those with mathematics as basic education?

Design

A CRF 2x2 experimental design is used to test the research hypothesis. There are two independent variables: the area of studies (Factor A: a1 = mathematics as major; a2 = mathematics as minor) and the length of study (Factor B: b1 = freshman, b2 = graduate). The dependent variable is a survey with ten different tasks to solve up to 1000.

Procedure and data analyses

The sample size is determined with a type II power analysis – N as a function of power (1-β), Δ, and α. The desired power (1-β) is 0.99, and medium effects (Δ = 0.50) in relation to the dependent
variable are classified as significant; the significance level is $\alpha=0.05$. Our sample were drawn from the university courses for freshmen and graduates, and comprised 77 freshmen and 75 graduates.

We used seven tasks with special problem characteristics (851–426, 960–320, 923–398, 906–891, 999–699, 672–335, 853–497). Firstly, every task was coded with a range zero to three. Therefore, the maximum benchmark for flexibly solving all seven tasks was 21, the minimum benchmark for correctly solving all seven tasks was seven. Secondly, we developed a score for flexible mental calculation from one to six. Score one represents high flexibility, and score six rigidity. This score for flexibility was subsequently mapped with the points of the coding.

For analyzing our empirical data four steps were carried out: (1) First, we screened the data and replaced missing data by the multiple imputation approach. (2) Second, we analyzed the data descriptively. (3) Third, we conducted a two-way ANOVA in accordance with the CRF 2×2 design. (4) We did a posteriori $t$-tests to analyze several factor levels.

**Results and Discussion**

The main effects A (area of studies) and B (length of study) were significant at the $\alpha$ level of .05 ($F_{1,148} = 4.38$, $p < .04$; $F_{1,148} = 14.40$, $p < .01$). The corresponding $H_0^A$ and $H_0^B$ were rejected in favor of $H_1^A$ and $H_1^B$. According to the mapping score, the results show as well that students with mathematics as a major subject start with the mean score of 3.95 and improve to 3.00. Likewise do students with mathematics as minor subject: They improve from 4.16 to 3.61 in mean. Regarding these score-levels, there is an improvement in both groups. However, the reached scores are not adequate at all for prospective primary teachers. Our results suggest a modification of the teacher training program in math courses as well as math education courses in terms of targeting teaching towards flexibility. This could be accomplished by focusing on inherent structures and discussing them on a meta-level in arithmetic and algebra classes, as well as by collaboration of teacher trainees and school students in classes which explicitly emphasize flexible mental calculation.

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**References**

