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Mental calculations with rational numbers across educational levels

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In this study primary, secondary and tertiary education students try to calculate mentally the very same tasks involving operations with rational numbers (mainly fractions and mixed numbers). The aim was to record the range of strategies used by the students and at the same time to examine how these strategies are distributed across these three educational levels. The findings give evidence that no matter the educational level, the dominant strategy is the use of mental form of the written algorithm. All the other strategies are underrepresented. Moreover, it seems that the use of rational numbers in mental calculation is in itself an obstacle for the students since a great percentage of them either did not give any answer or the answers they gave were incorrect or non-codable.

Keywords: Mental calculations, rational numbers, written algorithm.

Introduction

Mental calculations in arithmetic refers to the process of calculating the exact result of an arithmetical expression given that the solvers use neither a calculating device nor any kind of writing and which usually demands an application of mental strategies. In 1924, the well-known mathematician Carathéodory, during a lecture he delivered for the Hellenic Mathematical Society highlighted the vital importance of the students' ability "to calculate mentally instead of merely writing down, for example, simple subtractions or multiplications" (p. 87). The main interest of the majority of the relevant research studies lies particularly in identifying and organizing the strategies the solvers use. A considerable body of these studies examines mental calculation in the realm of whole numbers and their four basic operations (Rezat, 2011). Lately, researchers express a growth interest on the mental calculations in the set of rational numbers concerning either comparison of rational numbers (Yang, Reys & Reys, 2009) or operations with rational numbers (Caney & Watson, 2003).

In this study our focus lies on the strategies used by students across all the educational levels when they mentally make calculations that involve rational numbers. More precisely, our research questions are:

- What is the range of the strategies used by the solvers while they mentally make operations that involve rational numbers?
- How are these strategies distributed across the three different educational levels and what similarities or/and differences are identified?

Research literature

Cathy Seeley (2005), NCTM president for 2004-2006, in her "Do the Math in Your Head!" article is making a call to the educational community to realize the significance of the ability to perform mental calculations. She associates mental math with both the students' ability to perform

calculations quickly, and their conceptual understanding and problem solving. Furthermore, mental calculations constitute a useful means for calculations, given that they are used by almost everyone on a daily basis, as a direct and quick way of calculating. According to relevant studies (McIntosh, 2006) over three quarters of the mathematical operations performed by individuals in their daily lives are based on mental calculations. Moreover, they promote creative and independent thinking, develop sound number sense, constitute the basis for developing estimation skills, and finally they facilitate a natural progression through informal written methods to standard algorithms (Thompson, 2010).

As it has already been mentioned, the majority of the research studies focus on mental calculations involving whole numbers, and a wide variety of strategies associated to the four operations has been recorded. However, the corresponding body of research concerning mental calculations with rational numbers is rather limited. A systematic documentation of the strategies used by students in primary and secondary education in operations with fractions, decimals and percentages have been realized by Caney and Watson (2003). One main finding of their study was that many of the strategies used in operations with whole numbers are also used for operations with rational numbers. Furthermore, they identified two kinds of strategies for mental calculations with rational numbers: instrumental and conceptual strategies. The instrumental work refers to the use of procedural strategies learned by rote. Moreover, this work is not accompanied by an explanation displaying conceptual understanding of the involved processes. On the contrary, conceptual work takes place when students make use of their knowledge on the specific set of numbers and operations to calculate mentally. Finally, Caney and Watson (2003) added the 'mixed' strategy in case the students combine a rote procedure with a conceptual explanation. In their study they identified a series of strategies the students used to solve 12 problems. More specifically, the students '*changed operation*' (division to multiplication, subtraction to addition), '*changed representation*' (fractions to decimals and vice versa, percent to fractions), '*used equivalents*' (use of equivalent fractions), '*used known facts*' (times table), '*repeated addition/multiplication/division*' (by doubling, halving), '*used bridging*' (bridged to one/whole), '*worked with parts of a second number*' (split by place value or by parts), '*worked from the left/right*', '*used mental picture*', '*used mental form of written algorithm*' and '*used memorized rules*'. Carvalho and da Ponte (2015) examined grade-6 students' mental calculations involving fractions, decimals and percentages. They found that students mainly changed representations (fractions to decimals) or used the relationship between numerical representations when possible. Moreover, data analysis showed that pupils utilized depictive representations such as models or images and descriptive representations like the propositional representations in the sense that these mental models arise as a representation of the real world (i.e., the students used the context of money, clocks, pizzas, etc.). Callingham and Watson (2004) working with students in grades 3 to 10 found that fractions items were easier than decimals and percents and that multiplication and division operations with fractions were more difficult for students than addition and subtraction, thus mirroring similar findings for whole numbers. In the same spirit Lemonidis, Tsakiridou and Meliopolou (2018) found that teachers exhibit a lack of flexibility in the use of strategies in mental calculations with rational numbers. McIntosh (2006) examined the errors students make during mental calculations with rational numbers and identified two different kinds: procedural and

conceptual errors. Procedural errors are connected with simple operations between the terms of fractions. They are mostly careless errors or errors that appear while carrying a step of a strategy even though the steps are known to the student. For example, the student knows that multiplication of fractions means to multiply separately the numerators and denominators of the participating fractions but calculates mistakenly the product of these terms. So, if the numerators of the fractions are 6 and 7 the student might find that $6 \times 7 = 48$. Conceptual errors are connected with a limited understanding of the rational numbers or the operations they have to execute.

In this context our study focuses mainly on the strategies (rather than on the errors, although they have been recorded) used by primary, secondary and tertiary education students given that all of them had to calculate mentally the very same operations involving rational numbers.

Study design and methodology

The participants were 295 students on a voluntary basis from all the three educational levels (primary, secondary and tertiary education). More specifically, there were 78 grade-6 students [SP], 97 grade 10-12 students [SS] and 120 University students (63 from the Department of Primary Education [SU1] and 57 from the Mathematics Department and the Faculty of Engineering [SU2]). These last ones were chosen due to their strong mathematical background that would provide flexibility in their strategies. The tasks and the process were identical for all the three different groups.

Eleven tasks (Figure 1) were posed and the participants were asked to calculate mentally their results. The tasks included all the four operations and their design incorporated two main principles. First, the answers can be calculated in more than one way thus giving freedom to the participants to work in the way they feel more familiar. Second, the tasks are amenable to a fast and easy calculation if the correct mental strategy is chosen. For example, in task-a the answer is obvious if the solver can see that it is about the sum of two halves. Similarly, in task-f the wanted operation is 2×3.5 . Aiming to cover a variety of different combinations within the spectrum of the operations with rational numbers we included operations between fractions, whole numbers and fractions, mixed numbers and fractions, mixed and whole numbers. The criterion for each task was always to meet the standards for primary school students.

a) $\frac{1}{2} + \frac{4}{8}$	b) $4 \times \frac{3}{4}$	c) $4 \times \frac{2}{8}$	d) $\frac{5}{6} + \frac{4}{3}$
e) $1\frac{1}{2} - \frac{1}{4}$	f) $3\frac{4}{8} \times 2$	g) $3\frac{1}{4} - \frac{1}{2}$	h) $\frac{3}{4}$ of 20
i) $6 \div \frac{2}{6}$	j) $\frac{1}{2} + \square = \frac{7}{8}$	k) $1\frac{3}{4} \times \frac{1}{4}$	

Figure 1: Tasks posed to the students

Individual interviews were used and each participant was asked to solve one-by-one the whole collection of tasks in the order presented in Figure 1. The students were asked to vocalize their thoughts while solving the tasks and were prohibited from taking notes or making written calculations. There was no time limitations and the duration of the interviews varied from 4 to 9

minutes since the solvers could skip tasks they couldn't solve. Neither help nor any feedback were provided to them. The only clarification that was given concerned mixed numbers since they are used mainly in primary education and the older students confuse mixed numbers with the case of multiplication in algebra. Given that $2x$ means the multiplication 2 times x , in a similar way the $3\frac{1}{4}$ was mistakenly translated as $3 \times \frac{1}{4}$. All interviews were recorded and transcribed. These transcribed protocols constituted our data. The students' efforts were coded according to the following two levels: (i) whether the answer was correct, incorrect, left unanswered, or non-codable, and (ii) whether the strategy used by the student fits to a certain category of the Caney and Watson (2003) model.

The data were coded independently by the authors and validity and reliability were established by comparing sets of independent results, clarifying codes and re-coding data until agreement.

Results and Discussion

Table 1 summarizes the distribution of the total number of answers (3274 answers in total) across the various categories of strategies according to the categories of Caney and Watson (2003). It is worth mentioning that not all their strategies were identified in our sample and also a new one has been added. We've been able to identify the '*changed representation*', '*used equivalent*s', '*used known facts*', '*repeated addition/ multiplication*', '*worked with parts of a number*', '*used a mental picture*', and '*used mental form of written algorithm*' strategies. The new one is named '*used reduction*'. Finally, the incorrect, non-answered and non-codable strategies have also been recorded.

Strategy	Changed representation	Equivalent	Known facts	Repeated add/ multip	Parts of a number	Mental picture
Frequency	56 (1.71%)	59 (1.80%)	43 (1.31%)	3 (0.09%)	31 (0.95%)	30 (0.92%)
Strategy	Written algorithm	Reduction	Incorrect	Non-answered	Non-codable	
Frequency	1406 (42.95%)	177 (5.41%)	907 (27.7%)	519 (15.85%)	43 (1.31%)	

Table 1: Distribution of answers across strategies

The '*changed representation*' strategy refers to the conversion of a fraction or a mixed number to decimal. Some students considered $1\frac{1}{2} - \frac{1}{4}$ (task-e) as $1.5 - 0.25$. In task-b some preferred to use also decimal numbers to calculate $4 \times \frac{3}{4} = 4 \times 0.75 = 3$. In a similar way some students made use of this strategy to reach faster the answer in task-f converting the product $3\frac{4}{8} \times 2$ to $3.5 \times 2 = 7$. We identified the '*used equivalent*s' strategy when a fraction is replaced by an equivalent with an explicit reference to this. Some students' mental solution for task-a was $\frac{1}{2} + \frac{4}{8} = \frac{1}{2} + \frac{1}{2}$, but this was not because they made an immediate reference to the fact that there are two halves but to the fact that $\frac{1}{2}$ is equivalent of $\frac{4}{8}$. The '*used known fact*' strategy was mostly used in task-h ($\frac{3}{4}$ of 20) since

the solvers made use of their knowledge of the times table. They started reciting the multiples of 5 and they noticed that $4 \times 5 = 20$ and $3 \times 5 = 15$ which made them to say that the answer is 15. All the three strategies presented until now had a very low frequency (less than 2%). This frequency becomes even lower for the next three strategies. ‘*repeated addition/multiplication*’ was only used by two participants in three different tasks. The first example is from task-b. The solver’s reaction to the task was to calculate the product $4 \times \frac{3}{4}$ as follows: $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$. The next example is drawn from another student’s effort to cope with task-g ($3\frac{1}{4} - \frac{1}{2}$). The solver considered more convenient to substitute 3 with $\frac{4}{4} + \frac{4}{4} + \frac{4}{4}$. This substitution facilitated the addition that follows since all the fractions have now the same denominator.

SS14.7.1: I will substitute 3 with $\frac{4}{4} + \frac{4}{4} + \frac{4}{4}$.

SS14.7.2: Then, $\frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \frac{13}{4}$.

SS14.7.3: And then, $\frac{13}{4} - \frac{2}{4} = \frac{11}{4}$

The ‘*used parts of the numbers*’ strategy was recorded only in task-e and task-f. In the first case the solvers decomposed the mixed number to its parts, (i.e., the whole number and the fraction) and then made use of the equality $a + b - c = a + (b - c)$: $1\frac{1}{2} - \frac{1}{4} = 1 + (\frac{1}{2} - \frac{1}{4})$. In the second case the students decomposed again the mixed number and applied the distributive property of multiplication over addition: $3\frac{4}{8} \times 2 = (3 \times 2) + (\frac{4}{8} \times 2) = 6 + 1 = 7$. The ‘*mental picture*’ strategy is connected to the solvers’ effort to visualize the activity as a means to mentally calculate the result (Carvalho & da Ponte, 2015). So, some students while trying to solve task-h preferred to consider 20 as a whole unit, ‘like a circular pie’ which was divided mentally in 4 equal pieces and 3 of them were taken.

SS17.8.1: I am thinking of a circular pie and draw its $\frac{3}{4}$.

However, the strategy that dominated with extremely high percentage (42,95%) was the ‘*used mental form of written algorithm*’ strategy. Solvers worked mentally in a way that appeared to reflect the written algorithm. For task-d a relevant reaction was” “I will make sure that denominators are the same... $\frac{5}{6} + \frac{4}{3} = \frac{15}{18} + \frac{24}{18} = \frac{39}{18}$ ” (SP7.4.1). For task-f the students converted the mixed number to fraction and then they followed the rule for multiplying a whole number with a fraction: “ $3 \times 8 = 24$, plus 4...28, times...56, $56/8$...therefore, 7” (SU1.55.6.1). The ‘*used reduction*’ strategy was the one added by the research team. Instances of this strategy were identified in three tasks. The students simplified the whole number (in tasks b, c and h) with the denominator of the fraction. So, in task-b, they simplified the 4s: $4 \times \frac{3}{4} = 3$. In task-c, the simplification (4 and 8 were divided by 4) resulted in a fraction equal to one: $4 \times \frac{2}{8} = \frac{2}{2} = 1$. In task-h, the word ‘of’ was translated as ‘times’ and then the whole number was again simplified with the denominator: $\frac{3}{4}$ of 20, which means $\frac{3}{4} \times 20 = 3 \times 5 = 15$ (20 and 4 were divided by 4).

Sometimes it seems to exist an overlapping between the ‘*used reduction*’ strategy and strategies such as the ‘*used equivalence*’ or the ‘*written algorithm*’. But it was the solver’s aim as this was expressed verbally that made us to decide if the solution is associated with one strategy or another. For example, in task-a, the solution $\frac{1}{2} + \frac{4}{8} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$ might be considered as the ‘*use of*

equivalence'. However, the student's wording reveals pure use of fraction reduction: "I will simplify the fraction 4/8 which...I divide by 2... is 2/4 which in turn is 1/2. Therefore, 1/2 + 1/2" (SU1.45.1.1). In the same spirit, for the same task, the answer $\frac{1}{2} + \frac{4}{8} = \frac{4}{8} + \frac{4}{8} = \frac{8}{8} = 1$ sometimes was considered as 'use of equivalence' and others as 'written algorithm' (see below extracts SU1.9.1.1-2 and SU1.58.1.1-3 respectively)

SU1.9.1.1: I will make two equivalent fractions. I will multiply both the numerator and denominator by 4.

SU1.9.1.2: It makes $\frac{4}{8} + \frac{4}{8} = \frac{8}{8} = 1$

.....

SU1.58.1.1: I will make fractions with the same denominator and I will add them.

SU1.58.1.2: I need the Least Common Multiple which is 8.

SU1.58.1.3: It becomes $\frac{4}{8} + \frac{4}{8} \dots \frac{8}{8}$, therefore 1.

Errors were surprisingly the second most popular result (McIntosh, 2006). Both procedural and conceptual errors were detected. Procedural errors were mainly due to miscalculations, for example: $\frac{5}{6} + \frac{4}{3} = \frac{5}{6} + \frac{8}{6} = \frac{12}{6}$. Conceptual errors were mirroring students' limited understanding and misconceptions relevant to fractions. We could mention the flawed idea that when we add fractions we just add separately numerator and denominators, $\frac{5}{6} + \frac{4}{3} = \frac{9}{9}$ (task-d) or $\frac{1}{2} + \frac{4}{8} = \frac{5}{10}$ (task-a). We could also add the misconception that when we multiply a whole number with a fraction then the whole number must be multiplied by both the nominator and the denominator: $4 \times \frac{3}{4} = \frac{12}{16}$. Moreover, some the students hesitated or were not able to give an answer. Almost 16 out of 100 instances were actually non-answered. Finally, we sorted as non-codable answers that did not reveal any kind of thought. In this category we also included efforts that were based in a strategy that was not suitable for the specific task. In task-k some solvers tried to calculate the results as $1\frac{3}{4} \times \frac{1}{4} = 1.75 \times 0.25$. This last multiplication was quite demanding, and the students decided to abandon the task.

	SP (Primary)	SS (Secondary)	SU1 (stud. teachers)	SU2 (Math, Engin)
Changed representation	–	3 (0.28%)	17 (2.41%)	36 (5.63%)
Equivalentents	2 (0.23%)	3 (0.28%)	25 (3.55%)	29 (4.54%)
Known facts	12 (1.40%)	11 (1.03%)	13 (1.85%)	7 (1.10%)
Repeated add/multip	–	1 (0.09%)	2 (0.28%)	–
Parts of number	–	5 (0.47%)	22 (3.13%)	4 (0.63%)
Mental picture	8 (0.93%)	7 (0.65%)	13 (1.85%)	2 (0.31%)
Written algorithm	373 (43.42%)	330 (30.78%)	326 (46.31%)	377 (58.99%)
Reduction	12 (1.40%)	24 (2.24%)	40 (5.68%)	101 (15.81%)
Incorrect	296 (34.46%)	406 (37.87%)	147 (20.88%)	58 (9.08%)

Non-answered	151 (17.58%)	264 (24.63%)	88 (12.50%)	16 (2.50%)
Non-codable	5 (0.58%)	18 (1.68%)	11 (1.56%)	9 (1.41%)

Table 2: Number of total responses per strategy per group

It is more than evident the dominance of the written algorithm approach over the remaining strategies. This might raise questions about the effectiveness of the way we teach. This becomes more interesting if we take account of the incorrect, non-answered and non-codable answers. The total percentage of these three categories is 44.86% which actually is more than the percentage of the algorithm. If we consider together these two broad groups of participants (those who used the written algorithm and those who had difficulties in answering) then it can be said that almost 9 out of 10 participants either used the written algorithm or were unable to solve the tasks ($42.95\% + 44.86\% = 87.81\%$).

Table 2 presents in detail how students' answers are distributed across the groups in the different educational levels. The percentages in each column correspond to the specific group of students and not to the total sample and confirm the strong influence of the written algorithm in each group. A closer look reveals also some interesting issues. On the one hand it appears that both groups of university students rely more than primary and secondary students to the written algorithm (which by far dominates the landscape). Forty five out of 100 preservice teachers and 6 out of 10 Mathematics and Engineering students resorted back to the written algorithm to respond to the tasks. Moreover, it is seeming paradox that the highest percentage for algorithmic use (Table 2) is got by students from the Mathematics Department and the Faculty of Engineering [SU2] who are the ones with the strongest mathematical background.

On the other hand, it appears that as we move from primary to tertiary education there is an increased variety of the correct strategies used (though very small number of instances compared to the total number of answers). We refer to all but the '*written algorithm*' correct strategies. In total 3.96% of the primary school students' answers used these strategies. This percentage is more or less the same for secondary education students (5.04%). However, for the tertiary education students this becomes almost three times bigger (13.07%). This is not because the use of the '*written algorithm*' strategy has been reduced. On the contrary, the relevant percentages of the algorithmic use for SU1 and SU2 students are extremely high and greater than the corresponding ones for younger students. It is the number of the incorrect answers that became smaller.

Having said that, it remains impressive that the main finding of this study is the dominance of the algorithmic use across all the educational levels (Rezat, 2011). This students' preference becomes more interesting because the tasks were designed to be amenable to a variety of strategies for their solution. For example, task-f ($3\frac{4}{8} \times 2$) can be solved directly if one notices that the wanted is the double of 3 and half. But there are other possibilities also. According to the '*changed representation*' this can be seen as 3.5×2 . According to '*written algorithm*' it is $3\frac{4}{8} \times 2 = \frac{28}{8} \times 2 = \frac{56}{8} = 7$. There is also the case of the '*worked with parts*' strategy: $(3 \times 2) + (\frac{4}{8} \times 2) = 6 + 1 = 7$ or its modified version $(3 \times 2) + (\frac{1}{2} \times 2)$. In this last version the fraction $\frac{1}{2}$ might be the result of the '*equivalence*' or the '*reduction*' strategies. However, for this specific task, according to our data, 3 out of 4 who answered correctly preferred the '*written algorithm*' which obviously is not a

straightforward calculation. On the contrary it is rather demanding and time consuming. The situation is more or less similar for the total set of the tasks. This highlights the students' obsession with the use of the written algorithm which disqualifies the core idea of mental calculations.

Conclusions

Given the importance of the ability to execute mental calculations involving rational numbers it became evident that although the students exhibit a broad range of different strategies the majority of these strategies is underrepresented. The students mainly follow the steps of the written algorithm. Moreover, it seems that this is not an age-dependent reaction. Very interestingly, the 'written algorithm' strategy was the most frequent choice for primary, secondary, and tertiary education students. We do not doubt that possibly the solvers were aware of alternative strategies and able to use them for mentally calculating the results. What can be said on the basis of the data is that their first choice was the application of the written algorithm. Moreover, there was a great number of participants (44.86%) who either did not give any answer or the answer they gave was incorrect or non-codable. This makes evident the difficulty the students have with rational numbers especially when they are invited to handle them mentally. Additionally, it seems that university students exhibit flexibility in their choice of strategies but in an extremely small percentage. All these deepen our understanding of the topic but at the same time challenge us for a future study that will try to give answers to questions such as: Why do they choose these strategies? What alternative strategies do they know? Why do they not use those alternative strategies? Would the results be the same if the students were asked to solve written the same tasks?

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