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Interactions between pupils' actions and manipulative characteristics when solving an arithmetical task

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In this paper, we explore the use of manipulatives in the classroom to solve an arithmetical task via the concept of affordance. Manipulatives are part of the elementary class culture in different countries, and even if some studies question the efficacy of manipulatives, there seems to have a consensus around the necessity of using it. However, little is known about how mathematics is done with manipulatives. The analysis of pupils' actions helps put to light different affordances of two manipulatives: base-ten blocks and abacus, in a classroom setting where the operations of addition and subtraction are explored with pupils.

Keywords: Elementary School Mathematics, Arithmetic, Manipulatives, Affordance.

Introduction

Research and mathematics education communities at elementary level agree that using manipulatives promote pupil learning (Moyer, 2001). Some researchers see the use of manipulatives as a less abstract way of reasoning than with formal mathematical symbols (Lett, 2007, Özgün-Koca, & Edwards, 2011). According to Domino (2010), Piaget (1964), Bruner (1977) and Diènes' (1973) theories support most of the work on the uses of manipulatives. These theories of learning are based on the idea that manipulatives are necessary for the development of mental images that pupils may eventually summon in situations without manipulatives. In a way, the need, more over the benefit, of manipulatives seems to be taken for granted by many researchers. It leads to many other assumptions such as “doing math with manipulative is a concrete version of doing math” or “manipulatives should *support* the construct of mathematical objects”. In our research about manipulatives uses, we challenge these assertions. Doing math with or without manipulatives aren't the same activity. Based on this premise, we developed a project called *MathéRéaliser*, in which, we want to understand what it is to do mathematics with manipulatives in a school setting.

Indeed, research shows that the context used to do mathematics structures mathematical activities (e.g. Lave, 1988; Nunès, Schliemann, & Carraher, 1993). These studies, mainly conducted in nonacademic contexts, highlight the situated aspect of mathematical knowledge (Noss, 2002). For example, Pozzi, Noss, and Hoyles (1998) mention that in the professional context (nursing, banking, engineering, etc.), the tools and objects available to actors shape their mathematical activity. In other words, mathematical reasoning is developed in coordination with the “noise” of the situations in which it takes place (Noss, 2002). In this perspective, doing mathematics with manipulatives could be something very different from a concrete version of doing mathematics without them (Corriveau & Jeannotte, 2015). Kosko and Wilkins (2010) were able to show that the use of manipulatives colors the mathematical discourse developed with it. This aligns with a sociocultural point of view where mathematical learning is shaped by the historical culture of the community where the learning takes place and by the learners' culture and experiences themselves

(Sfard, 2008). Manipulatives when available during a mathematical activity bear a certain culture and shape the learner experiences.

In this paper, we aim to better understand what it is doing mathematics with manipulative in a calculation task at the elementary level. To do so, the concept of *affordance* is convoked to identify the potentialities of manipulatives in developing number sense through arithmetical operations.

Conceptual Clarification

Manipulatives and arithmetic

In the context of arithmetic, a plethora of manipulatives are available for elementary school. Poirier (2001) classified manipulatives used for developing number sense in three categories. The first category refers to manipulatives where units are visible and accessible. For example, if we use 3 free tokens and four transparent bags of 10 tokens to represent the number 43, the tens, represented by bags that contain 10 tokens, can be “broken” into 10 units (by ungrouping a bag). The second category refers to manipulatives where units are still visible but not accessible (not breakable). Base-ten blocks are a good example of this kind of manipulatives: we have to physically exchange one long for 10 units because the longs are usually unbreakable. The third category refers to symbolic manipulatives, where units are not visible in tens, nor in hundreds, etc. (e.g. abacus, money).

Furthermore, manipulatives can become a support or a constraint for the pupils’ mathematical activity. On the one hand, it can support pupils reasoning. For example, base-ten blocks may help to explain why an algorithm works. On the other hand, it can be considered as a constraint if we impose a certain manipulative to solve a task (Jeannotte & Corriveau, 2015). By imposing manipulatives, the task may gain in complexity.

Affordance

Affordances is linked to interactions between an individual and the environment. Environmental characteristics (classroom setting, properties of the manipulatives, etc.) cannot be detached from pupils (their perceptions, their experiences, etc.). They form an inseparable pair. According to Clot and Béguin (2004), affordances are characterized, on the one hand, by the fact that objects are significant, the user’s experience relates on this signification. On the other hand, by its praxis value: “an object is immediately associated with a signification for action” (p. 53). Also, “[w]hether or not the affordance is perceived or attended to will change as the need of the observer changes, but being invariant, it is always there to be perceived” (Gibson 1977, in Brown et al., 2004, p. 120). So, depending on the needs, the uses and the properties exploited of manipulatives may vary. e.g., the base-ten blocks have been designed to help pupils perceived the structure of our numeration system. As an adult, we can see these properties and exploit them to expose some mathematical patterns. “We are seeing concepts that we already understand” (Ball, 1992, p. 5). What pupils see is also related to what they know. Since manipulatives aren’t solely used by pupils, but also by teachers, what teachers do with manipulatives is also inherently part of pupils’ experiences. Thereby, the concept of affordance can not only offer an insight on learning processes but also on teaching processes.

Methodology

For the aims of this paper, we focus only on a part of the data collected from the *MathéRéaliser* Project. We conducted a collaborative research that solicits the participation of teachers. As Corriveau and Bednarz (2016) uphold:

[t]his perspective leads us to re-think [...] the relations between the researcher, as being the expert, and the teacher, as being the novice or the user, as frequently conceived in research in mathematics education [...] [T]he teacher is assumed to be reflective and knowledgeable (pp. 1–2).

In collaborative research, the concept of “double relevance” (Desgagné, 1998) is fundamental and refers to the construct of argumentation relevant to both communities, research and practice. A “double relevance” underlines every choice made in the collaborative work. For example, if a task is chosen to be experimented with pupils, it means that it makes sense for teachers and researches according to their respective sensible arguments.

Description of the Task

The task we experimented is inspired by Cobb (1994). Three similar questions to the following were answered by the pupils. The task was presented orally.

“Represent 1009 with your manipulative and then, I’ll ask you a question. [Wait for the pupil to represent 1009 with base-ten blocks or homemade abacus.] I had a number, I subtracted 453. [Write it on the blackboard.] I now have 1009. How much did I have at the beginning?”

Description of the co-constructed Lesson

The task was experimented with two grade 3 classes (9–10 years old) of 19 and 18 pupils. They already had learned an algorithm using drawing of base-ten blocks. They were not familiar with the abacus. Pupils worked by two using either base-ten blocks or homemade abacus. After teamwork, a whole class discussion took place about 1) the different answers obtained and 2) the strategies used to resolve the task. Table 1 presents the double-relevance of this task and its experimentation.

Plausible use of the task for teachers	Relevance of the task for researchers
<p>Task congruent with curriculum expectations (choice of operation, adding and subtracting 3–4 digit numbers).</p> <p>Curriculum fosters the use of manipulatives.</p> <p>Opportunity to reflect on the use of manipulative by their pupils.</p> <p>Manipulatives seen as a support to solve the task.</p>	<p>Task that could lead to mathematical argumentation (see Cobb, 1994).</p> <p>Manipulatives seen as a constraint in this particular task. Since pupils used to operate by drawing base-ten blocks, asking them to operate with real base-ten block or abacus is unfamiliar to them. This introduction of foreign elements may serve as a breaching experiment (Garfinkel, 1967) to put to light usual ways of doing.</p>

Table 1. Double-relevance of the task

Characteristics of the manipulatives

To perform the task, two manipulatives were available. Either pupils worked with base-ten blocks or with a homemade abacus (see Figure 1).



Figure 1: Base-ten blocks and homemade abacus

As each manipulative has his own properties, we can conjecture that each manipulative, in relation with pupils and the whole class generates different affordances in the classroom (see Table 2).

Base-ten blocks	Homemade abacus
<ul style="list-style-type: none"> • Proportional model; • Units visible but not accessible (not breakable); • The value does not depend on the arrangement of the manipulatives; • The same manipulative “object” can only take one value. 	<ul style="list-style-type: none"> • Non-proportional model; • Symbolic manipulatives (units not visible nor breakable); • The value depends on the arrangement of the manipulatives; • The same manipulative “object” can change value.

Table 2: Characteristics of each manipulatives

Analysis and Results

Data analysis has involved three stages. First, to explore the affordance in the use of manipulatives by the pupils, we watched the videos multiple times (Powell, Francisco, & Maher, 2003). Secondly, we extracted every pupil’s actions we observed. Every action was then described in association with the characteristics of the manipulatives. Finally, we grouped actions related to three different mathematical activities involved with the task and the classroom setting (the use of manipulatives).

One may think that the main difficulties of this task is to choose the good operation and we are aware that using manipulatives do not help the pupils with this choice. However, in this experimentation, it did not appear as an issue. Moreover, the use of base-ten blocks and the abacus helped the teacher to see at a glance who chose the right operation. Actually, most errors arose from counting strategies. Table 3 presents the analysis of pupils’ actions in relation to the characteristics of each manipulatives. We grouped the different actions according to the mathematical activity involved: representing, operating and interpreting.

When representing numbers, most pupils were able to exploit the manipulatives. For base-ten blocks, they associated the right blocks (units, longs, flats, etc.) to each position. Nevertheless, few of them disposed the blocks in a way they can “recognize” rapidly the number in front of them. For the abacus, some pupils struggle with choosing the right column when representing the second term of the operation. Furthermore, more pupils relied on visual patterning, thus helping them later interpreting the number.

When operating, other than counting mistakes, difficulties arose from converting strategies. Even if pupils referred to changing 10 ones for 1 ten (for example) in their discourse, we observed more than once pupils changing eleven units for 1 long. When using the abacus, to convert a ten into ones, they move one chip from the tens column to the ones column AND add ten new chips in the ones column.

One pupil worked with the abacus inconsistently: he represented one thousand nine from right to left, he managed the calculations, but when interpreting the result, he read the number from left to right. This gave the opportunity to the teacher to talk about communication in mathematics.

Maths Activities		Base-ten blocks characteristics and actions observed	Homemade abacus characteristics and actions observed
Representing		<ul style="list-style-type: none"> Counting the blocks needed for each “place value”: cubes, flats, longs, units. Disposing the blocks so the highest value is far left and the other ones on its right (no empty spaces for zero values). Piling the blocks (e.g. 9 units on a cube represent 1009). Using the space to differentiate positions: e.g. ten flats (instead of a cube), then the other flats, etc. 	<ul style="list-style-type: none"> Counting correctly or not the chips and disposing them in the right or wrong column. Disposing the chips in the same order we read numbers. Only one pupil disposed it in the opposite order. In each column, using or not visual pattern disposition when placing chips in each column (e.g. 3 rows of 3 to represent 9).
Operating			
	Algorithm	<ul style="list-style-type: none"> Place value dealing <ul style="list-style-type: none"> Dealing with positions where no exchange is required first Operating from left to right Following the taught algorithm (from units to cubes) Mixed strategies Intermediary calculations <ul style="list-style-type: none"> Counting (e.g.: when adding 4 longs to 8 longs, regroup all longs and count them) 	<ul style="list-style-type: none"> Place value dealing <ul style="list-style-type: none"> First, dealing with columns where no exchange is required Using the taught algorithm (from ones to thousands) Intermediary calculations <ul style="list-style-type: none"> Counting (e.g.: when adding 4 chips to 8 chips, regrouping all the chips in one column and count the result) Mental math, articulating manipulatives and number facts correctly or not.
	Converting	<ul style="list-style-type: none"> Changing one long into ten units (or ten units into one long) <ul style="list-style-type: none"> by counting correctly or not ten units; (from ten units to one long) by removing all the units counted or not; changing eleven units for a long. 	<ul style="list-style-type: none"> Changing one chip into ten chips in another column (vice versa) <ul style="list-style-type: none"> by counting correctly or not ten chips (from one ten to ten ones) by moving one chip from the tens to the ones column and adding 10 new chips. (From ten ones to one ten) by removing all the chips counted or not. By placing in the right or wrong column the chips exchanged.
Interpreting		<ul style="list-style-type: none"> Associating the right value to each sort of block. 	<ul style="list-style-type: none"> Associate the right column to each position Changing the reading direction from the one used when representing

Table 3: Actions made by pupils in relation to the characteristics of each manipulatives

Discussion

With the table presented above, we have tried to better understand what it means to do mathematics with manipulatives when developing calculation abilities at the elementary level. By examining the characteristics associated with action, we put to light different affordances. Even if some affordances are shared, some others are specific to only one of the manipulatives. In this section, we

discuss further one idea that emerged from our analysis. This idea allows to put to light some differences in pupils' actions that are related to manipulatives characteristics. Also, speaking of affordance is speaking of seeing what could be observed but is not. Both, what is observed and what is not can inform of the manipulative practices in the classroom.

Counting Over and Over

In the first grades of the elementary, children are used to count to solve problems. The teachers then try to enrich their number sense and to complexify their counting strategies (with number facts, adding to ten, visual pattern disposition, etc.). However, for most pupils, the counting aspect brought by the use of manipulatives clearly took over other calculation abilities and some control was lost in their mathematical activities. When using base-ten blocks, most pupils counted the blocks even for small quantities. For example, when adding three to nine units, we observed pupils counting three units, then counting nine units, putting them together, counting the total, obtaining twelve units, counting ten units to trade them for one long. This may seem trivial insofar as the counting strategy is achieved with success; it does not mean there is a lack of control. Nevertheless, we have observed several errors that arise from this way of doing even if pupils know their numerical facts. For example, a pupil took eight units, but counted nine. Thus, when he added three units and counted the total, he obtained eleven units (and not twelve). He continued his calculations with eleven units and obtained a very close answer, but the wrong one. However, we observed him mentioned to another pupil, further in the video, that nine and three give twelve.

Also, even if pupils are able to group by tens and ungroup, they do not rely on this property yet visible in the base-ten blocks. They rather count. For example, again, to add three to nine units, pupils could have taken one long and kept only two units directly, but they count as we described it above. However, while we observed the same way of doing with an abacus, we also saw more complex strategies. For example, when adding nine and three, a pupil added to ten. However, he was not able to coordinate his action with the abacus and the mental operation.

In short, the main action observed during the task was counting. Counting over and over seemed to divide the global task into counting sub-tasks. After each sub-task, most pupils had to think again about what they were doing sinking into a vicious cycle: thinking about the global task made them forget about the counting they did and counting again made them forget about the global task. With manipulatives, it is difficult, for pupils, to keep track of what is done and what has to be done.

Does this mean that using manipulatives only suggest this way of doing things for those pupils? There is more than one answer to that question. On the one hand, it does because even if it is not necessary all the time, there is always some counting to do. Using manipulatives to add or subtract involve counting physical objects. The physical aspect of manipulatives asks for this way of doing. However, counting is not always the most efficient strategy to use here. Of course, the environment, i.e. the usual class activities, certainly contributes to reinforcing this strategy. On the other hand, as mentioned years ago by researchers (e.g. Bednarz & Janvier, 1982), the tasks performed with the help of manipulatives relate essentially to number representation and "translation", a work on the number representation in our numeration system. As pupils presumably used base-ten blocks to count in the first place and less to operate coordinating other strategies as well, we can think that the praxis of this manipulative associate it immediately to counting. While we tend to believe that the

use of manipulatives supports mathematical reasoning, in this case, we rather observed that pupils refer to more basic reasoning than they could have used.

In fact, we observed more advanced strategies with the less familiar abacus than with base-ten blocks used since first grade. For McNeil & Jarvin (2007), working with familiar manipulatives might drive the attention of pupils in the wrong. In our case, it is not a wrong direction, but it seems to lead to more basic strategies and to some lack of control. We could conjecture that using a less familiar manipulative does not drive pupils to usual uses. Indeed, in terms of affordance, the meaning constructed in the interactions between the abacus and pupil is less constraint by the experiences. It gives leeway to integrate skills in a new situation (e.g. visual patterning, use of number facts, etc.).

Conclusion

Carbonneau, Marley & Selig (2013) said that “specific instructional variables either suppress or increase the efficacy of manipulatives suggests that simply incorporating manipulatives into mathematics instruction may not be enough to increase student achievement in mathematics” (p. 397). Looking at affordance help understand that not only the instructional setting, but the culture of the class and pupils’ experiences also play a role when talking about learning with manipulatives. As manipulatives used to solve this task are by definition physical object, counting can’t be detached from it. Doing math with manipulatives was not the same activity in this particular case. We observed that pupils rely mostly on what they used to do with a specific kind of manipulatives rather than what they used to do without them. The question is then how to help the pupils rely on other characteristics of these manipulatives? Drijvers (2003) stressed that the affordance that could be realized in the classroom rely not only on the tool itself, but rather on the exploitation of these affordances the educational context and the teacher drive that. As base-ten blocks are often used in grades 1 and 2 when pupils don’t master the table facts, the teachers’ role is here quite important. They have to help pupils go beyond counting and perceived the power of organizing manipulatives by exploiting the space to “see” table fact and not solely rely on it to count and to represent numbers. To do so, teachers could organize the educational context around manipulatives to provide wider learning opportunities for the pupils.

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References

- Ball, D. (1992). Magical Hopes: Manipulatives and the reform of math education. *American Education*. 16(2).
- Bednarz, N., & Janvier, J. (1982). The understanding of numeration in primary School, *Educational Studies in Mathematics*, 13, 33–57.
- Brown, J., Stillman, G., & Herbert, S. (2004). Can the notion of affordances be of use in the design of a technology enriched mathematics curriculum. In I. Putt, R. Faragher, & M. McLean (Eds.), *Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 119–126). Sydney, Australia: MERGA.
- Bruner, J. S. (1977). Process orientation. In D. B. Aichele, & R. E. Reys (Eds.). *Readings in*

Secondary School Mathematics (2nd ed.). Boston, MA: Prindle, Weber, & Schmidt.

- Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*, 380–400.
- Clot, Y., & Béguin, P. (2004). Situated action in the development of activity. *Activités, 1*(1–2).
- Cobb, P., Perlwitz, M. & Underwood, D. (1994). Construction individuelle, acculturation mathématique et communauté scolaire. *Revue des sciences de l'éducation, XX*(1), 41–61.
- Corriveau, C. & Bednarz, N. (2016). Collaborative research in mathematics education: approaching questions related to teaching practices. Paper presented at International Congress on Mathematical Education (ICME). Hamburg (Germany): University of Hamburg.
- Corriveau, C., & Jeannotte D. (2015) Quelques apports du matériel de manipulation sur l'activité mathématique au primaire. *Bulletin AMQ, LV*(3), 32–49.
- Desgagné, S. (1998). La position du chercheur en recherche collaborative: illustration d'une démarche de médiation entre culture universitaire et culture scolaire. *Recherches qualitatives, 18*, 77–105.
- Dienes, Z. P. (1973). *Mathematics through the senses, games, dance, and art*. Windsor, UK: The National Foundation for Educational Research Publishing Company Ltd.
- Domino, J. (2010). The effects of physical manipulatives on achievement in mathematics in grades K-6: a meta-analysis. (thèse non publiée). State University of New York at Buffalo.
- Drijvers, P. (2003). *Learning algebra in a computer algebra environment: design research on the understanding of the concept of parameter*. Utrecht, the Netherlands: CD-β Press
- Garfinkel, H. (1967). *Studies in ethnomethodology*. Englewood Cliffs, NJ: Prentice Hall. Paradigm Publishers.
- Jeannotte, D. & Corriveau, C. (2015). Analyse de l'utilisation d'un matériel symbolique en troisième année du primaire : raisonnement mathématique et accompagnement. In A. Adihou et al. (Eds.) *Proceedings of the 2015 GDM Seminar*, Québec: Université de Sherbrooke.
- Kosko, K. W., & Wilkins, J. L. (2010). Mathematical communication and its relation to the frequency of manipulative use. *International Electronic Journal of Mathematic Education, 5*(2), 79–90.
- Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.
- Lett, S. (2007). Using manipulative materials to increase pupil achievement in mathematics. *Rapport de recherche*, Marie Grove College.
- McNeil, N., & Jarvin, L. (2007). When theories don't add up: disentangling the manipulatives debate. *Theory into Practice, 46*(4), 309–316.
- Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175–197.

- Noss, R. (2002). Mathematical epistemologies at work. *For the Learning of Mathematics*, 22(2), 2–13.
- Nunes, T., Schliemann, A. D., & Carraher, D. (1993). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press.
- Özgün-Koca, S., & Edwards, T. (2011) Hands-on, minds-on or both? A discussion of the development of a mathematics activity by using virtual and physical Manipulatives. *Journal of Computers in Mathematics and Sciences Teaching*. 30(4), 389–402.