



HAL
open science

Use of conceptual metaphors in the development of number sense in the first years of mathematical learning

Natividad Adamuz-Povedano, Elvira Fernández-Ahumada, Teresa García-Pérez, Rafael Bracho-López

► **To cite this version:**

Natividad Adamuz-Povedano, Elvira Fernández-Ahumada, Teresa García-Pérez, Rafael Bracho-López. Use of conceptual metaphors in the development of number sense in the first years of mathematical learning. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02400808

HAL Id: hal-02400808

<https://hal.science/hal-02400808>

Submitted on 9 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Use of conceptual metaphors in the development of number sense in the first years of mathematical learning

Natividad Adamuz-Povedano¹, Elvira Fernández-Ahumada¹, Teresa García-Pérez² and Rafael Bracho-López¹

¹ University of Córdoba, Faculty of Education Sciences, Cordoba, Spain; nadamuz@uco.es

² Al-Andalus Public Primary School, Cordoba, Spain; teresagarcia20@gmail.com

Promoting experiences to develop the so-called number sense since early school years has become a focus of interest in mathematics education research. By number sense we mean a broad concept that refers to a deep understanding of the decimal number system, the relationships between numbers and operations, and the development of capabilities such as flexible mental calculation, numerical estimation and quantitative reasoning. This paper is an exploratory study (25 pupils), part of an on-going research, where a methodological approach, based on the use of manipulative materials for developing in six-year-old students' number sense is explored. Particularly, in this study, we present and analyse two activities, in which a jumping frog and a kangaroo are used as conceptual metaphors for the acquisition of calculation strategies that encourage number sense development.

Keywords: Number sense, Conceptual metaphor, Primary Education, Manipulative resource.

Introduction.

Expressions such as "number sense", "numerical consciousness" or "numerical thinking" are being imposed with force in the current studies on mathematical knowledge. In general terms it refers to several important abilities that involve mastering a wide range of skills with numbers. For McIntosh, Reys and Reys (1992) numerical thinking is a person's overall understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for managing numbers and operations.

In the Curriculum and Evaluation Standards for Mathematics Education of the National Council of Teachers of Mathematics (NCTM, 1989), five components that characterize numerical sense were identified: (1) correctly understand the meaning of numbers; (2) be aware of the multiple relationships that exist between numbers; (3) recognize the relative magnitude of the numbers; (4) know the relative effect of numerical operations; (5) have reference points for measurements of common objects and situations in the environment. In the same line, Sowder (1992) expresses that number sense is developed when students understand the size of numbers, they think about them and represent them in different ways, they use numbers as references and develop accurate insights into the effects of operations with numbers.

Although there is still no general consensus on the concept and operation of numerical thinking, many authors (Berch, 2005; Godino, Font, Konic, & Wilhelmi, 2009) are trying to define it, identify the keys to its development, establish criteria to evaluate it and analyse its repercussions on children's learning.

The school context is a privileged place to work with the possibilities offered by this powerful tool. Teachers must consider the number sense as an essential objective and articulate the resources and activities necessary to develop it.

Manipulative resources provide us with very advantageous visualizations to present, know and materialize both numbers and operations. To the visual properties of the resources must be added their sensory and motor properties, as they invite manipulation and movement. In this sense, another way of teaching and learning mathematics appears, because it is recognized that mathematical cognition is embodied and is intimately linked to our sensorimotor functioning (Gallese & Lakoff, 2005).

The matching of symbolic mathematics, initially quite abstract and apparently less close for students, with insights from the real world, helps to forge links between conceptual and procedural knowledge, which is critical to success in mathematics (Siegler, 2003). Conceptual metaphors are, among others, methodological strategies that facilitate neurological mechanisms that allow us to adapt the activated neuronal systems through the sensory-motor experience, making possible the seemingly complicated journey from the concrete to the abstract (Lakoff & Johnson, 2003). Thus, there arises a new way of conceiving the teaching and learning of mathematics associated with embodied mathematical cognition linked to sensory-motor functioning, departing from Piaget's theory of cognitive development, which separated motor-sensory development from conceptual development, basing the second on the first, but without the parallel development that we defend.

Conceptual metaphors provide a transition from one mode of representation to another or from a concept that is part of our knowledge to a more abstract one. Consequently, they value the three levels of representation of Bruner (1972): enactive, iconic and symbolic, putting into play and relating the four dimensions of mathematical thought (mathematical contents, non-rational processes, strategies and procedures and perception). Of course, this journey between the three types of representation does not have to be unidirectional and without return, from the enactive to the symbolic representation, passing through the iconic, but it can go back or advance through the levels on its way to conceptual consolidation. Conceptual metaphors do not only respond to a final objective of a cognitive nature, but also contribute to a didactic enrichment by providing resources and useful tools to address complex problem situations (Soto-Andrade, 2007), which makes them even more valuable for the construction of mathematical knowledge and the development of the competences associated with it.

This paper is part of an on-going research, where a methodological approach for developing six-year-old students' number sense is explored. The proposal is based on the use of manipulative materials, rigorously structured (García-Pérez & Adamuz-Povedano, 2016), that keep the relationship number-space very present, giving students a numerical support for the learning of the decimal numbering system, as well as an arithmetic support for operations. Here we present two activities, as specific examples of the use of a variety of conceptual metaphors (maps, routes, characters...) that play an important role for number sense development, as motivating and facilitating elements, both in the teaching and learning processes. In the design of these activities

we take into account the limits that Cole and Sinclair (2017) show relative to the use of conceptual metaphors.

Teaching materials

As we mentioned before, this proposal is based on the use of manipulative materials, rigorously structured, which we describe:

- *Number line*: it is a tape in which part of the set of natural numbers is presented. It starts with zero, a number that we will need to represent different situations: starting point, absence of counting elements, total loss, etc., and it ends in 100 example see figure 1.

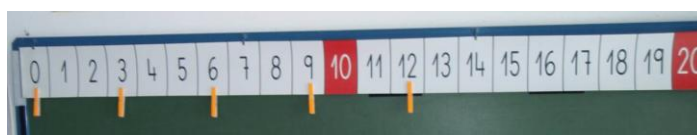


Figure 1: Detail of the number line

With it we can practice progressive and regressive recitation, form the first series and perform simple addition and subtraction operations. We can also compare positions on both sides of the 10 to begin the observation of regularities and changes.

The tape shows, with great consistency, the numerical order, the pattern that generates numbers and distances. The numbers colored in red, in addition to meaning exact tens, are privileged positions, strategic enclaves when planning routes that take us from one number to another.

- *Numerical panel*: it presents the numbers from zero to ninety-nine by families (figures 2 and 3). This format enhances the display of numbers on the tape expanding the possibilities of analysis and relation.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

50	51	52	53	54	55
60	61	62	63	64	65
70	71	72	73	74	75
80	81	82	83	84	85
90	91	92	93	94	95

Figure 2: Collective numerical panel Figure 3: Individual numerical panel

As in the case of the tape, the complete tens are highlighted with a red background, facilitating their location and indicating they are essential elements within our numbering system. The number-space association in the panel shows very clearly the patterns and regularities of our decimal number system. On this "precise map of the numbers" it is also possible to perform many addition and subtraction operations by tracing horizontal paths (we move forward or backward through the rows) and vertical (we raise or lower the columns by multiples of ten).

- *Numbering box*: this material facilitates full exploration and manipulation of numbers, favoring a correct understanding of decimal numeration system. It consists of abundant plastic sticks and red and green rubber bands, see figure 4 below. The work with the numbering box produces a qualitative leap in the understanding of the decimal structure of the number and its size, since it

provides a concrete and faithful model to the visible reality, which gives meaning to the use of written symbols and concepts. Regarding place value, in addition, we should move from that understanding to much more flexible forms. We can achieve this by removing the tens and units from the box and exploring other groups with total freedom, leaving or removing the rubber bands of the tens. Regarding the calculation, the numbering box directly connects with breakdown strategies and facilitates the graphic transcription that is derived from the manipulation of quantities.



Figure 4: Numbering box

Selected activities

In this paper we only show two activities focused on the number line resource. However, the methodology implies the implementation of other activities based on the use of the other resources. This provides students with greater flexibility in reasoning about numbers, an aspect directly related to the quality of their numerical sense.

Activity 1: Saltarina, the frog

The convenient breakdown of numbers is a fundamental skill to give agility and efficiency to thought calculation, so let children show some characteristics of development of their number sense. Young children find it difficult to acquire because they require complex skills such as observation and analysis of the complete operation, reflection and planning of the tactics to be used, and also a good handling of numbers at the mental and written level. To work on this awareness of what may be suitable for the proposed calculation we use the conceptual metaphor of the jumping frog, it is a frog that jumps forward and backwards through the number line. In addition, Saltarina has a fixation with the colour red: whenever she can, she boots in the numbers coloured in red (see figure 5 below).

Teacher: This is Saltarina, a very funny frog that comes and goes on the number tape jumping without stopping. Do you know what her favourite colour is? ... It's red! ... and the numbers that have this colour look like springboards to take energy to jump higher. You can be sure that whenever she has the opportunity she bounces on them. Today, Saltarina jumps eight, do you know how she will do that if she is on thirty-six?

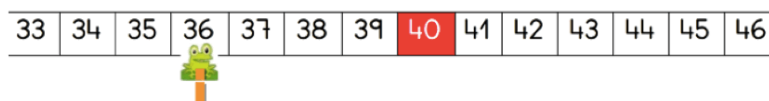


Figure 5: Saltarina is on 36

Student 10: She will jump four to land on forty and later she will jump another four.

Teacher: Very good! Let's check it!

The real movement is made, taking the clamp with the frog that bounces onto forty and ends the jump on forty-four.

Teacher: She has reached forty-four! Now let's say in writing what has happened.

Teacher: Say with me, please.

Teacher and students: Saltarina was on thirty-six [she writes 36 on the blackboard figure 6], first she has made a leap forward of four to reach forty [she writes +4] and then she has made another leap of four [she writes +4]. It has reached forty-four.

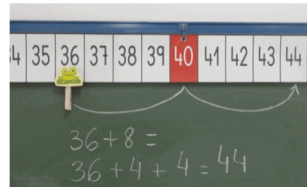


Figure 6: Saltarina's jump

Teacher: She has reached forty-four! ... What if she now adds eight again?

Student 10: Now she has to jump six to fifty and then two.

Teacher: And how do we express it with numbers and signs?

On the board, teacher writes the jump in two movements and the arrival number: $44 + 8 =$, and below $44 + 6 + 2 = 52$

In Figure , we have examples of two students' activities, one in which Saltarina adds eight and another which goes back six. In both, the empty number line has been used as a scheme for the representation of forward and backward:

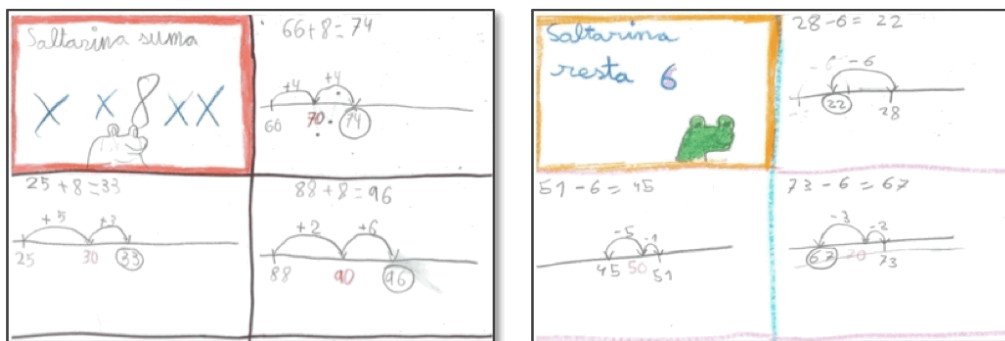


Figure 7: Saltarina's worksheet

Next, we analyse some examples showing different strategies that take part of what we have denominated *tactical calculation* (Adamuz-Povedano & Bracho-López, 2017), shows the operation $63 - 24$ see figure 8, the student rewrites the operation explaining the plan he will follow, removing three units, then two tens and finally one unit. Since he is not still self-confident in mental calculation he makes partial annotations.

$$63 - 24 = 39$$

$$63 - 3 - 10 - 10 - 1 = 39$$

$$45 + 29 = 74$$

$$45 + 5 + 20 + 4 = 74$$

Figure 8: Work out 63-24

Figure 9: Work out 45+29

In **Figure** the student rewrites the followed mental route to get the result, again Saltarina's strategy appears adding 5 to get 50 and then adds 24 in a convenience way.

Activity 2. Skip Kangaroo!

In this activity we will use the number tape as if it were a numbered track in which a kangaroo makes jumps of different magnitude passing from one family to another. The objective is to reinforce operations in which there is change of tens, i.e. operations of adding and subtracting with taken. The kangaroo should overcome a fence that is placed in an exact ten and can jump progressing in the numerical sequence (we will relate it to the expression "+") or going backwards (expression "-"). The connection between movement in space and numerical operation is being established. This activity places students in front of the control of phenomena and develops their relational thinking. With the tape and the Kangaroo, we configure a "mathematical scenario" in which we can experiment with different possibilities to relate the numbers of two families. Children should determine the starting point, the direction and magnitude of the jump, and the arrival number. They should then insert these data into an arithmetic structure. Very important processes such as estimation, decision making and the translation of each specific situation into the language of numbers and signs, which favour the development of the numerical sense, are involved in all this work.

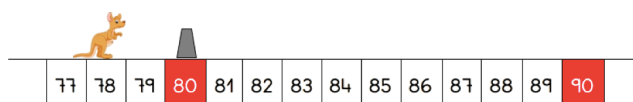


Figure 7: Kangaroo on the number tape

In this episode, see figure 10 above, the kangaroo is at number seventy-eight and it will jump until it reaches some number in the family of the eighties. First, we analyse the situation from a broader perspective, asking the group questions such as: What jumps shall not exceed the fence? It means, how far can he go without going over 80? What will be the minimum jump length? And the maximum?

Student 1: At least he has to jump three, up to eighty-one.

Student 1: And the most you can give is a jump from eleven to eighty-nine.

Then, with the contributions of all, we will be expressing verbally each of the possible solutions and their corresponding written expression.

Student 3: If it takes a jump of five, it reaches eighty-three.

We check on the number tape whether this is a correct solution

Teacher: How will we express this by writing what you have just said? (We write on the board at the same time we relate)

Teacher and Students: it begins at seventy-eight (78), jumps forward five (+5) and reaches eighty-three (= 83).

On the board will be written: $78 + 5 = 83$. We will continue exploring other possibilities from the same starting position. So, we will collect all the arithmetic structures on the board: $78+3=81$, $78+6=84$, $78+9=87$, $78+4=82$, $78+7=85$, $78+10=88$, $78+5=83$, $78+8=86$ and $78+11=89$. Then, we will observe them as a whole, becoming aware that with them we have translated each event into the language of numbers and signs. Moving forward with this observation, the teacher can bring the attention to the amounts that remain and to which they change, thus intuitively introducing the concept of a variable:

Teacher: From what number has the kangaroo always jumped? What jumps has it made? With what jumps did it stay close to the start? With which did it get very far? ...

Two sessions were developed with the dynamics showed; the kangaroo was introduced, working only with components in a sensorimotor and verbal level until we were sure that the whole group understood the proposal. In each participation, the student had to decide first the direction of the jump and the magnitude of the jump; then choose the exit number. These decisions inform the teacher about the students' number sense much more than the calculation itself. Then, the movement was actually made on the number tape verifying that it was feasible and, finally, the operation explaining the jump was verbalized. In the third session this activity is made on written level. To ensure the accessibility of all the students, an image of the tape with the corresponding numerical section was included in the worksheet. Below, we show two cases with interesting results. In figure 11 it is appreciated that the student discovers how to control the situation, the control of phenomena that is so present in the relational thinking, so that the number of arrival is always 41. In figure 12, the student ends, according to his own words, with "a jump from end to end"(that it is to say from 49 to 31)

Tramo 0-49
Escribe operaciones

$37+6=43$	$34+9=43$	$47-11=36$
$46-11=35$	$39+2=41$	$39+3=42$
$38+3=41$	$37+4=41$	$36+5=41$
$35+6=41$	$34+7=41$	$33+8=41$

Escribe operaciones de suma y resta para que el canguro pueda salir la casilla del 40.

Figure 8: Example of Kangaroo's worksheet

Tramo 0-49
Escribe operaciones

$37+6=43$	$34+9=43$	$47-11=36$
$46-11=35$	$39+2=41$	$44-5=39$
$44-11=33$	$48-13=35$	$38+4=42$
$31+10=41$	$35+8=43$	$49-78=31$

Escribe operaciones de suma y resta para que el canguro pueda salir la casilla del 40.

Figure 9: Example of Kangaroo's worksheet

The number tape section showed in this activity is a great help for students with difficulties because it reduces anxiety about the calculations (they can do the count directly) and focuses attention on the reflection and written expression of events. On the other side, there are students that are soon aware of ways to control numerical relationships (i.e. the student who experiments with the arrival at 41 from different positions in Figure 8). The implementation of these activities shows that the sensorimotor experience is very positive since practically the whole group correctly identifies the operation (addition or subtraction) that corresponds to each jump. Although there are students who

work more slowly and do not manage to complete the twelve required possibilities, those that do, do it correct.

Final comments

This work is a first approach to the development of number sense by using conceptual metaphors. Authors are aware that evidences should be tested empirically through wider research, but some interesting aspects already arise from this work. Among others, it may be said that (1) the activities proposed develop the relational thinking. They imply an active behaviour of children, provoking reflection on what they should do, and how they are going to solve it. This way of acting involves the knowledge that children have of numbers and how numbers relate to each other through operations, fostering relational thinking in flexible contexts that admit more than one solution to the challenge posed. (2) It also promotes transparent algorithms for calculation. Our approach, the tactical calculation, shows the reflection and planning designed by the student. (3) A communicative approach is given to mathematical language. The classroom becomes a place where students can talk, expose their plan and discuss the decisions they are making. This oral language gives shape and meaning to expressions with numbers and signs. Children are building an interactive dictionary full of meanings that all share. (4) Working memory is reinforced. Having the number line and the position of the characters in view gives confidence to the students and helps them to concentrate on the choice of tactics and graphic representation. Then, in the absence of resources, these visualizations (already in the mental plane) will be fundamental to solve problems and calculations.

References

- Adamuz-Povedano, N., & Bracho-López, R. (2017). *La aritmética del siglo XXI*. Madrid: Catarata.
- Berch, D. B. (2005). Making Sense of Number Sense: Implications for Children With Mathematical Disabilities. *Journal of Learning Disabilities*, 38(4), 333–339.
- Coles, A. L. F., & Sinclair, N. (2017). Re-thinking place value: From metaphor to metonym. *For the Learning of Mathematics*, 37(1), 3–8.
- Gallese, V., & Lakoff, G. (2005). The Brain's concepts: the role of the Sensory-motor system in conceptual knowledge. *Cognitive Neuropsychology*, 22(3–4), 455–479.
<https://doi.org/10.1080/02643290442000310>
- García-Pérez, T., & Adamuz-Povedano, N. (2016). Conocemos los números. Una experiencia basada en la manipulación. *Epsilon-Revista de Educación Matemática*, 33(1), 73–88.
- Godino, J. D., Font, V., Konic, P., & Wilhelmi, M. R. (2009). El sentido numérico como articulación flexible de los significados parciales de los números. In J. Cardeñoso & M. Peñas (Eds.), *Investigación en el aula de Matemáticas. Sentido Numérico* (pp. 117–184). Granada: SAEM THALES y Departamento de Didáctica de la Matemática de la Universidad de Granada.
- Lakoff, G., & Johnson, M. (2003). *Metaphors we live by*. Chicago: The University of Chicago Press.
- McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A Proposed Framework for Examining Basic Number Sense. *For the Learning of Mathematics*, 12(3), 3–8, 44.

- NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston: NCTM.
- Siegler, R. (2003). Implications of Cognitive Science Research for Mathematics Education. In J. Kilpatrick, W. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 219–233). Reston, VA: National Council of Teachers of Mathematics.
- Soto-Andrade, J. (2007). La cognición hecha cuerpo florece en metáforas. In D. Cosmelli (Ed.), *Nuevos enfoques de la Cognición: Redescubriendo la dinámica de la acción, la intención y la inter subjetividad* (pp. 71–90). Chile: Universidad Diego Portales.
- Sowder, J. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of Research in Mathematics Teaching and Learning* (pp. 371–389). New York: Macmillan Publishing Co.