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Phase driven modal synthesis for forced response evaluation

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Abstract A new definition is proposed for the Nonlinear Normal Modes, close to the one developed by Bellizzi & Bouc [1]. These NNMs are the used to evaluate the forced responses using a modal phase parametrization rather than the classical forcing frequency parametrization.

The basic dynamic equation considered for nonlinear dynamics writes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{f}_{\text{nl}}(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{f}_e(t) \quad (1)$$

where \mathbf{f}_{nl} gathers nonlinear forces while \mathbf{f}_e denotes a periodic external forcing.

Damped nonlinear normal modes (dNNMs) are the solutions of Eq. (1) when the forcing \mathbf{f}_e is nullified [3]. Several methods to compute these solutions were proposed. The one exposed and used here is close to the amplitude and phase parameterization described by Bellizzi and Bouc [1]. Displacements \mathbf{u} and velocities \mathbf{v} have the same dependency to an amplitude α and a dimensionless time τ than in [1] but the amplitude decay function η and the pseudo circular frequency ω only depend on amplitude here:

$$\mathbf{u}(t) = \alpha(t)\boldsymbol{\psi}^u(\alpha(t), \tau(t)), \quad \mathbf{v}(t) = \alpha(t)\boldsymbol{\psi}^v(\alpha(t), \tau(t)), \quad \dot{\alpha}(t) = \eta(\alpha(t))\alpha(t), \quad \dot{\tau}(t) = \omega(\alpha(t)) \quad (2)$$

Once injected in Eq. (1), and adding $\mathbf{v} = \dot{\mathbf{u}}$ condition leads to

$$\alpha\boldsymbol{\psi}^v(\alpha, \tau) = \eta(\alpha)\alpha\boldsymbol{\psi}^u(\alpha, \tau) + \alpha(D_\alpha\boldsymbol{\psi}^u(\alpha, \tau)\eta(\alpha)\alpha + D_\tau\boldsymbol{\psi}^u(\alpha, \tau)\omega(\alpha)) \quad (3a)$$

$$\mathbf{M}(\eta(\alpha)\alpha\boldsymbol{\psi}^v(\alpha, \tau) + \alpha(D_\alpha\boldsymbol{\psi}^v(\alpha, \tau)\eta(\alpha)\alpha + D_\tau\boldsymbol{\psi}^v(\alpha, \tau)\omega(\alpha))) + \mathbf{C}(\alpha\boldsymbol{\psi}^v(\alpha, \tau)) + \mathbf{K}(\alpha\boldsymbol{\psi}^u(\alpha, \tau)) + \mathbf{f}_{\text{nl}}(\alpha\boldsymbol{\psi}^u(\alpha, \tau), \alpha\boldsymbol{\psi}^v(\alpha, \tau)) = \mathbf{0} \quad (3b)$$

Instead of seeking for the various quantities as a power series in α and a Fourier series in τ which leads to a very large system of equations, a ‘‘point-by-point’’ approach is preferred in the α dimension: a branch is defined by successive points gathering $\alpha^{(i)}$ (modal amplitude), $\mathbf{Q}^{u^{(i)}}$ (Fourier coefficients for $\boldsymbol{\psi}^{u^{(i)}}$), $\mathbf{Q}^{v^{(i)}}$ (Fourier coefficients for $\boldsymbol{\psi}^{v^{(i)}}$), $\eta^{(i)}$ (modal amplitude decay function) and $\omega^{(i)}$ (modal circular frequency). While $D_\tau\bullet = \partial\bullet/\partial\tau$ quantities can be evaluated exactly via Fourier series derivation, $D_\alpha\bullet = \partial\bullet/\partial\alpha$ is evaluated using a linear interpolation between the previous and the current points. The two necessary normalization conditions are defined by

$$\boldsymbol{\psi}^u(\alpha, 0)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, 0) + \boldsymbol{\psi}^u(\alpha, \pi/2)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, \pi/2) = 1 \quad (4a)$$

$$\boldsymbol{\psi}^u(\alpha, 0)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, \pi/2) = 0 \quad (4b)$$

Lastly, points on the branch are indexed by their (discrete) arclength $s^{(i)}$:

$$s^{(i)} = s^{(i-1)} + \left((\alpha^{(i)} - \alpha^{(i-1)})^2 + (\eta^{(i)} - \eta^{(i-1)})^2 + (\omega^{(i)} - \omega^{(i-1)})^2 + \left\| \mathbf{Q}^{u^{(i)}} - \mathbf{Q}^{u^{(i-1)}} \right\|^2 + \left\| \mathbf{Q}^{v^{(i)}} - \mathbf{Q}^{v^{(i-1)}} \right\|^2 \right)^{1/2} \quad (5)$$

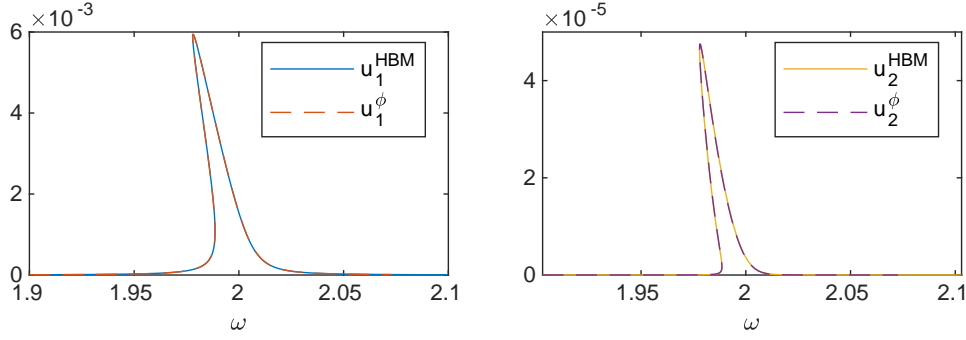


Figure 1: Illustration: modal synthesis around first mode for a 2-dofs system.

Once a dNNM is calculated, it offers a first understanding of the structure as well as a rough prediction of its behavior when forcing is introduced. It can also be used to compute the forced response effectively using modal synthesis.

Let us assume that $\mathbf{f}_e(t) = \mathbf{f}_{e_0} \cos(\omega t)$. Using a dimensionless time $\tau = \omega t$ and denoting $\mathbf{u}_\tau(\tau) = \mathbf{u}(t)$, $\bullet' = d\bullet/d\tau$, Eq. (1) becomes

$$\omega^2 \mathbf{M} \mathbf{u}_\tau'' + \omega \mathbf{C} \mathbf{u}_\tau' + \mathbf{K} \mathbf{u}_\tau + \mathbf{f}_{nl}(\mathbf{u}_\tau, \omega \mathbf{u}_\tau') = \mathbf{f}_{e_\tau}(\tau) \quad (6)$$

Then, \mathbf{u}_τ is naturally sought as

$$\mathbf{u}_\tau(\tau) = \tilde{\mathbf{u}}(s, \tau + \phi) \quad (7)$$

where the 2 unknowns are s which defines the location on the dNNM branch and ϕ , the phase with respect to the excitation as in the linear case.

Equations used to find these 2 unknowns are

$$\int_0^{2\pi} \mathbf{r}(\tau) \tilde{\mathbf{u}}(s, \tau + \phi) d\tau = 0 \quad \text{and} \quad \int_0^{2\pi} \mathbf{r}(\tau) (\omega \tilde{\mathbf{u}}'(s, \tau + \phi)) d\tau = 0 \quad (8)$$

with $\mathbf{r}(\tau)$ being the residue of the dynamical equation (6):

$$\mathbf{r}(\tau) = \omega^2 \mathbf{M} \mathbf{u}_\tau'' + \omega \mathbf{C} \mathbf{u}_\tau' + \mathbf{K} \mathbf{u}_\tau + \mathbf{f}_{nl}(\mathbf{u}_\tau, \omega \mathbf{u}_\tau') - \mathbf{f}_{e_\tau}(\tau) \quad (9)$$

This system can be solved using any continuation method in the variables ω, s, ϕ .

Another approach is to consider that, as in the linear case, ϕ will vary from 0 to $-\pi$ with a continuous decrease along the frequency function response (FRF). Hence, the FRF can be computed by solving for ω and s only for discrete values of $\phi \in]-\pi, 0]$ avoiding the use of a continuation scheme. This approach was applied to compute the first mode and the FRF around this first mode for the 2-dofs example used by Touz e and Amabili [4] and return very accurate results as illustrated in Figure 1 for which reference results are HBM results with up to 5 harmonics. This phase parameterization can be very interesting in the stochastic case to link points of different realizations as explained in [2] for the linear case.

References

- [1] S. Bellizzi and R. Bouc. “An amplitude-phase formulation for nonlinear modes and limit cycles through invariant manifolds”. In: *Journal of Sound and Vibration* 300.3–5 (2007), pp. 896–915. DOI: [10.1016/j.jsv.2006.09.004](https://doi.org/10.1016/j.jsv.2006.09.004).
- [2] E. Sarrouy. “Phase driven study for stochastic linear multi-dofs dynamic response”. In: *Mechanical Systems and Signal Processing* 129 (2019), pp. 717–740. DOI: [10.1016/j.ymsp.2019.04.042](https://doi.org/10.1016/j.ymsp.2019.04.042).
- [3] S.W. Shaw and C. Pierre. “Normal Modes for Non-Linear Vibratory Systems”. In: *Journal of Sound and Vibration* 164.1 (1993), pp. 85–124. DOI: [10.1006/jsvi.1993.1198](https://doi.org/10.1006/jsvi.1993.1198).
- [4] C. Touz e and M. Amabili. “Nonlinear normal modes for damped geometrically nonlinear systems: Application to reduced-order modelling of harmonically forced structures”. In: *Journal of Sound and Vibration* 298.4–5 (2006), pp. 958–981. DOI: [10.1016/j.jsv.2006.06.032](https://doi.org/10.1016/j.jsv.2006.06.032).