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# Phase driven modal synthesis for forced response evaluation

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**Abstract** A new definition is proposed for the Nonlinear Normal Modes, close to the one developed by Bellizzi & Bouc [1]. These NNMs are the used to evaluate the forced responses using a modal phase parametrization rather than the classical forcing frequency parametrization.

The basic dynamic equation considered for nonlinear dynamics writes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{f}_{\text{nl}}(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{f}_e(t) \quad (1)$$

where  $\mathbf{f}_{\text{nl}}$  gathers nonlinear forces while  $\mathbf{f}_e$  denotes a periodic external forcing.

Damped nonlinear normal modes (dNNMs) are the solutions of Eq. (1) when the forcing  $\mathbf{f}_e$  is nullified [3]. Several methods to compute these solutions were proposed. The one exposed and used here is close to the amplitude and phase parameterization described by Bellizzi and Bouc [1]. Displacements  $\mathbf{u}$  and velocities  $\mathbf{v}$  have the same dependency to an amplitude  $\alpha$  and a dimensionless time  $\tau$  than in [1] but the amplitude decay function  $\eta$  and the pseudo circular frequency  $\omega$  only depend on amplitude here:

$$\mathbf{u}(t) = \alpha(t)\boldsymbol{\psi}^u(\alpha(t), \tau(t)), \quad \mathbf{v}(t) = \alpha(t)\boldsymbol{\psi}^v(\alpha(t), \tau(t)), \quad \dot{\alpha}(t) = \eta(\alpha(t))\alpha(t), \quad \dot{\tau}(t) = \omega(\alpha(t)) \quad (2)$$

Once injected in Eq. (1), and adding  $\mathbf{v} = \dot{\mathbf{u}}$  condition leads to

$$\alpha\boldsymbol{\psi}^v(\alpha, \tau) = \eta(\alpha)\alpha\boldsymbol{\psi}^u(\alpha, \tau) + \alpha(D_\alpha\boldsymbol{\psi}^u(\alpha, \tau)\eta(\alpha)\alpha + D_\tau\boldsymbol{\psi}^u(\alpha, \tau)\omega(\alpha)) \quad (3a)$$

$$\mathbf{M}(\eta(\alpha)\alpha\boldsymbol{\psi}^v(\alpha, \tau) + \alpha(D_\alpha\boldsymbol{\psi}^v(\alpha, \tau)\eta(\alpha)\alpha + D_\tau\boldsymbol{\psi}^v(\alpha, \tau)\omega(\alpha))) + \mathbf{C}(\alpha\boldsymbol{\psi}^v(\alpha, \tau)) + \mathbf{K}(\alpha\boldsymbol{\psi}^u(\alpha, \tau)) + \mathbf{f}_{\text{nl}}(\alpha\boldsymbol{\psi}^u(\alpha, \tau), \alpha\boldsymbol{\psi}^v(\alpha, \tau)) = \mathbf{0} \quad (3b)$$

Instead of seeking for the various quantities as a power series in  $\alpha$  and a Fourier series in  $\tau$  which leads to a very large system of equations, a ‘‘point-by-point’’ approach is preferred in the  $\alpha$  dimension: a branch is defined by successive points gathering  $\alpha^{(i)}$  (modal amplitude),  $\mathbf{Q}^{u^{(i)}}$  (Fourier coefficients for  $\boldsymbol{\psi}^{u^{(i)}}$ ),  $\mathbf{Q}^{v^{(i)}}$  (Fourier coefficients for  $\boldsymbol{\psi}^{v^{(i)}}$ ),  $\eta^{(i)}$  (modal amplitude decay function) and  $\omega^{(i)}$  (modal circular frequency). While  $D_\tau\bullet = \partial\bullet/\partial\tau$  quantities can be evaluated exactly via Fourier series derivation,  $D_\alpha\bullet = \partial\bullet/\partial\alpha$  is evaluated using a linear interpolation between the previous and the current points. The two necessary normalization conditions are defined by

$$\boldsymbol{\psi}^u(\alpha, 0)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, 0) + \boldsymbol{\psi}^u(\alpha, \pi/2)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, \pi/2) = 1 \quad (4a)$$

$$\boldsymbol{\psi}^u(\alpha, 0)^T \mathbf{M} \boldsymbol{\psi}^u(\alpha, \pi/2) = 0 \quad (4b)$$

Lastly, points on the branch are indexed by their (discrete) arclength  $s^{(i)}$ :

$$s^{(i)} = s^{(i-1)} + \left( (\alpha^{(i)} - \alpha^{(i-1)})^2 + (\eta^{(i)} - \eta^{(i-1)})^2 + (\omega^{(i)} - \omega^{(i-1)})^2 + \left\| \mathbf{Q}^{u^{(i)}} - \mathbf{Q}^{u^{(i-1)}} \right\|^2 + \left\| \mathbf{Q}^{v^{(i)}} - \mathbf{Q}^{v^{(i-1)}} \right\|^2 \right)^{1/2} \quad (5)$$

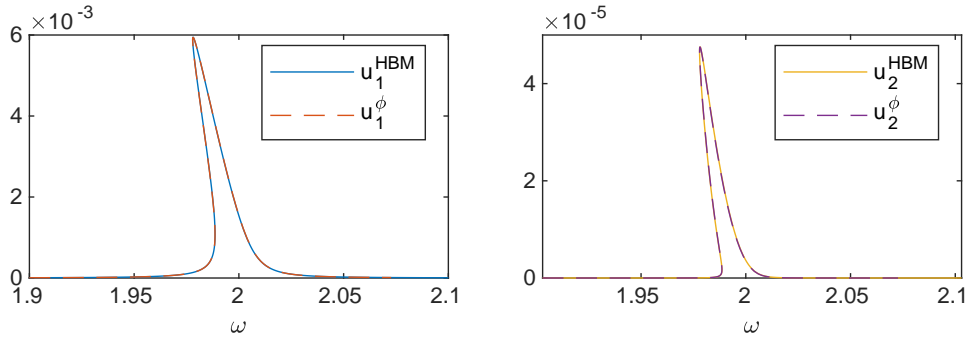


Figure 1: Illustration: modal synthesis around first mode for a 2-dofs system.

Once a dNNM is calculated, it offers a first understanding of the structure as well as a rough prediction of its behavior when forcing is introduced. It can also be used to compute the forced response effectively using modal synthesis.

Let us assume that  $\mathbf{f}_e(t) = \mathbf{f}_{e_0} \cos(\omega t)$ . Using a dimensionless time  $\tau = \omega t$  and denoting  $\mathbf{u}_\tau(\tau) = \mathbf{u}(t)$ ,  $\bullet' = d\bullet/d\tau$ , Eq. (1) becomes

$$\omega^2 \mathbf{M} \mathbf{u}_\tau'' + \omega \mathbf{C} \mathbf{u}_\tau' + \mathbf{K} \mathbf{u}_\tau + \mathbf{f}_{nl}(\mathbf{u}_\tau, \omega \mathbf{u}_\tau') = \mathbf{f}_{e_\tau}(\tau) \quad (6)$$

Then,  $\mathbf{u}_\tau$  is naturally sought as

$$\mathbf{u}_\tau(\tau) = \tilde{\mathbf{u}}(s, \tau + \phi) \quad (7)$$

where the 2 unknowns are  $s$  which defines the location on the dNNM branch and  $\phi$ , the phase with respect to the excitation as in the linear case.

Equations used to find these 2 unknowns are

$$\int_0^{2\pi} \mathbf{r}(\tau) \tilde{\mathbf{u}}(s, \tau + \phi) d\tau = 0 \quad \text{and} \quad \int_0^{2\pi} \mathbf{r}(\tau) (\omega \tilde{\mathbf{u}}'(s, \tau + \phi)) d\tau = 0 \quad (8)$$

with  $\mathbf{r}(\tau)$  being the residue of the dynamical equation (6):

$$\mathbf{r}(\tau) = \omega^2 \mathbf{M} \mathbf{u}_\tau'' + \omega \mathbf{C} \mathbf{u}_\tau' + \mathbf{K} \mathbf{u}_\tau + \mathbf{f}_{nl}(\mathbf{u}_\tau, \omega \mathbf{u}_\tau') - \mathbf{f}_{e_\tau}(\tau) \quad (9)$$

This system can be solved using any continuation method in the variables  $\omega, s, \phi$ .

Another approach is to consider that, as in the linear case,  $\phi$  will vary from 0 to  $-\pi$  with a continuous decrease along the frequency function response (FRF). Hence, the FRF can be computed by solving for  $\omega$  and  $s$  only for discrete values of  $\phi \in ]-\pi, 0]$  avoiding the use of a continuation scheme. This approach was applied to compute the first mode and the FRF around this first mode for the 2-dofs example used by Touz e and Amabili [4] and return very accurate results as illustrated in Figure 1 for which reference results are HBM results with up to 5 harmonics. This phase parameterization can be very interesting in the stochastic case to link points of different realizations as explained in [2] for the linear case.

## References

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