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Phase driven modal synthesis for forced response evaluation

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Abstract A new definition is proposed for the Nonlinear Normal Modes, close to the one developed by Bellizzi & Bouc [1]. Theses NNMs are the used to evaluate the forced responses using a modal phase parametrization rather than the classical forcing frequency parametrization.

The basic dynamic equation considered for nonlinear dynamics writes

$$\ddot{\mathbf{u}} + C\dot{\mathbf{u}} + K\mathbf{u} + f_{nl}(\mathbf{u}, \dot{\mathbf{u}}) = f_e(t)$$

(1)

where $f_{nl}$ gathers nonlinear forces while $f_e$ denotes a periodic external forcing.

Damped nonlinear normal modes (dNNMs) are the solutions of Eq. (1) when the forcing $f_e$ is nullified [3]. Several methods to compute these solutions where proposed. The one exposed and used here is close to the amplitude and phase parameterization described by Bellizzi and Bouc [1]. Displacements $\mathbf{u}$ and velocities $\mathbf{v}$ have the same dependency to an amplitude $\alpha$ and a dimensionless time $\tau$ than in [1] but the amplitude decay function $\eta$ and the pseudo circular frequency $\omega$ only depend on amplitude here:

$$\mathbf{u}(t) = \alpha(t)\psi^u(\alpha(t), \tau(t)), \quad \mathbf{v}(t) = \alpha(t)\psi^v(\alpha(t), \tau(t)), \quad \dot{\alpha}(t) = \eta(\alpha(t)\alpha(t)), \quad \dot{\tau}(t) = \omega(\alpha(t))$$

(2)

Once injected in Eq. (1), and adding $\mathbf{v} = \dot{\mathbf{u}}$ condition leads to

$$\alpha\psi^u(\alpha, \tau) = \eta(\alpha)\alpha\psi^u(\alpha, \tau) + \alpha (D_\alpha\psi^u(\alpha, \tau)\eta(\alpha) + D_\tau\psi^u(\alpha, \tau)\omega(\alpha))$$

(3a)

$$M(\eta(\alpha)\alpha\psi^u(\alpha, \tau) + \alpha (D_\alpha\psi^u(\alpha, \tau)\eta(\alpha) + D_\tau\psi^u(\alpha, \tau)\omega(\alpha))) + C(\alpha\psi^u(\alpha, \tau)) + K(\alpha\psi^u(\alpha, \tau)) + f_{nl}(\alpha\psi^u(\alpha, \tau)) = 0$$

(3b)

Instead of seeking for the various quantities as a power series in $\alpha$ and a Fourier series in $\tau$ which leads to a very large system of equations, a “point-by-point” approach is preferred in the $\alpha$ dimension: a branch is defined by successive points gathering $\alpha^{(i)}$ (modal amplitude), $Q^{u(i)}$ (Fourier coefficients for $\psi^{u(i)}$), $Q^{v(i)}$ (Fourier coefficients for $\psi^{v(i)}$), $\eta^{(i)}$ (modal amplitude decay function) and $\omega^{(i)}$ (modal circular frequency). While $D_\tau\bullet = \partial \bullet / \partial \tau$ quantities can be evaluated exactly via Fourier series derivation, $D_\alpha\bullet = \partial \bullet / \partial \alpha$ is evaluated using a linear interpolation between the previous and the current points. The two necessary normalization conditions are defined by

$$\psi^u(\alpha, 0)^T M\psi^u(\alpha, 0) + \psi^u(\alpha, \pi/2)^T M\psi^u(\alpha, \pi/2) = 1$$

(4a)

$$\psi^v(\alpha, 0)^T M\psi^v(\alpha, \pi/2) = 0$$

(4b)

Lastly, points on the branch are index by their (discrete) arclength $s^{(i)}$:

$$s^{(i)} = s^{(i-1)} + \left( (\alpha^{(i)} - \alpha^{(i-1)})^2 + (\eta^{(i)} - \eta^{(i-1)})^2 + (\omega^{(i)} - \omega^{(i-1)})^2 \right)^{1/2}$$

(5)
Figure 1: Illustration: modal synthesis around first mode for a 2-dofs system.

Once a dNNM is calculated, it offers a first understanding of the structure as well as a rough prediction of its behavior when forcing is introduced. It can also be used to compute the forced response effectively using modal synthesis.

Let us assume that \( f_e(t) = f_{e_0} \cos(\omega t) \). Using a dimensionless time \( \tau = \omega t \) and denoting \( u_\tau(\tau) = u(t), \ \dot{\cdot} = d \cdot /d\tau \), Eq. (1) becomes

\[
\omega^2 M u''_\tau + \omega C u'_\tau + K u_\tau + f_{nl}(u_\tau, \omega u'_\tau) = f_e(\tau)
\]  

Then, \( u_\tau \) is naturally sought as

\[
u_\tau(\tau) = \tilde{u}(s, \tau + \phi)
\]  

where the 2 unknowns are \( s \) which defines the location on the dNNM branch and \( \phi \), the phase with respect to the excitation as in the linear case.

Equations used to find these 2 unknowns are

\[
\int_0^{2\pi} r(\tau) \tilde{u}(s, \tau + \phi) \ d\tau = 0 \quad \text{and} \quad \int_0^{2\pi} r(\tau) (\omega \tilde{u}'(s, \tau + \phi)) \ d\tau = 0
\]  

with \( r(\tau) \) being the residue of the dynamical equation (6):

\[
r(\tau) = \omega^2 M u''_\tau + \omega C u'_\tau + K u_\tau + f_{nl}(u_\tau, \omega u'_\tau) - f_e(\tau)
\]  

This system can be solved using any continuation method in the variables \( \omega, s, \phi \).

Another approach is to consider that, as in the linear case, \( \phi \) will vary from 0 to \(-\pi\) with a continuous decrease along the frequency function response (FRF). Hence, the FRF can be computed by solving for \( \omega \) and \( s \) only for discrete values of \( \phi \in [-\pi, 0] \) avoiding the use of a continuation scheme. This approach was applied to compute the first mode and the FRF around this first mode for the 2-dofs example used by Touzé and Amabili [4] and return very accurate results as illustrated in Figure 1 for which reference results are HBM results with up to 5 harmonics. This phase parameterization can be very interesting in the stochastic case to link points of different realizations as explained in [2] for the linear case.

References


