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Alternating TLM symmetrical condensed node

S. Le Maguer and M.M. Ney

A new alternating scheme for the transmission line matrix (TLM) method is proposed. As shown, this scheme is no longer based on the alternate use of TLM cells in the computational volume, but rather on an alternating process in the cell itself. This feature allows easy treatment of boundaries, which is the usual drawback of alternating schemes.

Introduction: The transmission line matrix (TLM) method is a wellknown numerical technique for electromagnetic field simulations. Using a time and space discretisation, this method allows a wideband characterisation in a single run. Since Johns' paper [1], that introduced the symmetrical condensed node (SCN), much effort has been put into decreasing the computational cost. A major insight in memory reduction resulted from the alternating-TLM scheme (ATLM) [2]. This model allows the use of any type of TLM node (SCN [1], HSCN [3, 4] or SSCN [5]) while decreasing the computational effort by a factor of two. Unfortunately, this technique has a major drawback in that boundaries have to be placed at the centre of the TLM cell. This feature breaks the generality of the TLM scheme since special nodes have to be created for each type of boundary. Later, the ATLM scheme was associated with rotated-TLM (RTLM) [6], to yield the AR-TLM scheme [7] that leads to a further 50% computer-effort economy. However, boundary treatment in some configurations has not yet been solved.

In this Letter, a new alternating TLM scheme is proposed. Unlike usual alternating TLM techniques, it is no longer based on the alternate use of TLM cells in the computational volume, but rather on an alternating process in the cell itself. To this purpose, an alternate direction implicit (ADI) [8] discretisation of Maxwell's equations is used. As shown, numerically and theoretically, the alternating scheme obtained is equivalent to the SCN scheme. Furthermore, memory expenditure is divided by a factor of two and boundary conditions are treated according to the standard TLM algorithm.



Fig. 1 Two sub-cells a Step 1 b Step 2

Scheme derivation: Let us start with the ADI scheme (see [8] or [9]). It is a two-step algorithm, which usually leads to an implicit method by approximating Maxwell's equations in a convenient manner. These two sub-steps are equivalent to one iteration (time step: Δt) of a classical scheme. Let us describe the ADI approximation for the E_x component of the field in a Cartesian grid:

$$\varepsilon \frac{\partial E_x}{\partial t} \bigg|_{n+(1/4)} = \frac{\partial H_z}{\partial y} \bigg|_n - \frac{\partial H_y}{\partial z} \bigg|_{n-(1/2)}$$
(1)

$$\varepsilon \frac{\partial E_x}{\partial t} \bigg|_{n+(1/4)} = \frac{\partial H_z}{\partial y} \bigg|_n - \frac{\partial H_y}{\partial z} \bigg|_{n+(1/2)}$$
(2)

where 'n' is the time-step index. The approximation of (1) (which corresponds to step 1) and (2) (step 2) can be derived in a TLM form using the technique proposed by Peña and Ney [10]. In the present Letter the technique is developed without a stub (i.e. a cubic cell in an homogenous medium at maximum time-step). Equations (1) and (2) lead, respectively, to the following expressions of the E_x field component at the centre of the cell:

$$\Delta l E_x^{n-(1/4)} = (a_2 + a_9)^{n-(1/2)} \tag{3}$$

$$\Delta l E_x^{n+(1/4)} = (a_1 + a_{12})^n \tag{4}$$

where Δl is the space discretisation. *a* denotes an incident voltage and the subscript is the number of the port with respect to Johns' notation [1]. Furthermore, reflected voltages (denoted *b*) are obtained from the field at the centre of the cell. For instance, at the end of step 2 one can obtain the following equation:

$$b_2^{n+(1/2)} = \frac{1}{2} \Delta l [E_x^{n+(1/4)} - Z_0 H_y^{n+(1/4)}]$$
(5)

where Z_0 is the impedance of free space. From (3), (4) and (5), it can be observed that the classical 12-port SCN degenerates in a pair of six-port nodes. These nodes are used one after the other in the iterative scheme. This implies that memory storage is divided by a factor of two. Furthermore, the scheme remains explicit and boundary conditions are treated as is usually done with TLM (using voltage reflection coefficients). The algorithm is summarised in Fig. 1, where the two six-port nodes are illustrated.

Equivalence between new scheme and SCN: Proving that fields at the centre of the cell and reflected voltages have the same expression as that of the SCN can show the equivalence between the new scheme and the SCN algorithm. Let us first examine the field at the centre. Since the alternating scheme is a two-sub-step algorithm where field components at the centre of the cell are calculated at $[n - (1/4)]\Delta t$ for step 1 and at $[n + (1/4)]\Delta t$ for step 2, the equivalent field component with SCN is evaluated at $n\Delta t$. Thus, from (3) and (4):

$$\Delta I E_x^{(n)} = \frac{1}{2} \left(\Delta I E_x^{n-(1/4)} + \Delta I E_x^{n+(1/4)} \right)$$
$$= \frac{1}{2} \left[\left(a_2 + a_9 \right)^{n-(1/2)} + \left(a_1 + a_{12} \right)^n \right]$$
(6)

From (6), it can be observed that it is the exact expression of the component calculated with SCN (compare with expressions in [1]) except that incident voltages are staggered in time. Furthermore, if one takes into account that:

$$Z_0 \Delta l H_v^{n+(1/4)} = (a_{10} - a_6)^n \tag{7}$$

it is obvious using (5) that:

$$b_2^{n+(1/2)} = \frac{1}{2} \Delta l \left[E_x^{n+(1/4)} - Z_0 H_y^{n+(1/4)} \right] = \frac{1}{2} (a_1 + a_{12} - a_{10} + a_6)^n$$
(8)

which is the exact expression of this reflected voltage as extracted from the usual SCN scattering matrix (see [1]). As a result, the schemes are equivalent except that voltages are staggered in time. This equivalence has to be confirmed numerically.



Fig. 2 Longitudinal magnetic field component Fourier transform magnitude obtained in hollow cavity; comparison between new alternating scheme and SCN-TLM

To this purpose, the new TLM scheme is tested in the case of a hollow metallic cavity composed of a rectangular waveguide (20×10 mm cross-section) short-circuited at each extremity (total length: 50 mm). The TE_{10m} modes are excited and, after a 4000 iteration simulation, Fourier transform is performed. The results are

shown in Fig. 2 where they are compared with an SCN-TLM simulation. The space discretisation used in the simulations is $\Delta l = 10^{-3}$ m. As can be observed, the staggered nature of the new algorithm does not appear to add any numerical dispersion compared to the SCN scheme. This tends to confirm that both schemes are numerically equivalent.

Conclusion: A new alternating TLM scheme (ATLM) is proposed. It was developed without a stub (i.e. cubic cell in vacuum at maximum time-step). As a result, the new scheme is composed of two nodes used one after the other in the iterative scheme. Consequently, the memory storage is divided by a factor of two (six voltages compared to 12 with SCN). Numerical results and theory show the perfect match between the new scheme and classical SCN-TLM. Furthermore, the new scheme is explicit and boundary conditions are treated as in standard TLM. This last point is a major advantage compared to previous alternating schemes. Future work will concern extension of this algorithm to general SCN situations (i.e. any medium and any time-step) while keeping its explicit form.

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