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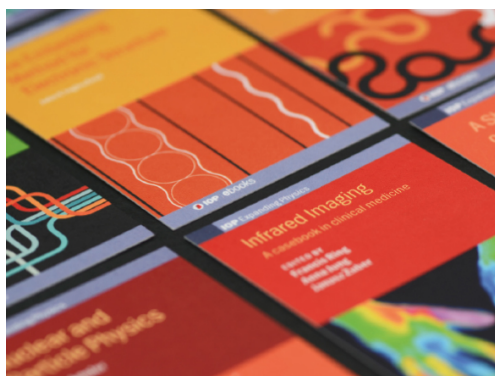
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# Modal analysis of anapoles

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**Abstract.** The resonant interaction between light dielectric scatterers can yield to a resonant light scattering process and to a strong enhancement of the near field intensities at a deep sub-wavelength scale. While Fano resonances can be observed in the scattering spectrum of Mie scatterers, the spectrum of the internal field enhancement is composed of a set of peaks that feature Lorentzian shapes. The question is to know how to optimize the internal field enhancement and how to predict the maximum of the internal field enhancement with respect to the far field response. Here, we derive a pole expansion of the scattering operator to perform a modal analysis of the external and internal field spectra. This pole expansion provides a suitable mathematical description of the Fano resonances observed in the scattering spectrum and of the Lorentzian peaks observed in the internal field spectrum. This modal analysis is also used to predict the maxima of the internal field, *i.e.* the frequency that yields the strongest field intensity inside the dielectric scatterer.

## 1. Introduction

Mie resonances have been studied for several decades in hyperfrequencies and have recently attracted a growing attention due to their ability to enhance light matter interaction in the visible spectrum [1, 2]. The high refractive index of semi-conductors in the visible and near infrared spectrum can be used to design subwavelength sized particles that resonantly interact with light [3, 4, 5, 6]. Another source of interest comes from the possibility to excite both electric and magnetic resonances in spherical particles while spherical plasmonic particles excite electric resonances only (under a subwavelength size condition). When calculating or measuring the scattering spectrum of Mie scatterers, Fano resonances characterized by a minimum followed by a sharp maximum of the scattering efficiency can be observed [7, 8, 9, 10]. Anapole conditions describe this pronounced minimum of the scattered efficiency [11, 12]. A question is to know how is the fact that the particle that does not scatter light while it is excited close to its resonance condition trap light in a more efficiency way, and if this minimum of the scattering efficiency is associated with a maximum of the internal field enhancement. To address this problem, we follow a modal analysis of the  $S$  and  $T$  scattering operators.



## 2. Modal expansion of the scattering operator

Following a previous work published in 2013 in which we derived a modal expansion of the  $S$ -matrix, The  $T$  matrix relates the scattered field to the incident field defined by :

$$\mathbf{E}_{exc}(k\mathbf{r}) = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n e_{n,m}^{(h)} \mathbf{M}_{n,m}^{(1)}(k\mathbf{r}) + e_{n,m}^{(e)} \mathbf{N}_{n,m}^{(1)}(k\mathbf{r}) \quad (1)$$

$$\mathbf{E}_{scat}(k\mathbf{r}) = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n f_{n,m}^{(h)} \mathbf{M}_{n,m}^{(+)}(k\mathbf{r}) + f_{n,m}^{(e)} \mathbf{N}_{n,m}^{(+)}(k\mathbf{r}). \quad (2)$$

The T-matrix coefficients are defined by  $T_n^{(i)}(z) = \frac{f_{n,m}^{(i)}(z)}{e_{n,m}^{(i)}}$  where  $i = e, h$ . They can be cast in a ratio:  $T_n^{(i)}(z) = -\frac{N_{T,n}^{(i)}(z)}{D_n^{(i)}(z)}$  with: [13]:

$$N_{T,n}^{(e)}(z) = z \frac{\varepsilon_s j_n(n_s z) \psi_n'(z) - \psi_n'(n_s z) j_n(z)}{i n_s}, \quad (3)$$

$$D_n^{(e)}(z) = z \frac{\varepsilon_s \xi_n^{(+)}(z) j_n(n_s z) - \psi_n'(n_s z) h_n^{(+)}(z)}{i n_s}, \quad (4)$$

$$N_{T,n}^{(h)}(z) = -iz \left( \psi_n'(z) j_n(n_s z) - \psi_n'(n_s z) j_n(z) \right), \quad (5)$$

$$D_n^{(h)}(z) = -iz \left( \xi_n^{(+)}(z) j_n(n_s z) - \psi_n'(n_s z) h_n^{(+)}(z) \right), \quad (6)$$

On the other hand, the internal field coefficients are defined as:

$$\mathbf{E}_{int}(k_s \mathbf{r}) = E_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n u_{n,m}^{(h)} \mathbf{M}_{n,m}^{(1)}(k_s \mathbf{r}) + u_{n,m}^{(e)} \mathbf{N}_{n,m}^{(1)}(k_s \mathbf{r}) \quad (7)$$

The  $\Omega$ -matrix coefficients linking the internal field to the incident field is defined as follows,  $\Omega_n^{(i)}(z) = \frac{u_{n,m}^{(i)}(z)}{e_{n,m}^{(i)}}$ .

Modal expansions of the  $T_n^{(i)}$  and  $\Omega_n^{(i)}$  coefficients are [13, 14]:

$$T_n^{(i)}(z) = \frac{S_{nr,n}^{(i)} e^{-2iz} - 1}{2} + \frac{e^{-2iz}}{2} \sum_{\alpha=-M}^{+M} \frac{r_{n,\alpha}^{(i)}}{z - p_{n,\alpha}^{(i)}}, \quad (8)$$

$$\Omega_n^{(i)}(z) = \frac{1}{2} \sum_{\alpha=-M}^{+M} \frac{r_{\Xi,n,\alpha}^{(i)}}{z - p_{n,\alpha}^{(i)}}. \quad (9)$$

In the latter expression,  $r_{n,\alpha}^{(i)}$  is the residue of  $T_n^{(i)}$  at the poles  $p_{n,\alpha}^{(i)}$ , and  $r_{\Xi,n,\alpha}^{(i)}$  is the residue of  $\Omega_n^{(i)}(z)$  and  $S_{nr,n}^{(i)} = 1 + \sum_{\alpha=-M}^{+M} \frac{r_{n,\alpha}^{(i)}}{p_{n,\alpha}^{(i)}}$ .

These expansions of the  $T$  and  $\Omega$  matrices are different since the  $T$  matrix expansion unveils the existence of a non-resonant term while the  $\Omega$  matrix depends only on resonant terms. As the scattering and internal field coefficients depend on the  $T$  and  $\Omega$  matrix elements respectively, we can get the modal expansions of these coefficients. These expansions are very powerful since they allow the modal expansion of (i) the scattering efficiency (related to the scattering Mie coefficients) and (ii) of the average of the electric field intensity (related to the internal field coefficients) [13, 15].

These modal expansions of the scattering efficiencies and average of the electric field intensity provide unique tools to study the link between the Fano anomalies observed in the scattered efficiency and the enhancement of the light intensity inside the scatterer. In particular, they highlight the interplay of the eigen-modes in the formation of the Fano resonance and the crucial role of a high quality mode for the enhancement of the internal field [15].

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