Inertial single-phase flow before and after the first Hopf bifurcation in 2D model porous media

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1. Introduction

Inertial one-phase flow in porous media is usually classified under the steady laminar regime (Darcy or creeping flow, weak inertia, strong inertia), the regime beyond strong inertia [1] [2]), the unsteady laminar regime, and the turbulent flow regime [3], including transitions between them. The first Hopf bifurcation [4] can be considered to characterize the critical Reynolds number at which unsteadiness may appear. This work is a continuation of previous studies on the inertial flow pre- and post-bifurcation with a focus on the impact of the pore-structure previously identified as an important parameter [1, 2].

2. Physical model and inertial correction

The dimensionless Navier-Stokes equations have been solved using both the steady solver in COMSOL and unsteady solver in OpenFOAM in order to determine the correction, $-\mathbf{F}\cdot\langle\mathbf{v}^*_\beta\rangle$, to Darcy's law defined as

$$\langle\mathbf{v}^*_\beta\rangle = -\mathbf{K}^*\cdot\nabla\langle p^*_\beta\rangle^\beta - \mathbf{F}\cdot\langle\mathbf{v}^*_\beta\rangle \tag{1}$$

In this equation, $\mathbf{K}^* = \mathbf{K}/l^2$ (see Fig. 1(a) for notations) is the dimensionless permeability and for the isotropic strucures under concern, $\mathbf{K}^* = k^*\mathbf{I} = k/l^2\mathbf{I}$. The analysis of this correction is carried out in terms of the dimensionless correction vector $\mathbf{f}_c$ [1]

$$\mathbf{f}_c = \frac{-\mathbf{F}\cdot\langle\mathbf{v}^*_\beta\rangle}{|\langle\mathbf{v}^*_\beta\rangle|} = \frac{\langle\mathbf{v}^*_\beta\rangle + \mathbf{K}^*\cdot\nabla\langle p^*_\beta\rangle^\beta}{|\langle\mathbf{v}^*_\beta\rangle|} \tag{2}$$

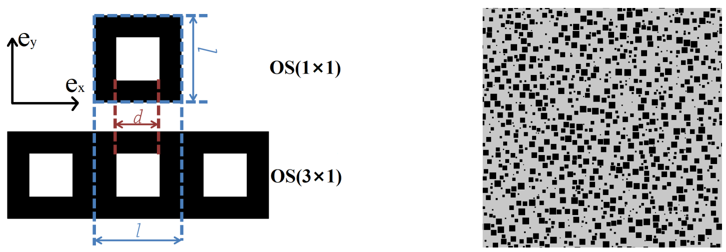
which is obtained from averaging the solution of the flow fields. Simulations have been performed in a wide interval of the Reynolds number, $Re_k = \rho_\beta|\langle\mathbf{v}_\beta\rangle|\sqrt{k}/\mu_\beta = \rho_\beta l^3|\nabla\langle p_\beta\rangle^\beta|\sqrt{k^*}|\langle\mathbf{v}^*_\beta\rangle|/\mu^2_\beta$, including the critical value characteristic of the first Hopf bifurcation.

From Eq. 2 and the expression of Re_k, one can clearly see that, when $\mathbf{f}_c$ scales as $\langle\mathbf{v}^*_\beta\rangle^2$, the relationship between $\mathbf{f}_c$ and Re_k is linear while, when $\mathbf{f}_c$ scales as $\langle\mathbf{v}^*_\beta\rangle^3$, $\mathbf{f}_c$ depends linearly on Re^2_k. Therefore the correlations between the x-component, f_{cx}, of $\mathbf{f}_c$ and Re_k have been studied to investigate inertial flow before and after the first Hopf bifurcation.

The pressure gradient (which is such that $|\nabla\langle p^*_\beta\rangle^\beta| = 1$, see details in [1], section II) was aligned with the principal axes of the structures (e.g. along $\mathbf{e}_x$ so that $f_{cy}\approx 0$). Periodic boundary conditions have been applied in both the x and y directions and the porosity was taken to be 0.75.

3. Numerical results

Two types of 2D structures, namely Ordered structure with Square (OS) cylinders (OS(1×1), OS(3×1) in Fig. 1(a)) and Globally Strongly Disordered structure (GSD, Fig. 1(b)), have been chosen as the Representative Elementary Volumes (REVs) of the flow domains. The critical Reynolds number values for OS(1×1) and OS(3×1) structures were found to be $Re_{kc} = 37.33\pm 0.39$ and $Re_{kc} = 23.02\pm 0.57$ respectively, values that are in agreement with results reported in a previous work

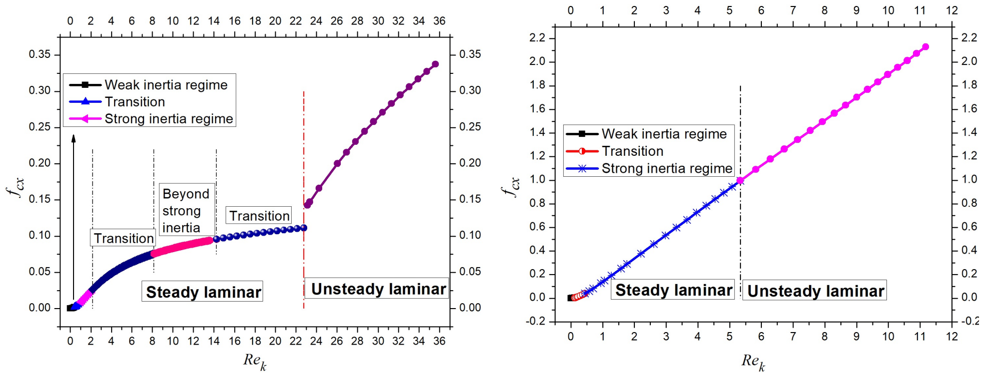


(a) OS (b) GSD

Figure 1: OS (a) and GSD (b) pore structures.

[5]. Due to disorder, the critical Reynolds number for the GSD structure is significantly smaller, as expected, and was computed to be $Re_{kc} = 6.28 \pm 0.45$.

Different flow regimes in the OS(3×1) and GSD structures have been identified, according to the relationship between f_{cx} and Re_k represented in Fig. 2(a) and Fig. 2(b). It can be seen that the transition from steady to unsteady laminar flow is extremely sharp for OS(3×1), which differs from the gradual transition for the GSD structure also observed in a previous work [6]. In addition, when Re_k is larger than the critical Reynolds number, Re_{kc}, an increase of the x-component, $\langle v^*_{\beta x} \rangle$, of the averaged velocity, $\langle \mathbf{v}^*_\beta \rangle$, has been observed for the OS structures during the transition period when the first Hopf bifurcation occurs in the flow. Initiating the simulation with a zero velocity, the time at which the bifurcation occurs is observed to be decreasing while increasing Re_k. Moreover, no such increase on $\langle v^*_{\beta x} \rangle$ at the bifurcation to the unsteady flow is observed for the GSD structure.



(a) f_{cx} vs. Re_k for OS (b) f_{cx} vs. Re_k for GSD

Figure 2: f_{cx} vs. Re_k pre- and post- bifurcation for OS (a) and GSD (b) structures.

This work completes previous analyses on the inertial flow regimes in ordered and disordered model porous materials and confirms that the structural disorder plays a significant role on the inertial flow. In particular, the nature of the flow governing the transition between steady and unsteady laminar flow, corresponding to the first Hopf bifurcation, is different for OS and GSD structures.

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