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Metacognitive activities of pre-service teachers in proving processes

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Proving processes are often complex and difficult. Problems during the phase of proof finding are not unheard of. The ability to find out which knowledge and strategies we need to prove a given statement and how to use knowledge and strategies in proving is called metacognition. A study with pairs of pre-service teachers shows that content knowledge is an important factor whether or not metacognitive activities like monitoring can help handling problems in the proving process.

Keywords: Metacognition, monitoring, proving processes, pre-service teachers.

Introduction

To be able to successfully prove a geometrical statement, one needs more than just subject-specific knowledge and methodological knowledge of proving procedures (Heinze & Reiss, 2003). It is important to know if what one is doing makes sense, is promising and productive. This ability to reflect on one's own knowledge is called metacognition, which is important in general life as well as in mathematics and allows one to find out what knowledge and strategies are needed to solve a problem and how to use knowledge and strategies in proving.

In mathematics education a lot of research exists about metacognition in problem-solving (e.g. Kuzle, 2013; Schoenfeld, 1987, 1992). But there is little research about the significance of metacognitive activities for proving processes, in particular there is nearly no research on pre-service primary school teachers' metacognitive activities, even though they will later in their classrooms be responsible to establish the basis for argumentation. Our research project addresses the need of future teachers to be able to understand and perform proving processes, as well as to perceive their argumentation basis. Metacognitive competences can in our view be useful and fruitful, if taken care of in a sensible way.

The metacognitive activity of monitoring is the focus of this paper. How monitoring is in play during the proving process of students, particularly in what we call loops, elongations, circular reasoning and wanderings is of interest to us. We look specifically at what role content knowledge plays in this and what impact content knowledge has on monitoring.

Theoretical background

The colloquial meaning of metacognition is approximately "reflections on cognition or thinking about your own thinking" (Schoenfeld, 1987, p. 189). Schraw (1998) divides metacognition into two big fields. *Knowledge of cognition* is the knowledge about one's own cognition or about cognition in general. The *regulation of cognition* is an accumulation of activities that help to control one's own learning. It includes *monitoring*, which is the activity of controlling and evaluating one's understanding and one's own performance during a process, in our case the proving process. According to Heinze and Reiss (2003) the ability to prove a statement includes metacognition. Wittmann (2014) focusses on two things that are important for proving statements, factual

knowledge and metaknowledge about proofs. According to Wittmann students often do not have sufficient metaknowledge about proof, which includes knowledge of the functions of proof, characteristics of proving as well as knowledge of how to prove.

The difference between proving as a process and proof as a product is important as well. Wittmann (2014) sees the process of proving as the activity of finding a proof. The process is complex, needs creativity and problem solving competencies. It can go in leaps and bounds and includes sudden success as well as phases where one does not know what to do. The proof product on the other hand is the final product of the proving process. It is succinctly verbalised and neatly written down. Looking at the written proof one cannot see the efforts of the proving process.

Empirical methods

The base of this research project are case studies of four pairs of pre-service primary school teachers. These students participated in a lecture course on elementary geometry in the third semester of their Master Studies. Participation in the study was voluntary and took place in the spring of 2018. Video recorded interviews are the basis of the case studies. The students worked in pairs on proof tasks so that discussions between them provided insight into their metacognitive activities. In the first part of the semi-structured interview, questions about their own proving experiences and proving in general were asked. Then the students were given two statements from plane geometry, one after the other, which could be proven with knowledge from the lecture. Afterwards the process of finding the proof was discussed with the students.

The transcripts of the interviews were looked through for obstacles that can occur in the proving process. In a second step these obstacles were scanned for monitoring activities, which were then coded with a system for categorizing metacognitive abilities developed by Cohors-Fresenborg and Kaune (2007a), that divides monitoring into eight subcategories, e.g. the controlling of calculations and the controlling of argumentation. This allowed us a differentiated view on monitoring activities. After coding the data, the effect of monitoring in the identified phases with obstacles were analysed. Particularly the influence of content knowledge was focused on in order to clarify the impact of the assumed relationship between monitoring and content knowledge for the proving processes and specifically their successful performance.

Prototypes for obstacles in proving processes

Proving processes are rarely linear. Having difficulties, getting stuck or trying something that is not constructive is normal. After looking through the data we can distinguish four specific obstacles that the students encountered in their proving processes: *loops*, *elongations*, *circular reasoning* and *wanderings*. In the following we are going to characterise these obstacles with data from all four groups. With these examples we explain to what extent monitoring influences these obstacles and show the important role of content knowledge.

Loops

Loops are one type of obstacle in proving processes. In a discussion one might have a kind of side discussion that is still part of one's proof idea and attempt, but that is not really constructive for the

proof. To resolve a *loop* situation one must rein in this discussion and pursue the original proof idea instead.

A typical example for a *loop* is the following discussion of Pia and Charlotte (Group 3). The two students attempt to prove the second statement: “A parallelogram is a rectangle if and only if its diagonals have the same length.” During their discussion they draw a rectangle and a parallelogram and argue whether or not they should use the drawing of the parallelogram to prove the statement or if they should use the drawing of the rectangle instead. While Pia is arguing that in a parallelogram the diagonals cannot have the same length, Charlotte suddenly has a new idea; here a *loop* starts. Charlotte wants to change the statement they have to prove to the following: “A parallelogram is a square if and only if its diagonals have the same length”. She describes her thinking.

At that moment Pia uses the definitions of a square and a parallelogram and the connections and differences between the two (every square is a parallelogram, but not every parallelogram is a square). She intends to make clear to Charlotte why her idea, to substitute the rectangle in the statement with a square, is not helpful for the proof of the original statement. Pia’s behaviour is a reaction to a monitoring activity, which is the examination of her partner Charlotte’s understanding of mathematical objects and their role for their argumentation. According to the system of Cohors-Fresenborg and Kaune (2007b) this can be categorized as the monitoring activity M4c: “Controlling of Argumentation – uncovering mistakes in the argumentation” or M2b: “Controlling of an assessment of (assumed) mistakes – uncovering one’s own false ideas or those of others”. Charlotte then applies this knowledge to squares and rectangles and understands why her idea will not work out as a proof. Both students are back at the “starting point” (arguing about which drawing to use for their argumentation), the *loop* is closed. Their side discussion ends, they do not lose themselves in it. They are back on to their original path.

In this example of a *loop*, Pia’s monitoring activities match the “problem” Charlotte has (Why can we not use a square in our argumentation?) and helps them to end their side discussion. They go back to their original proof idea. Their monitoring was quite precise regarding the comprehension of the geometrical objects square, rectangle and parallelogram. Pia showed that she possessed the content knowledge necessary to make use of the monitoring activity and improve their work.

Elongations

An *elongation* in a proving process occurs when one does not pursue the shortest possible way, but instead substitutes several steps for one step.

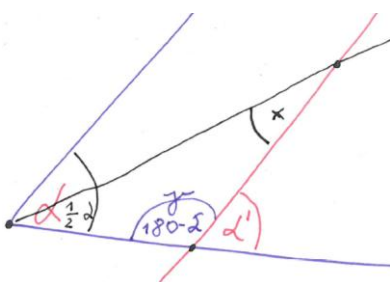


Figure 1: Drawing for the first statement (Group 4)

Elongation simply means to lengthen an action or a process. Often these are hard to determine in data, as a shortest possible way needs to be defined first, in order to be able to find variations of it. To define this, one has to consider (theoretically) available knowledge which may differ between people and in time. The shortest way of a grade 5 student may differ from that of a university professor; the shortest way in a first proof of a new theorem may be an elongation later on, when shorter ways have

evolved.

The first statement our students had to prove was: “Given is an angle with arms g and h . If you draw a line k parallel to arm g that crosses arm h , then h and k together with the angle bisector form an isosceles triangle”. The shortest way to prove the statement that one might follow is using the Alternate Interior Angles Theorem with the angle $\frac{\alpha}{2}$ to show that the base angles have the same size, then the Isosceles Triangle Theorem can be used. Nina and Maja (Group 1), as well as Daria and Leonie (Group 4), proved the statement using the Corresponding Angles Postulate with the angle α (see Figure 1). Then supplementary angles and the Triangle Postulate are used to be able to show that the base angles are equal. This is an example of an *elongation*, as the students had the same basic idea, but for the elongated proof more steps are used than needed.

Neither in the proving process of Group 1 nor in the process of Group 4 a monitoring activity could be found that triggered an *elongation*. The cause of the observed *elongation* seems instead to be the angle the students focussed on (α instead of the angle $\frac{\alpha}{2}$) and therefore more steps were needed to show that the base angles of the triangle are equal. No indication was found why a group decided to focus on α instead of $\frac{\alpha}{2}$. Sufficient content knowledge seems to be the reason that the groups were still able to prove the statement. The students applied all necessary theorems and did not get to a point where they did not know how to proceed.

Circular reasoning

While involved in the proving process *circular reasoning* is hard to see for the students engaged in proving. *Circular reasoning* makes it impossible to produce a correct proof, because one falsely assumes what is meant to be proven by the proof.

Dennis and Julius (Group 2) showed *circular reasoning* in their ‘proof’ for the first statement. After the exploration and discussion of the statement, Dennis and Julius began to write down their ‘proof’. In the first part of their text the students attempted to show that two of the angles in the triangle had the same size. Julius had noted that the angle bisector divided the angle α into two angles of the same size, labelled as base angles of the triangle in his drawing (see the two “ $\frac{1}{2}\alpha$ ” written in red, Figure 2), instead of labelling them as two halves of α (two equal parts of an angle bisector). His argumentation was questioned by Dennis: “Then the question is, whether we can assume that they [Dennis is pointing more at the base angles than at the sides h and k] have the same length”.

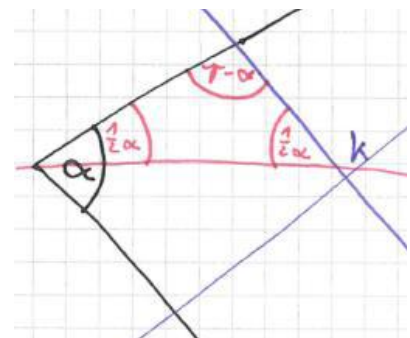


Figure 2: Drawing for the first statement with “red angles” (Group 2)

His question shows a monitoring activity, as Dennis is checking what they are doing while they are still in the proving process. Both students have just started writing down the proof. With his question Dennis controls the effectiveness of their approach and if they are still on target. According to the system of Cohors-Fresenborg and Kaune (2007b) this is the monitoring activity M4a: “Controlling of Argumentation – local examination”. As a reaction to the monitoring question Julius writes a second explanation about why the base angles have the same size. As a reason he

mentions the equality of the two sides of the triangle. This explanation is unfortunately circular as he assumes that the length of the two sides of the triangle are equal, which still needs to be proven.

In *circular reasoning* monitoring affects the proving process differently than in the loop. Dennis and Julius end up with a circular argument in their proof of the first statement because of a monitoring activity. While in the loop the monitoring brings the students back to a reasonable way. Dennis' monitoring activity is also rather imprecise, more like a bad feeling instead of a well-founded suspicion, and the reason for the *circular reasoning*. Both students lack the necessary content knowledge and cannot use monitoring activities to their advantage. Their monitoring activity did not improve the quality of their proof, but instead guided the students into *circular reasoning*, which also demonstrates their lack of knowledge about logic and the structure of a proof.

Wanderings

The abandoning of a good proof idea in favour of ideas that are not constructive is what we call *wandering*. An example of *wandering* is shown by Pia and Charlotte (Group 3). During their discussions Pia and Charlotte have a productive idea to prove the first statement. They use the Alternate Interior Angles Theorem to show that the base angles of the triangle are equal. For the next step in their proof they need to show that a triangle with two equal base angles has two sides of the same length. For this they have the idea to use the Isosceles Triangle Theorem. But Pia is not sure, if the theorem can be used in both directions and asks: "Can you say that?", which is actually a monitoring question. According to the system of Cohors-Fresenborg and Kaune (2007b) this would be the monitoring activity M5a: "Determination of position: naming of Deficiency of Understanding – defined step". This labelling of a deficiency in her understanding is an important monitoring activity as Pia can only follow and support their argumentation if she understands the concepts and the connections between them.

In this case the monitoring question is the starting point for the *wandering*. Because of Pia's monitoring question her partner Charlotte is now insecure in her belief and understanding of the Isosceles Triangle Theorem. Even though Pia affirms that she believes that they are allowed to use it, Charlotte abandons this proof idea in favour of another idea. Their new idea does not work out; their written proof is more a construction protocol with a proof idea but not a proof. For this statement, both first have a good idea how to prove it, but then they discard their idea and start with new ideas which in the end do not work out. They end in a *wandering*.

Like in the loop, and in contrast to the circular reasoning, Pia's monitoring here is quite precise. She questions the use of one specific theorem, the Isosceles Triangle Theorem, which is important for their proof. Unfortunately, both students lack the necessary content knowledge, in this case the content of the theorem, to make use of their monitoring.

Preliminary results

One can raise the question whether or not metacognitive activities, especially monitoring activities, can help students to prevent problems in their proving processes. In theory, monitoring one's work and progress should help students to prevent or find problems and mistakes.

Our prototypes can be systematized in a fourfold table along the dimension “monitoring” and “content knowledge” (see Table 1). Loops are in the field of monitoring and content knowledge, elongations in the field of content knowledge without monitoring. Circular reasoning and wanderings are in the area of monitoring, but with no content knowledge. In the area without monitoring or content knowledge we suspect another prototype, *dry spells*. A dry spell is a part of one’s proving process where one hits a barrier and does not know how to go on for quite some time. Not actively working at the proof anymore can be a sign for a dry spell, as well as remaking drawings, calculating something or recapping what one already knows in the hope of getting a new idea. Looking at the success that is (or not) shown in the proving process, it is possible to group the observed obstacles (loops, elongations, circular reasoning and wanderings). Both loops and elongations are parts of a proving process where one still reaches the proof in the end, but not in the shortest or easiest way as one’s ideas would have allowed. These two are *detours* in the proving process. Circular reasoning and wanderings are parts of one’s proving process where one follows an idea that does not end in a proof. These are *drifts* in the proving process. *Dry spells* can be seen as a third group where the proving process pauses, but this still remains to be looked at in our data.

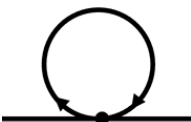

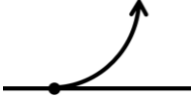

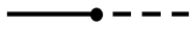
		Content knowledge	
		yes	no
Monitoring	yes	<p><i>loops:</i> side discussion that gets back to the original discussion and proof idea</p> 	<p>drifts</p> <p><i>circular reasoning:</i> using things that are yet to be proven</p>  <p><i>wanderings:</i> abandoning good ideas in favour of proof ideas that do not work out</p> 
	no	<p><i>elongations:</i> using several steps when one step would have been possible.</p>  <p>detours</p>	<p><i>dry spells:</i> parts of a proving process where one does not know how to go on or what to do for a longer period</p>  <p>dry spells</p>

Table 1: Overview of the obstacles and the relation to monitoring and content knowledge

Whether or not one follows detours or drifts in a proving process seems to depend on one's *monitoring activities*. Looking at the monitoring activities described above, one can see that monitoring has many different facets, for example controlling an argument, naming problems in the comprehension of arguments and their connection or uncovering deficiencies in understanding. But our examples show that the mere existence of monitoring activities is not enough to indicate the success of a proving process. There seem to be differences in the way that monitoring activities are expressed. Monitoring formulated in a general way like "Is that possible? Is that right?" like the example of a circular reasoning by Dennis and Julius (Group 2) seem to be about a gut feeling that something is not quite right, without knowing exactly what. This type of monitoring activities we shall call *lower-level monitoring*. It is not specific enough to help pinpoint a concrete mistake and correct it. Dennis' question "Then the question is, whether we can assume that they have the same length" is one example for this. What we call *higher-level monitoring* is a metacognitive activity that is more precise and aims at specific parts of an explanation or argumentation. For example, if one can use a specific theorem the way it is used in the argumentation or if the comprehension of a specific geometric object is correct. Pia and Charlotte (Group 3) showed higher-level monitoring in both cases, in the loop and in the wandering. In one case Pia questioned the use of a specific theorem, the Isosceles Triangle Theorem, at a specific point of their argumentation. In the other case Pia monitored and corrected Charlottes' understanding of the definition of and the connection between squares, rectangles and parallelograms. These interventions were precise and specific and therefore higher-level monitoring.

The level of monitoring does not seem to be the only factor influencing the quality of a proof as one reason for ending up in drifts can be monitoring activities. In cases where the monitoring could not be used profitably, monitoring activities seem to have a negative effect on the proving process. Looking at the two examples of drifts, in one case, Dennis and Julius (Group 2) show lower-level monitoring, the other students, Pia and Charlotte (Group 3) demonstrate higher-level monitoring. Nevertheless, both groups ended up in a drift. *Content knowledge* is the other factor that influences the success of the proving process. The circular reasoning in Dennis and Julius' proof of the first statement happens because they do not know how to prove the equality of the base angle in the triangle (and lack the logical and methodological awareness that you cannot use what you want to prove). Pia and Charlotte end up in a drift after Pia's monitoring activity, because both cannot remember all of the content of the Isosceles Triangle Theorem, which led them to abandoning their proof idea. Looking at detours, monitoring activities can also show a positive effect, for example ending detours by closing loops. Here the monitoring and the content knowledge used afterwards ended the side discussion and brought the students back to their original proof idea, thereby helping them not to end in a drift. Thinking about these examples, content knowledge seems to be what enables one to find a solution to the obstacles that monitoring activities reveal. One can only benefit from monitoring activities if one has the content knowledge necessary to "fix the problems", for example to discern the problematic parts, to recognise what is wrong or missing or to see possible corrections. Without content knowledge one is less likely to find and correct mistakes, misunderstandings or errors through monitoring.

Monitoring is important in mathematics and especially in proving. It allows to control and evaluate one's own performance and understanding during the process. In the proving process three different categories of obstacles can appear: detours, drifts and dry spells. The interviews with the pre-service primary school teachers show that monitoring activities have a relevant influence. But not every monitoring activity has the same influence on proving processes. One can differentiate between monitoring of a lower and a higher level. Whether monitoring activities support or inhibit the success of the proving process is also highly dependent on one's content knowledge.

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