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# Abduction in argumentation: Two representations that reveal its different functions

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*Abduction can function in argumentation and proving processes in different ways. In this paper we focus on the ways in which abductive arguments, taking place in argumentation and proving processes, can be reconstructed in the frame of a Global Argumentation Structure (GAS). We propose two different ways to model abductive arguments within a GAS, with a forward or a backward flow, depending on the function of these arguments in the whole argumentation.*

*Keywords: Abduction, argumentation, proof, Toulmin-model, Global Argumentation Structure.*

## Introduction

Researchers in mathematics education have seen abduction either as process of reasoning backwards (a kind of reversed deduction) (Pedemonte, 2002; Knipping, 2003) or as a process for creating hypotheses (Peirce, 1878; Pease & Aberdein, 2011). The purpose of this paper is to examine the ways in which abductive arguments have been modelled up to now and to propose two different ways of modelling them, depending on their function in the argumentation. We are interested in modelling abductive arguments in the frame of the whole Global Argumentation Structure (Knipping, 2003) of the argumentation or proving process taking place, rather than in isolation from the other arguments taking place (as done for example by Pedemonte, 2002).

## Abduction in argumentation and proof

Abduction is an area of research that has received increasing emphasis in mathematics education, in reference to a number of different processes. Here we discuss abduction in mathematical argumentation. As we consider mathematical proof to be a kind of argumentation, our discussion includes also abduction in mathematical proof.

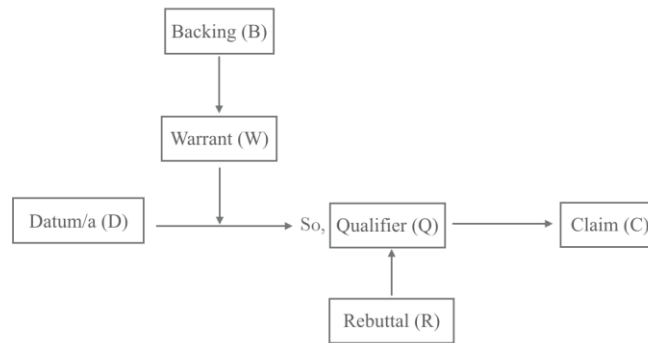
A simple way to distinguish between abduction and deduction is to consider how they connect a case to a result via a rule. An example often cited in the mathematics education literature comes from an article Charles Saunders Peirce published in *Popular Science Monthly* in 1878. At this time he referred to abduction as “hypothesis”, and characterised the two processes as shown in Table 1 below. From this example it is evident that abduction can be thought of as a reversed deduction, reasoning from a Result via a Rule to a Case, rather than from a Case via a Rule to a Result.

Deduction.	Hypothesis. [Abduction.]
<i>Rule.</i> —All the beans from this bag are white. <i>Case.</i> —These beans are from this bag. $\therefore$ <i>Result.</i> —These beans are white.	<i>Rule.</i> —All the beans from this bag are white. <i>Result.</i> —These beans are white. $\therefore$ <i>Case.</i> —These beans are from this bag.

**Table 1: Peirce’s syllogisms for deduction and abduction (1878, p. 472; CP 2.623)**

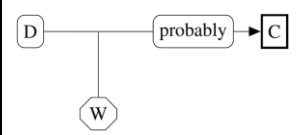
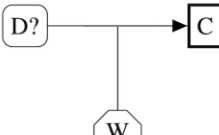
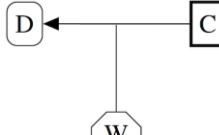
Toulmin (1958) provides a model that can be used to analyse arguments in general, not only deductive arguments. In his model (see Figure 1), an argument includes the following elements:

C (claim or conclusion) the statement of the speaker, D (data) are data justifying C, W (warrant) the inference rule that allows data to be connected to the claim, B (backing) is reason to believe W, Q (qualifier) expresses *how much* to believe C, R (rebuttal) tells you *when* belief in C is *not* supported



**Figure 1: The Toulmin model of an argument (1958)**

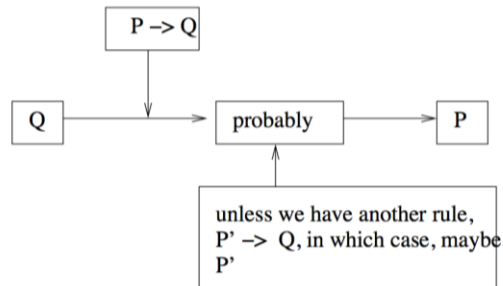
Toulmin considered only two types of reasoning when he created his model, namely deduction and induction. There are however, several cases in the literature of researchers modelling abduction using Toulmin’s model in different ways. In Table 2 we use Peirce’s example and terminology as a common reference in order to describe and depict the different ways in which abduction has been modelled using Toulmin’s terminology.

Peirce	Pease & Aberdein (2011)	Pedemonte (2002)	Knipping (2003)
Rule	W 	W 	W 
Result	D	C	C
Case	C	D?	D
Function of abduction	Generating hypothesis	Reasoning backwards	Reasoning backwards
Structure of abduction (Flow)	Forward	Forward	Backwards

**Table 2: Peirce’s abduction modelled with Toulmin’s terminology**

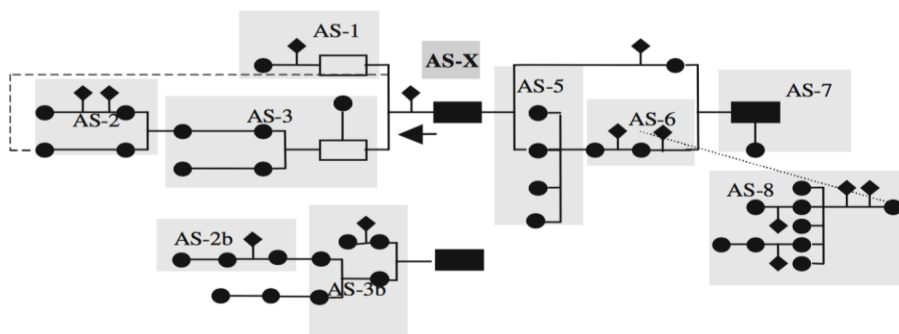
Pease and Aberdein (2011) keep the forward flow (arrows pointing from left to right) of Toulmin’s model but they characterise as a datum the Result that is known and needs no justification, and the claim as the Case that is inferred. They express the uncertainty of the claim by employing a qualifier (the word “probably”) before the claim (see Figure 2). Pedemonte (2002) was the first researcher in mathematics education to use Toulmin’s model to describe abduction. She too models an abductive argument with a forward flow but reverses Pease and Aberdein’s (2011) placement of data and claim. She considers a claim to be the known target of the inference (Result) and the datum to be the unknown fact (Case), which must hold in order for the claim to stand. She denotes the

status of the unknown datum with a question mark next to the letter D representing the datum. The difference between Pease and Aberdein's, and Pedemonte's modelling of abduction may arise because of the different functions they attribute to it. Pease and Aberdein see abduction as a process of generating a hypothesis, whereas Pedemonte sees it as a reversed deductive process, as "reasoning backwards". It is also important to note that these researchers model individual arguments, in isolation from other arguments.



**Figure 2: Structure of an abductive argument, Pease and Aberdein (2011, p.12)**

In contrast Knipping (2003, 2008) describes a way of linking together arguments analysed using the Toulmin model to describe larger structures of argumentation. She reconstructs the whole argumentation of students, building a Global Argumentation Structure (GAS), which reveals a complete overview of the argumentation that took place. For example, Figure 3 shows her analysis of the argumentation in a lesson in which the Pythagorean theorem was proven. Knipping (2003), like Pedemonte, sees the function of abduction as "reasoning backwards". She reconstructs complex deductive argumentation processes, which may contain some abductive arguments. The flow of the overall GAS is a forward one, but the flow of the abductive arguments is a backward one (see Figures 3 and 4). Like Pedemonte (2002) she considers the claim to be the Result of the inference and the datum to be the Case, which must hold in order for the claim to stand. But she marks the abduction differently, by reversing the arrow to indicate the backward flow, and she does not use a question mark next to the letter D to indicate the unknown fact.



**Figure 3: GAS of discussion on the Pythagorean theorem, Knipping (2008, p. 437)**

The arrow in AS-X in Figure 3 marks the abduction. The students know they want to conclude that ABCD is a square, and they use abduction to go from this desired conclusion to the datum ( $BCD=90^\circ$ ) needed. The dotted line marks the shift in focus to finding an argument for the conclusion ( $BCD=90^\circ$ ). Through AS-2 and AS-3 they establish  $BCD=90^\circ$  and combining this

datum with the previous established statement ABCD is a rhombus, they deduce the desired conclusion, ABCD is a square.

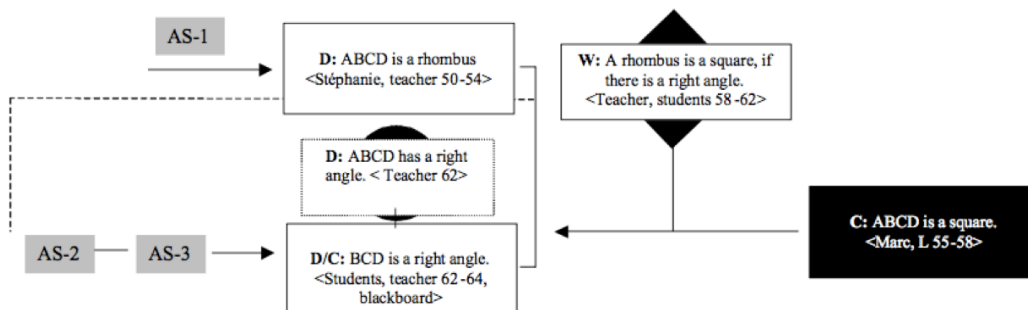


Figure 4: Abductive argument reconstructed with a backward flow, Knipping (2003, p.8)

## Methodology of the data analysis

In the following we compare two Global Argumentation Structures developed following the procedures outlined in Knipping and Reid (2013). They describe a three-stage process: reconstructing the sequencing and meaning of classroom talk; analyzing arguments and argumentation structures; and comparing argumentations to reveal their rationale.

Here we are chiefly interested in comparing two Global Argumentation Structures that emerged in two mathematics classrooms and which have some interesting differences regarding the abductive arguments taking place in them. We will describe each context and the abductions that occurred before comparing the two.

### Ms James' classroom

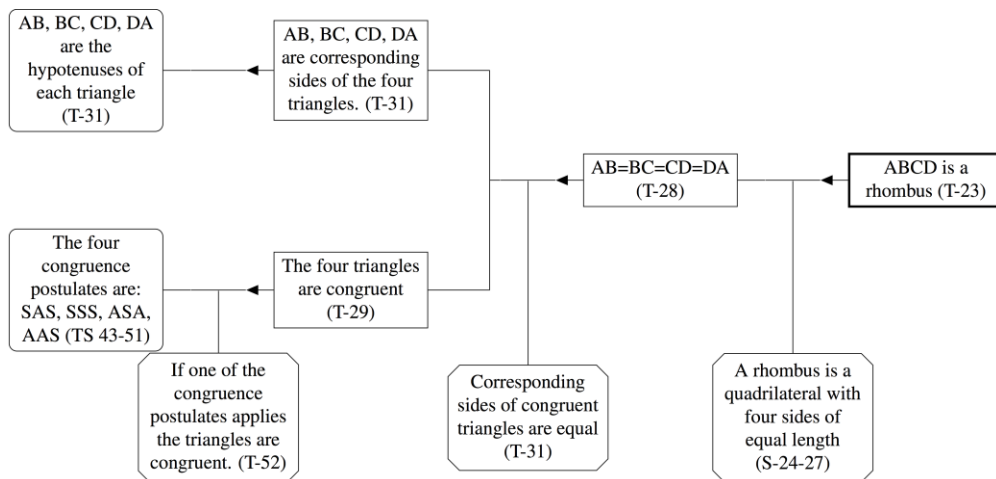
The abduction described here occurred in Ms James' grade 9 (age 14-15 years) classroom in Canada. The class was trying to explain why two diagonals that are perpendicular and bisect each other define a rhombus. The students had discovered and verified this property while working in a Dynamic Geometry Environment (DGE).

Figure 5 shows the reconstructed argumentation stream that occurred after the class had identified the given information in the situation and had recorded it in the diagram Ms James was drawing on the board. She then asked the class what information was needed to show that a shape is a rhombus, given the definition. This stream is interesting because the statements that were made first chronologically, appear last, on the right hand side, and vice versa. This is similar to the abductive stream described by Knipping (2003) in that the argument goes backwards from the intended conclusions to the data needed to deduce that conclusion. It differs, however, as it involves several abductive steps connected in a stream.

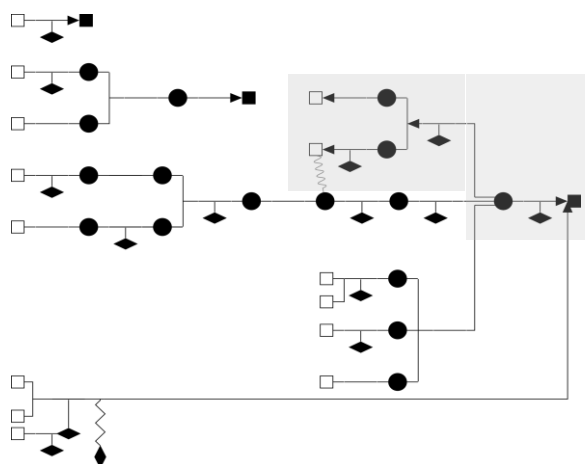
There are linguistic markers of what is going on in the transcript. For example, Ms James says:

Ms James: If I can prove that that is the same length as that, is the same length as that, is the same length as that [draws in the segments AB, BC, CD, and DA]. If I can prove that, I'm done. Rhombus. [Statement T-28 in Figure 5]

The phrase “If I can” marks this as an abduction. In a classroom proving process, where the main flow of the argument is deductive, it makes sense to diagram such abductions in this way, as if they were deductions, but marking them with arrows showing the flow of the argument in the opposite direction. This is shown in the GAS in Figure 6.



**Figure 5: The abductive argumentation stream from Ms James’ class (the parenthetical comments after each statement indicate speaker and transcript line)**

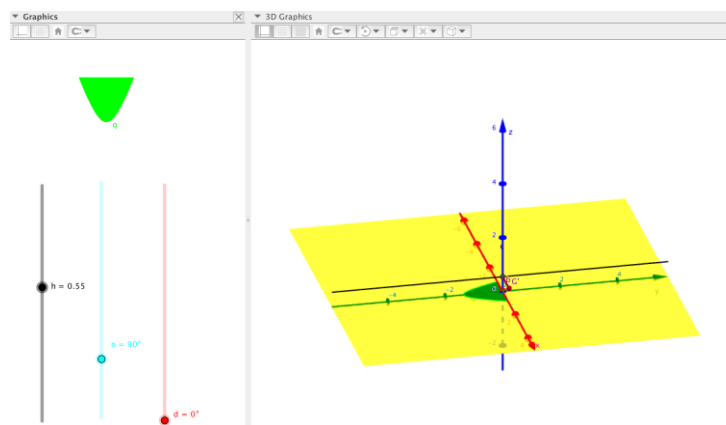


**Figure 6: The GAS from Ms James’ class. The shaded area is the abductive stream shown in Figure 5**

## Axel and Dave

In the case presented here two 10<sup>th</sup> grade students (Axel and Dave) in a German classroom worked together on a geometry task designed in a DGE. All the students in the classroom worked on the geometrical tasks in pairs and at the end all the tasks were discussed in a whole-class discussion with the focus being the argumentation the students provided to justify their answers. In the task the DGE window was divided into two sub-windows (see Figure 7). On the right sub-window (3D Graphics) there was a 3D coordinate system in which a solid had been constructed and hidden, and the plane  $xOy$ . On the left sub-window there were three sliders ( $h$  for height,  $n$  for tilt and  $d$  for spin) that the students could manipulate in order to move the invisible solid, and a two-dimensional

depiction of the intersection of the solid with the  $xOy$  plane. The main question set to the students was: “What solid do you think this could be, judging from its cross-sections?”. The students were provided with a worksheet, which included an Exploration Matrix with specific  $(h, n, d)$  slider positions they could use if they wanted. They were asked to justify their answers in writing. The students were not told how to justify their conclusions, only that they should attempt to explain the validity of their conclusions and to be as complete (leaving no doubts) as possible. There was no requirement that they follow a deductive structure, or that they produce a formal proof.



**Figure 7: Task in DGE – Position  $(h=0,55, n=90^\circ, d=0^\circ)$  of hidden solid**

The GAS presented here (Figure 8) represents Axel and Dave’s argumentation that took place in their discussion during their pair work alone, before the classroom discussion. Axel and Dave used the positions in the Exploration Matrix for their explorations. They started by generating four hypotheses and during their exploration they refuted step-by-step three out of the four hypotheses. The remaining hypothesis ( $H_{16}$ ) became then a claim ( $Cl_{23-25}$ ), which they would later provide an argument for, in order to verify it. After the verification the epistemic value (Duval, 2007) of the statement “The solid is a cone” changes and from a claim it becomes a conclusion ( $C_{118.2-120}$ ). This GAS has two phases (Argumentation Streams). In the first phase (AS-1) belongs the process of generating hypotheses, during which the students suggest possible solutions to the problem, basing their decisions on the data they gather. This is then followed by the second phase (AS-2), during which more data are provided to support the claim ( $Cl_{23-25}$ ). With the support of these new data, the claim is then treated by the students as a conclusion ( $C_{118.2-120}$ ).

The abduction here (AS-1) is not represented as reasoning backwards, rather a process of suggesting multiple possible solutions to a problem all of which, at least initially, seem plausible. Here the chronological flow of the statements made corresponds to their logical function in the argumentation. For example, the conclusion  $C_{118.2-120}$  (which is usually the last inference drawn in an argumentation) comes last both chronologically (in the argumentation) as well as structurally (in the GAS). In such cases, in which speculation is required and the students engage in hypothetical reasoning, it makes sense to diagram abduction with a forward flow.

In the case of Axel and Dave, unlike in that of Ms James’ classroom, there are no warrants in the GAS. This should not be interpreted as that the students had no warrants in mind when creating hypotheses or drawing conclusions. The absence of warrants is merely the result of our

methodology according to which, what is depicted in the reconstructed argumentation is only what the students state explicitly, and not what they may have been thinking but have not said.

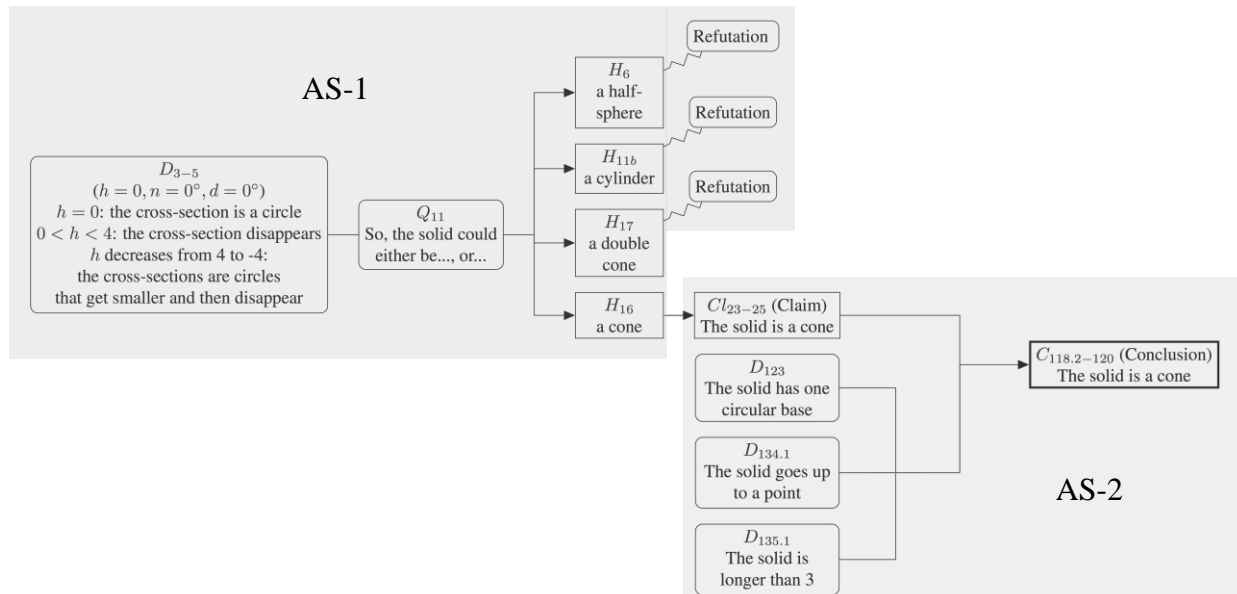


Figure 8: The GAS in Axel and Dave's discussion

## Conclusions

Abduction has two different functions, *explaining* and *exploring*, and depending on the function that it has in argumentation, the reconstructed Global Argumentation Structure is also different.

In Ms James' classroom abduction was used to reason backwards from a desired conclusion to the data needed to deduce it. The students know that the shape is a rhombus and they are seeking data that can connect that conclusion to what they know about the diagonals. They are using abduction in order to *explain* the known conclusion. Most of the arguments in the GAS are deductive, and it is important to note that the final 'official' proof omits the abductive steps. Having abducted the necessary data, they then establish it deductively from other data they already know, and then use it to deduce their desired conclusion. In the case of Axel and Dave, abduction is used to generate hypotheses. They are *exploring* to discover something new. Their hypotheses are then tested in the second phase of their argumentation, through comparison with data generated in the DGE.

The differences in the GASs in these two situations reveal an important difference in the way abduction functions in them. This shows how such an analysis can reveal not only the presence of an abduction but also the role it plays in the overall argumentation. The ways in which the abductions in Ms James' classroom and Axel and Dave's pair work were represented, as flowing backwards or forwards, reflect the functions of the abductions. When abductions are used to explain, the flow is backwards to the facts that explain the conclusion. When abductions are used to explore, the flow is forwards, from data that has been collected to possible results, and includes qualifiers to mark the epistemic value of the inference. The role of qualifiers is vital in order to keep a forward flow for the abduction in an overall abductive GAS.



These differences in function, in turn, depend on the nature of the mathematical tasks. In the case of Axel and Dave the geometrical object the students are asked to work with is unknown and they are not told that they need to prove anything. The focus of their argumentation is the generation and justification of hypotheses, rather than proof production. It is not even possible for them to deductively prove a hypothesis is correct; they can only disprove false hypotheses. In Ms James' case the task was explicitly to find a proof of a given statement. The status of the statement as a conclusion is already established, by the students empirical explorations earlier and by the teacher's authority. In such a proof production task it is clear that the overall structure must be deductive, and abductive argument is incidental, not essential to the argumentation. The final proof could have been produced without using abduction.

We have shown in this paper how Pease and Aberdein's (2011) way of modelling abduction (forward flow of the abductive argument) can be embedded into GASs reconstructed using Knipping's (2003, 2008) methodology, when the function of the abduction is exploring. We are not claiming that the two ways that abduction has been modelled in the GASs we described are all the possibilities. Research in other contexts will no doubt reveal other ways in which abduction is embedded in argumentation structures. However, our describing and distinguishing these two ways, along with the associated functions of the abductions in the argumentation, is a contribution to research in this area.

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