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# Factors influencing teachers' decision making on reasoning-and-proving in Hong Kong

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*A project is ongoing to develop an intervention that promotes the teaching and learning of reasoning-and-proving in teacher education in Hong Kong. Since the area of reasoning-and-proving in Hong Kong has not been widely studied, this paper reports the first research cycle of the project: to investigate the potential factors that limited reasoning-and-proving activities in Hong Kong classrooms. In this cycle, seven secondary mathematics teachers were interviewed. The participating teachers' perceptions of reasoning-and-proving, students and learning cultures in Hong Kong were examined, and the results indicate factors that may contribute to the marginal situation of reasoning-and-proving in Hong Kong school mathematics.*

*Keywords: decision making, Hong Kong, reasoning-and-proving, teachers' perceptions*

## Introduction

Proof can verify the truth of a mathematical statement and may also promote students' sense making and conceptual understanding in mathematics by providing epistemological justifications (Harel, 2018; Knuth, 2002). Consequently, many mathematics educators suggest that teaching and learning proof and proof-related reasoning activities should spread through school mathematics at different levels and in different content areas. Noting that teachers are the key decision makers of what the students will experience in the classrooms, many studies have been conducted to examine teachers' knowledge and perceptions of proofs (e.g. Knuth, 2002). Yet, reasoning-and-proving (RP, see Stylianides, 2008) tends to have a marginal place in school mathematics. Whilst Western mathematics educators attribute this situation to teachers' counterproductive mathematical knowledge and beliefs about RP, and established mathematical cultures of the schools (e.g. Stylianides, Stylianides, & Shilling-Traina, 2013), studies on factors that influence teachers' decisions around RP activities are scarce in East-Asian countries (e.g. Hong Kong, Singapore, South Korea). The author has begun a project aimed at developing an intervention that promotes teaching and learning RP in teacher education in Hong Kong. Since the area of RP in Hong Kong has not been widely studied, the first research cycle focused on the context, Hong Kong. This paper intends to report the findings of this cycle. This cycle aimed at investigating the factors that limited the teachers' decisions on teaching and learning RP in the situation of Hong Kong classrooms.

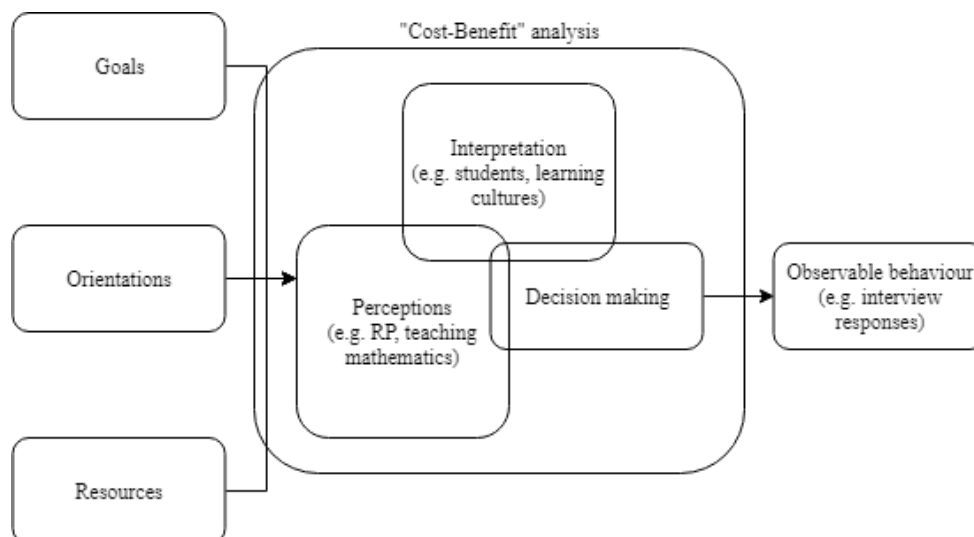
## The context: Hong Kong

According to the official documents of the Education Bureau, the mathematics education curriculum in Hong Kong aims to develop students' "(a) ability to think critically and creatively, to conceptualise, inquire and reason mathematically, [...]; (b) ability to communicate with others and express their views clearly and logically in mathematical language; [...]" (CDC & HKEAA, 2015, p. 2). Despite the lack of an explicit statement, readers may find some implicit indications of teaching RP among these curricular aims.

Like other East-Asian countries, teachers in Hong Kong are often the primary decision makers of what students would experience in mathematics class through lesson preparation and in-the-moment teaching approaches. Teachers' decisions can be influenced by their perceptions and interpretation of the situation, for example, mathematics and RP, students, and the learning culture (Stylianides et al., 2013). Whilst learning culture in Hong Kong is said to be examination-oriented, which may have influenced decisions on school curriculum and assessments (Brown, Kennedy, Fok, Chan, & Yu, 2009; Choi, Lam, & Wong, 2012), studies have not been widely conducted in Hong Kong to investigate how a teacher's lesson preparation and in-the-moment teaching approaches are influenced by this culture. This study aims at examining the factors limiting or promoting teachers' decisions around teaching and learning RP, with respect to their perceptions of teaching RP, students and the learning culture. As a research cycle of a design-research project, the findings in this study will inform the development of an intervention.

### Theoretical backgrounds

To investigate teachers' decision making around teaching, researchers often examined the relationship between teacher's beliefs, knowledge and practice. Conner (2017) examined the differences in argumentation between two U.S. classes, and suggested these differences might be attributed to the teachers' intentions regarding the components of rationality. Kempen and Biehler (2015) investigated 12 pre-service teachers' perceptions of the generic proof, and reported that participants' preference for formal proof over generic proof in a task might be attributed to the situations that they adopted the construction of a formal proof as a norm for solving the task during their time in school, or they (mis)interpreted that construction of a formal proof was asked in the task. These results indicated that teachers' belief constructs could be complex and influenced teachers' teaching practices. However, the process of decision making and transition from teachers' belief constructs to practices were not thoroughly discussed.



**Figure 1: Model of teacher's decision adopted from Blömeke et al (2012) and Schoenfeld (2011)**

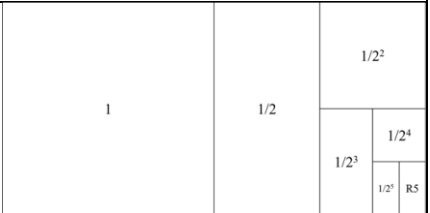
Blömeke, Gustafsson, and Shavelson (2012) and Schoenfeld (2011) attempted to model teachers' decision making. In the model of Blömeke et al. (2012), when teachers' perceptions are affected by their belief and knowledge constructs, their interpretation of the context may mediate between their

perceptions and decision making (and practices). Schoenfeld (2011) further elaborated the process of transition and decision making using the conceptualisation of goals, orientations and resources; he defines resources (with a focus on knowledge) as “the set of intellectual, material, and contextual [information] available to the teacher” (p. 10) and orientations as “an inclusive term encompassing a group of related terms such as *dispositions, beliefs, values, tastes, and preferences*” (p. 29, italics added). I adopted the models of Blömeke et al. (2012) and Schoenfeld (2011): if teaching RP is a routine practice of a teacher, the teacher will continue this routine practice; otherwise, if teaching RP is an unfamiliar practice of the teacher, the teacher will perform a *cost–benefit* analysis on different options using the variables (perceptions and interpretation), and the decision will be based upon the outcomes of the *calculation of expected values* on the options. Figure 1 depicts a model of a teacher’s decision adopted from Blömeke et al. (2012) and Schoenfeld (2011).

## Methods

Seven secondary mathematics teachers participated in this study. These teachers were selected because: (a) they obtained related undergraduate degrees and gained their teacher qualification through postgraduate teacher training programmes; (b) they have been serving local schools differed on locations, school cultures and students’ academic achievements: in Hong Kong, secondary public/government-aided schools and students are divided into three school bands regarding their academic achievement. One participant was serving high-achieving schools, two in mid-achieving schools, two in low-achieving schools, and two were serving private schools that do not belong to any school band; (c) they had worked at schools for at least three years; and (d) they had served at least two schools. Their diverse backgrounds resulted in responses that covered a range of concerns of teachers when considering teaching and learning RP in the situation of Hong Kong classrooms.

The teachers participated in task-based interviews. The tasks and questionnaire items were adopted from the studies of Almeida (2000), Keceli-Bozdog, Ugurel and Bukova-Guzel (2014), Kotelawala (2007), and Yoo (2008) to capture teacher’s knowledge, attitudes and beliefs about RP and their instructional practices. Table 1 lists some sample items. Each interview lasted about an hour, and was conducted and audio-recorded in Cantonese, the teachers’ mother tongue to enable them to express their opinions easily and effectively. The teachers were asked to complete the questionnaire and tasks using think-aloud strategy and then respond to follow-up queries.

Origins of the items	Sample items
Almeida, 2000	<p>Decide which ‘proofs’ you find (a) convincing, (b) not convincing, or (c) incorrect.</p> <p>Proposition: <math>(1 + 1/2 + 1/2^2 + 1/2^3 + \dots + 1/2^n) \rightarrow 2</math> as <math>n \rightarrow \infty</math>.</p> <p>Assumptions/assertions that may be used in the proofs: A1. If <math> a  &lt; 1</math>, then <math>an \rightarrow 0</math> as <math>n \rightarrow \infty</math>.</p> <p>‘Proof’ 1 Let <math>S_n = 1 + 1/2 + 1/2^2 + 1/2^3 + \dots + 1/2^n</math>. <math>S_5</math> can be represented in the figure as a portion of 2 squares. <math>S_5</math> is 2 less a remainder <math>R_5</math> as shown: Clearly, the figure shows that <math>R_5</math> becomes smaller as <math>n</math> increases. That is, <math>R_n \rightarrow 0</math> as <math>n \rightarrow \infty</math>. It now follows that <math>S_n \rightarrow 2</math> as <math>n \rightarrow \infty</math>. This proves the proposition.</p> 

**Table 1: Sample items of the tasks**

	<p>'Proof' 2  Let <math>S = 1 + 1/2 + 1/2^2 + 1/2^3 + \dots + 1/2^n + \dots</math> (1)  Multiplying both sides of the equation above by <math>1/2</math> gives  <math>S/2 = 1/2 + 1/2^2 + 1/2^3 + \dots + 1/2^n + \dots</math> (2)  Subtracting equation (2) from (1) gives: <math>S/2 = 1 \rightarrow S = 2</math>.  This proves the proposition.</p>					
Keceli-Bozdag et al., 2014	I think that theorems and proofs are the foundations of mathematics.					
	Strongly agree	Agree	Neutral	Disagree	Strongly disagree	
Kotelawala, 2007	Please divide 100 units of time among the following activities, consider a typical junior secondary mathematics class (12–15 years old) that you teach.					
	Items				Units of time	
	1. Receiving instructions from teacher on how to solve a problem					
	2. Practising calculations for recall, accuracy, and/or speed					
	3. Student solving problems using steps from the day's lesson or the weeks lessons					
	4. Students formulating conjectures					
	5. Student explaining to group why their answers or steps make sense					
	6. Students working on their own methods for solving unfamiliar problems					
Yoo, 2008	Ideally in a proof-based mathematics lesson, students should (a) make conjectures, investigate them, and construct their own proofs, or (b) practise proving statements or conjectures that they know have been proven to be true before by an expert.					
	(a)	towards (a)	both	towards (b)	(b)	neither
	1	2	3	4	5	6

**Table 1(con't): Sample items of the tasks**

The audio-recorded data were transcribed into written Chinese. The transcripts were then analysed using the model of the teacher's decision (Figure 1) regarding two themes: (a) teachers' perceptions of RP and teaching and (b) their interpretation of the contexts. Based on the teachers' responses, their interpretations of the contexts were further divided into (i) interpretation of students as learners and (ii) interpretation of learning cultures. To report the findings, selected excerpts were further translated into English using word-to-word strategy (that is, English grammatical rules are not strictly followed) assisted with appropriate interpretation.

## Results

### Perceptions of RP and teaching

All participants related proofs to logical reasoning and its promotion. For example, when asked which items of learning activities are related to proof, Participant 2 stated, "if the activities enable to develop [students'] logical reasoning, I think the options are [formulating conjecture], [explaining why answers or steps make sense] and [working on one's own methods for solving unfamiliar problem]." He also added that making a conjecture may be a precursor of a proof.

Participant 2: Like Pythagoras' theorem, he (the prover) makes a conjecture. If he did not have a conjecture, he would not have tried to prove [it]. Therefore, [we] must first have conjectures.

Participant 4 perceived that, through teaching and learning proofs, students may be provided with epistemological justification of a mathematical statement for promoting conceptual understanding in

mathematical facts and ideas. When discussing ‘proofs’ of “square root of 6 is an irrational number”, Participant 4 expressed his intention and motivation of teaching the proof: to foster students’ conceptual understanding of irrationality of a number:

Participant 4: ... because learning a proof like this means to understand what “irrational” [an irrational number] is... because something is true... like this then [we] can dig out the meaning of “irrational”. Therefore, I will teach [the proof].

The realisation of educational values of proofs may *initiate* teachers’ motivation, like Participant 4, of teaching RP: the teachers initiated the process of *benefit–cost* analysis on teaching RP, but their interpretation of students and learning cultures may mediate between their perceptions, and decisions.

### **Interpretation of students as learners**

Most participants perceived that their students were not capable of nor interested in learning any proof of a mathematical proposition in general.

Participant 1: ... because [in] my school, not many people “accept” (prefer) proofs. They often [prefer], for example Pythagoras’ theorem... it is enough for them to use [the theorem]. Yes, they barely care or think about how this formula was derived. Since they do not want to know, I am not urged to explain. Also, their capability is weak; it will be a trouble if I finish explaining [it] but they still do not understand. Therefore, proofs are relatively limited.

Participant 7: ... Let me say it this way, in my views, [the] ideal [RP lessons] should train (develop) their (students’) that (RP-related) reasoning. Training their logical [reasoning] But in actual lessons, this [training], however, was not emphasised... I think students’ attitudes [towards learning mathematics] are important. In fact, many students are not... enthusiastic [over learning mathematics]. So, I think [if I] keep on asking them (students) to make a conjecture, in fact they are not really making any conjecture. Yes... but if ideally they really can... that is, [if] I ask them to do... use [their] brains to think (make a conjecture)..., then they think. Then, I think [this] is fine. But, in real life, then... in fact, perhaps, not the whole class [would] do (make a conjecture). [I] sometimes, perhaps, find some smarties, and then ask them to do this conjecturing. Therefore, the proportion [of making conjectures] would be far less... in the real life (my actual teaching practices).

To facilitate some RP activities in classroom, the teachers tended to select proofs that they found easy to understand, but there were no systematic selection criteria; instead, the teachers selected a proof by their feelings of its length and complexity.

Participant 5: [I prefer proofs that are] easy to “see”. [The students] all can “see”; that is, easy to explain and understand.

Participant 1: [I choose to teach proofs which are] simple and easy to follow. [Interviewer: What do you mean by simple and easy to follow?] This depends on [my] feelings... This is sometimes my feelings... If, if I am a teacher and have a judgement, if I see this [proof] and I think this can be understood by the majority, from my gut feeling...

Of course, I have to know the characters of my class, then I will evaluate if they can understand what I say... evaluate if they will “accept” (be convinced by and/or understand) this proof. If, after I complete the evaluation, I think [it is] worth saying (teaching) [it], then I will try to say (teach) [it]. If, after evaluation, I think [it is] not worth saying (teaching) [it], then I won't.

Whilst teachers' realisation of the educational values of teaching RP (perceptions) may promote their use of RP when teaching (decisions), the participants demonstrated how teachers' counterproductive interpretation of students' learning capabilities and attitudes may mediate between their perceptions and practices of teaching RP. Participant 1 demonstrated that his counterproductive interpretation interacted with his perception of what students needed to learn/master and teacher-centred disposition. The choice of a “simple and easy” proof (decision) may refer to his interpretation that (a) the length and complexity of the selected proof were aligned with students' capabilities, and/or (b) the main purpose of the selected proof was to convince students of the truth of a proposition with less effort. Participant 7 demonstrated that he developed a positive perception of teaching RP and an ideal practice, but his counterproductive interpretation contributed to investing less in involving students in RP activities (e.g. making a conjecture). As a result, teaching RP was limited in their classes.

### **Interpretation of learning cultures**

Apart from their interpretation of students, teachers' interpretation of the learning cultures where they are situated may also mediate between their perceptions of RP and teaching, and decision making. Like other East-Asian countries, examination-oriented culture is well established in Hong Kong (Brown et al., 2009; Choi et al., 2012). The participating teachers' responses revealed that this examination-oriented culture greatly influenced their decisions on teaching RP and mathematics.

Participant 4: [My lessons tend to be] exam-oriented! Even if I do not want to be like this, students' expectation and parents' expectation are like this... because [we] have examination; [so, we] have drilling exercises... The tasks in examination are not proof-related, so [we] cannot spend much time [on proofs]. If [students] can understand, then understand; if not, then keep on drilling, or solve problems...

Participant 4's responses suggested a connection between his interpretation of students' expectation (attitudes) and that of the examination-oriented learning culture. Participant 7 added that the examination-oriented culture has also contributed to the discrepancy between his ideal RP lesson (perceptions of RP and teaching) and his actual implementation.

Participant 7: ... But [the actual lessons focused] more on whether they could “do the maths” (solve the problem tasks), which is more about examination. Of course, I am not completely saying [that my lessons are] examination-oriented; my lessons are really not so examination-oriented. However, [my lessons] in fact tend more to be the side of calculation. Yes, therefore, when having the term “ideally” here, my... my [ideal] practice will be a bit different [from the actual practice].

The participating teachers perceived that preparing students for public examinations is a main role of being a teacher in Hong Kong (a goal); they also held an interpretation that the public examination in

Hong Kong was calculation-oriented. Any decision that may strengthen students' accurate calculation and problem-solving skills was prioritised over the teachers' ideal practices if the decision and the ideal practices were different. For example, Participant 4 perceived that the examination-oriented culture was deeply rooted in his teaching environments, and, as a result, students expected mathematics class to be oriented towards examination preparation. He prioritised the goal of preparing his students for examination highly over the goal of fostering students' conceptual understanding of mathematics. Since teaching RP was not his routine practice, according to Schoenfeld's (2011) model, a "cost-benefit" calculation was performed by Participant 4. His cost of practising RP in his mathematics class outweighed the benefits that he perceived when teaching RP. Therefore, he found it difficult to practise any RP activity in his mathematics class. Therefore, they invested more in involving students in calculation and problem-solving activities, rather than RP.

## **Discussion**

This study examined the three *interrelated* factors that may influence Hong Kong secondary mathematics teachers' decisions around RP: teacher's perceptions of teaching RP, interpretation of their students as learners, and interpretation of the learning cultures. Whilst the teachers' productive perceptions of teaching RP, for example, recognising that teaching proofs may foster students' conceptual understanding of mathematical ideas and logical reasoning, may promote and develop their ideal practices, their counterproductive interpretation of students' learning needs: in particular, they did not think proofs are needed or suitable for all students (cf. Frasier & Panasuk, 2013; Varghese, 2009), and the learning cultures (examination-oriented culture) may interact with each other and mediate between their perceptions (of students' learning needs) and actual practices.

The teachers perceived examination-oriented culture to be a main factor that limited their decisions on teaching RP; this culture influenced the teachers' interpretation of students' attitudes and their learning needs. Examination-oriented culture is deeply embedded in Hong Kong school mathematics and teachers' interpretation, and the public examination in Hong Kong is interpreted as calculation-oriented. As a result, the teachers deviated from researchers' expectation of making RP spread through school mathematics, and prioritised calculation-oriented teaching materials over RP. If teachers avoid RP in their teaching because they perceive that proofs are less important and/or not suitable for all students in understanding mathematics and preparing for examination, professional development should address teachers' interpretation of students' learning needs and the social contexts (school and learning cultures) and be focused on fostering teachers' reflection on their teaching philosophy and broadening their horizons for learning other trends and visions. Educational reforms including frameworks of public examination and teacher education may also be considered when RP activities should be central of mathematics education in Hong Kong, and worldwide.

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