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# Questions and answers ... but no reasoning!

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*Statistical reasoning has often been used synonymously with statistical thinking. In this paper, we focus on the reasoning part and we analyse mathematic lessons about statistics in a primary school class, using a construct from the Anthropological Theory of Didactics approach, called Study and Research Paths. By comparing the a priori tree-diagram of the course with the a posteriori tree-diagram of the observed teaching, it becomes clear that the teacher's questions never make the students engage in statistical reasoning and the students' questions are more concerned with practical and organizational issues than with obtaining a greater understanding of statistical reasoning.*

*Keywords: Mathematical reasoning, statistical reasoning, study and research path.*

## Introduction

Teaching reasoning and justification requires more than just asking students to explain their answers or to pose open problems (Ball & Bass, 2003). Teachers' and students' questions and answers are crucial. This paper contributes further insight into teaching reasoning in elementary mathematics classrooms by analysing a teacher and students' questions and answers during a statistics course. We aim to contribute to a more systematic methodology to study what actually takes place in the classroom and to determine if it is possible to do this by looking at the questions and answers of students and teachers during their lessons.

To analyse the students' reasoning processes in the classroom, we will use a tool from the Anthropological Theory of Didactics (ATD), namely, Study and Research Paths (SRP). SRP provides to model mathematical knowledge from a didactical perspective (Chevallard, 2006). The analysis needs detailed information about context, contents, order of questions and answers. In the last decade several studies has focused on the potentials of SRP; Winsløw, Matheron, and Mercier (2013) examine how SRP and a new diagrammatic representations can be used to analyse didactic processes; Barquero, Bosch, and Romo (2015) illustrate how SRP can be used in professional programs for teachers; and (Jessen, 2017) studies how SRP can support the development of knowledge in bidisciplinary settings.

We investigate whether the teacher and the students ask questions and answer questions in a way that allows the students to engage in mathematical reasoning related to statistics; we also reflect on the potentials and limitations we found when using SRP as an analytical tool. More precisely, we ask the following research question: What is the content of the students' and teachers' questions and answers in the "Youngsters and ICTs" intervention, and does it support the students' opportunities to produce statistical reasoning?

## **Theoretical considerations: mathematical and statistical reasoning**

Researchers and educators around the world have advocated that a primary goal of mathematics education for all grades should be the development of mathematical reasoning (Ball & Bass, 2003). Nevertheless, there is no consensus about the definition of mathematical reasoning in the research literature (Jeannotte & Kieran, 2017; Mariotti, Durand-Guerrier, & Stylianides, 2018). Reasoning in statistics can be seen as a particular form of reasoning in mathematics. del Mas (2004) argues that mathematical and statistical reasoning should place similar demands on a student and should display similar characteristics when the students are asked to reason with highly abstract concepts. At first, mathematical and statistical reasoning appear to be similar, but the nature of the tasks in statistics and mathematics are somewhat different. In mathematical reasoning, context may not play a large role, but in the practice of statistics, the inquiry will always be dependent on data and typically grounded within a context (del Mas, 2004). Statistical reasoning is often used to define the same capabilities as statistical thinking, but Ben-Zvi and Garfield (2004) try to separate the two concepts. They define statistical reasoning as “understanding and being able to explain statistical processes and being able to fully interpret statistical results” (p. 7). Statistical thinking, on the other hand, involves an understanding of why and how statistical investigations are conducted, and also when to use appropriate methods of data and analysis. Both statistical thinking and reasoning can be involved when working on the same task, so the two types of activities cannot necessarily be separated. del Mas (2004), however, writes that it is possible to distinguish them through the nature of the task: “For example, a person who knows when and how to apply statistical knowledge and procedures demonstrates statistical thinking. By contrast, a person who can explain, why a conclusion is justified demonstrates statistical reasoning.” (p. 85). Brousseau and Gibel (2005) argue that the teaching of reasoning used to be conceived as a presentation of model proofs, which then had to be faithfully reproduced by the students. Teachers today see reasoning as an activity, which cannot be learned as a simple recitation of a memorized proof; instead, it is necessary to confront students with problems, where they naturally engage in reasoning. If students are presented with model-proofs today, they are meant to serve as a model of others’ reasoning, which students then can use to produce their own original or creative forms of reasoning. However, as Brousseau and Gibel (2005, p. 14) noted, “There is always the risk of reducing problem solving to an application of recipes and algorithms, which eliminates the possibility of actual reasoning”. When it comes to statistical reasoning, most teachers tend to teach concepts and procedures and hope that reasoning will develop as a result (Cobb & McClain, 2004). Cobb and McClain (2004) argue that, in statistical reasoning, students must reason about data rather than attempt to recall procedures for manipulating numerical values.

## **Study and Research Paths**

Study and Research Paths (SRP) is a recent construct in the Anthropological Theory of Didactics (ATD) (Chevallard, 2006). Within ATD SRP were introduced as a design tool for teaching within the paradigm of “Questioning the world” (Chevallard, 2006). The aim is to focus on important and meaningful “big” questions and not just “visit monuments”, meaning a set of rules prescribing, what is to be studied with no place to raise “What for?” or “So What?” questions (Chevallard, 2006).

The fundamental dialectics between questions and answers are at the root of the idea of SRP (Winsløw, 2011). A group or an individual develops knowledge as a result of working with an overall question, Q. Students identify the “official” knowledge that can help them answer Q; the students use this “studying” to justify their answers to Q by engaging in reasoning, which is the “research” about Q. Elaborating on Q generates the “path” (Winsløw et al., 2013). SRP is normally used to design lesson plans but Winsløw et al. (2013) introduce SRP as a modelling tool to analyse didactic processes. Jessen (2014) argues that a tree-diagram of the SRP is a strong tool for analysing didactical processes. We will use a tree-diagram (an example can be seen in figure 1) to make a SRP analysis in an elementary mathematics classroom. Conducting a SRP a priori analysis entails exploring what questions and answers could occur from one particular overall question (the generating question, called Q0). The tree-diagram refers to the possible path the students could follow after generating Q0. The Q0 must be so strong that students can derive new questions, Qi, from it. The answers to the derived questions add up to an answer to the original question, Q0. The Q0 must be of real interest to the students. This continues with more questions and more answers and could leads to a tree-diagram of pairs of questions and answers (Jessen, 2014). The questions in black are ask by the teachers, and the questions in white are made by the students. The grey-coloured questions are those that are created in collaboration between the teacher and the students. The numbers next to the questions and answers indicate the order of the questions. In the SRP-process the media-milieus dialectics must be taken seriously: the information the teacher brings into the class; the answers available through different books, articles, videos or online resources; and the classroom milieu where the teacher and students manage to establish meaningful actions.

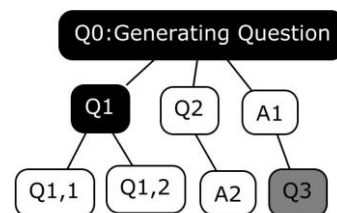


Figure 1: An example of a SRP tree diagram

### “Youngsters and ICTs”

“Youngsters and ITCs” is a 15-lesson statistics course, which is taught over 3 weeks and intended for grade 6 students; it was designed by C. K. Skott and the second author of this paper, (Skott & Østergaard, 2016)<sup>1</sup>. Before teaching the course, the teacher participated in a professional development workshop (6 lessons) that focused on statistical reasoning and digital technologies. The purpose of the course is to improve students’ reasoning in statistics. The course frames and proposes ways that teachers can engage students in statistical investigations; formulate statistical problems; generate, analyse, and reason about data; interpret results; and disseminate them both inside and outside of a school context. The emphasis of the course is to create new habits of classrooms interactions, in which the students raise questions and explore the context with their teacher, who challenges them to come up with new questions and reflect further on possible answers. This approach breaks away from teaching a succession of more or less independent “chapters” where only “small” questions are raised.

<sup>1</sup> The lesson plan (in Danish) is available at:

<http://auuc.demonstrationsskoler.dk/materialer/innovation/forloeb3/faser/hvordan-bruges-statistik-i-hverdagen>.

## **Methodological approach**

We employ a micro ethnographic design; this kind of approach is well-suited for describing, analysing, and interpreting a specific aspect of a group's shared behaviour (Garcez, 1997). We use a case study research design to obtain a thick description of the observed teaching and to understand, how all the questions and answers in the classroom operate together in this context, which is part of a complex system (Stake, 1995).

Within a two-year period, we observed a teacher in 31 classroom lessons; 16 observations from the course and 15 from before or after the course. All the observed lessons were video-recorded. Four audio-recorded semi-structured interviews with the teacher were conducted. The choice of semi-structured interviews was chosen to get a deep understanding of the background of the classroom context. All the interviews were transcribed, and 15 lessons selected from the observations were transcribed.

### **"Children and young people spend too much time on media ... or?"**

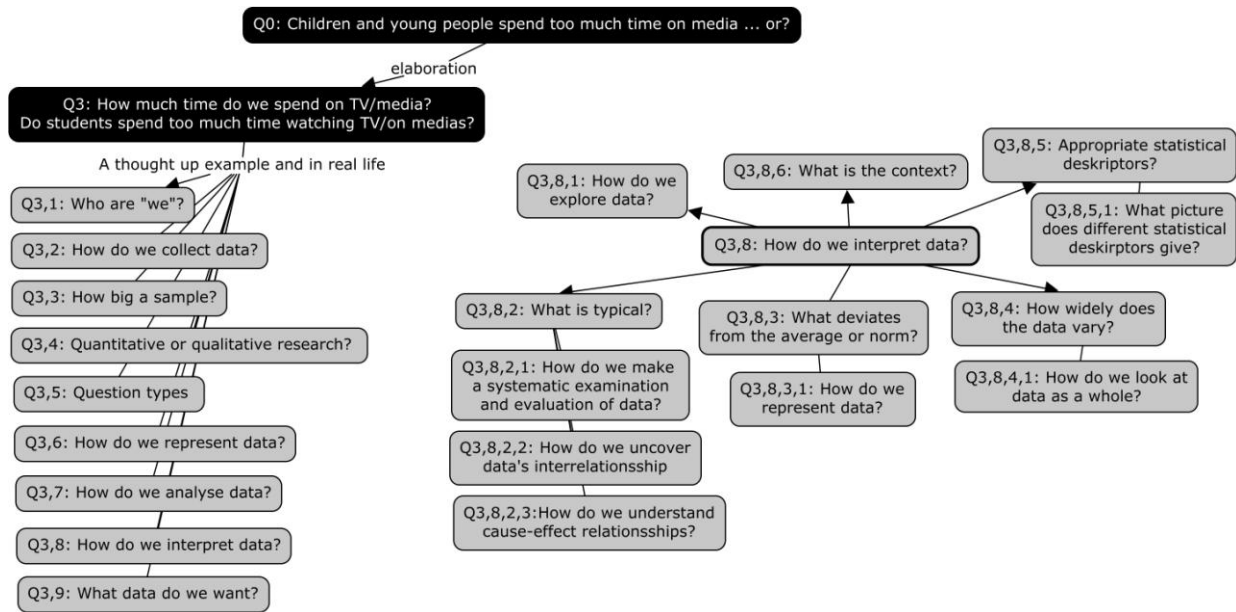
"Youngsters and ITCs" was not originally designed as an SRP project. The course was designed by the five design principles by Cobb and McClain (2004) including using technology to support students development of statistical reasoning, establishing norms for students statistical argumentation and the importance of making students explore realistic data - in this case questioning the students media habits, which can be compared with "Questioning the World". The overall question that "Youngsters and ITCs" asks is: "Children and young people spend too much time on media ... or?" (Q0). To answer the overall question, the students must explore the TV patterns and the media habits of each student in their class through a circular working process, which entails collecting data, analysing data, and making conjectures about habits to discuss whether or not the students spend too much time on media. A really important part of the work is to justify how and why the students' conjectures are accurate or true.

The institutional framework for the lessons analysed and presented in this paper is a small elementary school in Denmark. The teacher, Ea, is an experienced mathematics teacher. Her normal teaching is dominated by working with skills framed by a textbook or by iPad exercises. Ea characterizes herself as "a bit old-fashioned. I think it is most important they have skills". When describing dialogs in the classroom, Ea explains, "I fail to tell them anything in twenty minutes ... I do not think they listen when I stand at the blackboard".

In the present article we analyse two lessons (2x45 minutes) of the course (Q3). In this section, the students investigate a set of data that includes 49 students' TV-consumption. They analyse and interpret the data, and make a conclusion, based on that background, about whether the students watch too much TV. The idea is, that the students can use their experiences from the smaller dataset in (Q3), when they work with the authentic and more motivational question Q0. The students are explicitly asked to explain and justify their conclusions.

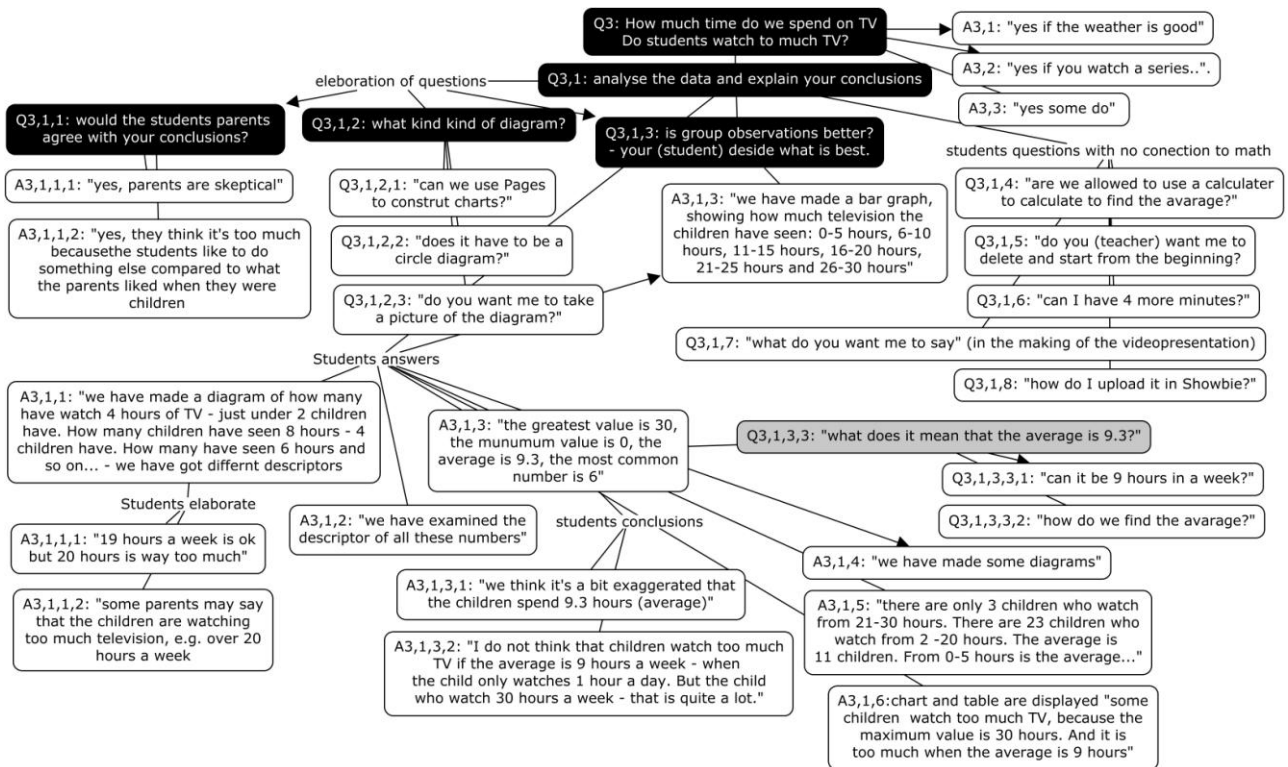
### **Presentation of the a priori and a posteriori diagrams**

A comparison between the a priori and a posteriori diagrams illustrates (respectively, Figure 2 and Figure 3) how the intended teaching goals and the actual enacted lessons are different:



**Figure 2: The a priori tree-diagram of the course, Youngsters and ICTs, for Q3**

In Figure 2, it is evident that we expect the students to consider and ask many questions in collaboration with the teacher. All the questions from Q3,1 to Q3,9 have extensions like the one shown for Q3,8 to the right. The expectation is that the students' answers will include an explanation of their results and of their choices of models/representations, and that students will use statistical arguments to justify their claims when they generalise about conjectures.



**Figure 3: The a posteriori tree-diagram for Q3 in the observed classroom**

The posteriori tree-diagram of the observed classroom is constructed by analysing the collected observations from the classroom. Q3 is a question from the teacher Ea, which give rise to three quick answers from the students (A3,1, A3,2, & A3,3) and one new question from Ea (Q3,1). Q3,1 produces six answers (A3,1,1- A3,1,6), and six new questions (Q3,1,1 – Q3,1,8), all with no connection to mathematics. We only see a few questions from the students that focus on specific solutions and procedures (e.g. Q3,1,3,3,1 – Q3,1,3,3,2).

### **Comparing the a priori tree-diagram with the posteriori tree-diagram**

In comparing the two tree-diagrams we do not see any of the expected questions (Q3,1-3,9) from the a priori tree-diagram. Ea does not ask many questions in this lesson, although she does ask a interpretative questions such as Q3,1.1: “Would the students’ parents agree with your conclusions?” The questions Ea asks, do not enable the students to elaborate, explain, or justify their choices. Instead the students answer Ea’s questions with examples from their everyday life and their beliefs about what parents think. In the posteriori tree-diagrams, it also becomes clear that the students ask many questions that are organizational and procedural, including Q3,1,8 (“How do we upload in Showbie?”) and Q3,1,7 (“What do you want me to say [in the presentation]?”). Studying the students’ answers in the posteriori tree-diagram, it is possible to see that there are not any answers that include statistical argumentation like the questions in the a priori tree-diagram Q3,8. Mostly, the students use their own rationale as argumentation: “19 hours a week is ok, but 20 hours is way too much” (A3,1,1,1). However, some answers include calculations of different descriptors, like A3,1,3: “The greatest value is 30, the minimum value is 0, the average is 9,3 and the most common number is 6.”

### **Discussion**

The tree-diagrams reveal a lot of answers that indicate that the students may be unaware of the purpose of the generating question, how to ask questions, and how to answer those questions. The students answer the question, Q3, immediately without any inquiry and without any mathematical argumentations. For example, they say, “yes, [children watch too much TV] if the weather is good” (A3,1), or “yes, [children watch too much TV] if they watch a series” (A3,2). These quick answers could indicate that the students have not understood the premise about “inquiry” that is crucial to this type of investigative work, and the SRP does not contribute to establish a milieu with new norms for working with statistical argumentation. The tree-diagram indicates that the students do not discuss appropriate statistical descriptors. The students simply repeat an approach to analyse data that they have made in earlier lessons by calculating descriptors. Most of the students found the average number of hours spent on media without using any argumentation whatsoever for why it makes sense to calculate the average number of hours or what the average actually says about the dataset. The interpretation of what actually happens in this class may be characterized as ‘model reasoning’ (Brousseau & Gibel, 2005). Overall, the a posteriori tree-diagram does not indicate any joint dialogue in which the students raise questions and explore the context in collaboration with their teacher, who challenges them to create and reflect further on their statistical reasoning. However, we do see that the students apply statistical knowledge and procedures and demonstrate statistical thinking, e.g. when the students draw up charts (A3,1,6), fill tables (A3,1,4), and calculate

descriptors (A3,1,3,2). Therefore, this approach can be seen more as “visiting monument” than “questioning the world” (Chevallard, 2006). To understand the students’ development of statistical reasoning, it was rewarding to use SRP as an analytical tool. It helped us to distinguish different strategies for raising and answering questions and to interpret the content in the questions and answers. However, the SRP model has not helped us to clarify the amount of time spent on the different questions and answers and to know if certain areas received a greater focus. The teacher's treatment of students’ answers and the possible feedback of students' answers are not visible. Finally, in the model we do not explicit focus on medias and the lack of integrating new medias, and we do not include the teacher’s explanations and introductions; e.g. Ea explicitly explained how to make a frequency-table: “So you start counting how many one hours there are in the survey, and then you type the number here...[points at the Excel sheet].” Ea also explicitly explains how to calculate the average: “... to figure out how much the average is in this week per day - you just sum it together and divide it by seven.” To make the diagrams, the teacher explains: “Just get Excel to make the diagrams for you by plotting the numbers in the sheet.” These instructions can possibly be seen as a basis for ‘model reasoning’ and are therefore very important for the interpretation of the students’ reasoning processes.

The gap between the a priori and the a posteriori tree-diagrams is in many ways not surprising. Ea participated in the 6-lesson professional development workshop and she worked collaboratively with her colleague during this program, but the intended statistical course is very different compared to Ea’s normal teaching method and the milieu established in the classroom, which mostly focuses on mathematical skills. It is also important to notice, that the students also experience a very different way of learning mathematics; an approach, which no matter what, stresses some adaptation. The statistical course challenged Ea’s view of classroom dialogs and it was difficult for her to establish new practices: “We'll take the arguments afterwards, because we'll first see all the presentations.” However, in the end of the lesson Ea did not prioritize to hear the argumentation from the students.

## **Conclusion**

The comparison of the two SRP tree-diagrams displaying the teacher and the students’ questions and answers show that the content was focused more on ‘model reasoning’ and statistical thinking and that the course in many ways did not support the students' opportunities for production of statistical reasoning. The SRP tree-diagram has been found to be a convincing tool for analysing classroom practice. A comparison of the two diagrams provides a rich view of the questions and answers in the classroom, which is closely connected to the students’ development of statistical reasoning; however, the use of the model had limitations, in the sense that we did not directly include media aspects as feedback, explanations, and introductions. It could nevertheless be interesting to design a “tree diagram” including not only question and answers, but which also include feedback, explanations and introductions and study if this model in any way gives a deeper understanding. Furthermore, it could be interesting also to focus on the processes and relations between the questions and the answers – to explore why the students respond as they do?



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