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► **To cite this version:**

Simone Jablonski, Matthias Ludwig. Mathematical Arguments in the Context of Mathematical Giftedness – Analysis of Oral Argumentations with Toulmin. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02398107

HAL Id: hal-02398107

<https://hal.science/hal-02398107>

Submitted on 6 Dec 2019

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Mathematical Arguments in the Context of Mathematical Giftedness – Analysis of Oral Argumentations with Toulmin

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Being defined as a main learning goal of mathematical education, arguments and argumentative skills are of high relevance. The development of these skills in the context of a deeper understanding is an important aspect for mathematical teaching and the support of mathematically gifted students, whose interest in mathematics should be increased. To focus more in detail on arguments in the context of mathematical giftedness, the paper poses the question whether it exists a relation between mathematical giftedness on the one hand, and mathematical argumentations on the other hand. For this purpose, the arguments of primary students from an enrichment program for mathematically gifted and interested students are gathered in an interview setting and analyzed by means of the Toulmin scheme. The results show that giftedness might influence the content and quality of arguments, but not the need for argumentations.

Keywords: argumentation, giftedness, Toulmin

Introduction

For curricular, societal and mathematical reasons, argumentative skills are claimed as central learning goals in mathematical education. Especially the view on mathematics as a deductive organized system with theorems and proofs forces fundamental skills in argumentation (e.g. Hanna, 2000). Nevertheless, former research shows that (German) students have deficits in formulating arguments in written and oral form (Cramer, 2011). It is therefore an important aim to support the development of argumentative competences, not solely, but also with regards to mathematical giftedness. The idea that gifted students perform better in the formulation of mathematical arguments seems legitimate, even though the relation is not clear (Fritzlar, 2011). In this context, the study focuses on this relation through analyzing gifted students' oral arguments.

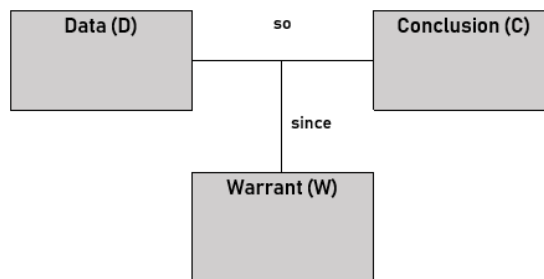
Theoretical Framework

Arguments in Mathematical Education

Not only in every day's communication, but also in mathematical education, arguments are of high relevance. The aim of an argumentation is to convince the communication partner with the help of shared respected statements (Cramer, 2011). Nevertheless, real disputes in mathematical education are rare, so that argumentations are best described in the context of a problem solving process as "a type of dialogical or dialectical game [...] that is associated with collaborative meaning-making." (Baker, 2003, p. 48). In mathematical education, the process of an argumentation involves activities such as to formulate assumptions on mathematical characteristics, to reason on relations, as well as to question assumptions (Bezold, 2009). The paper defines the term 'argument' as a product which results from the processes of former described activities. With this product-oriented focus, it is possible to analyze arguments among different categories, e.g. structure and content.

According to Toulmin (1958/2003), every argument can be structured through different functional elements. Toulmin describes *Data* (D) as “facts we appeal to as a foundation for the claim” and *Conclusion* (C) as claim “whose merits we are seeking to establish” (Toulmin, 2003, p. 90). The step from D to C often requires further considerations, asking for “general, hypothetical statements, which can act as bridges, and authorize the sort of step to which our particular argument commits us” (Toulmin, 2003, p. 91). These kind of statements are called *Warrant* (W). Figure 1 presents the core of the Toulmin scheme.

Figure 1: Core of the Toulmin Scheme (Toulmin, 1958/2003)



Apart from a structural analysis, it is furthermore necessary to analyze arguments according to their content as the analysis by means of the Toulmin scheme does not state anything on the argument’s quality, e.g. whether a warrant is based on authority or a mathematical rule (Koleza, Metaxas & Poli, 2017). Following Toulmin’s definition, an argument is called *analytic* if W includes all relevant information for the step from D to C (Toulmin, 2003), e.g. mathematical rules or laws that are adequate in the specific context. Arguments that do not fulfil this condition are claimed as *substantial* (Toulmin, 2003). These arguments leave open or fail to answer (further critical) questions (Koleza, Metaxas, & Poli, 2017), e.g. “*My calculation is correct, because Anna has the same result*” could be rebutted by the question “*But what if both of you did a wrong calculation?*” (Fetzer, 2012).

Mathematical Giftedness

Mathematical giftedness is a complex construct that lacks a standardized definition and diagnosis as it is not directly observable (Bardy, 2013). It is more a potential that might develop into an outstanding mathematical performance by means of an advantageous interplay of genetic and environmental factors (Käpnick, 1998). Therefore, the construct is approximated by means of lists of characteristics, including special skills such as the memorizing and transferring of mathematical structures, creativity and problem solving competences (Käpnick, 1998).

State of the Art

A main interest of studies on argumentations in the educational context is on the analysis of arguments with help of frameworks and models in order to make characteristics observable (e.g. Nussbaum, 2011). Mostly, studies observe arguments in classroom interactions and collaborative argumentations (e.g. Forman, McCormick, & Donato, 1997). For this purpose, the Toulmin scheme, and especially its core, is often used (e.g. Koleza, Metaxas, & Poli, 2017). Further studies focus on the evaluation of argumentative skills by means of models and methods to improve them (e.g. Bezold, 2009). The studies show quite consistently that arguments in primary school have a low

significance and real disputes are rare, so that the teacher has to initiate arguments and show the need for arguments (Schwarzkopf, 2000). With regards to Toulmin, arguments by primary school children are typically characterized by the missing of a warrant and by substantial arguments (Fetzer, 2012). Koleza, Metaxas & Poli (2017) assume that primary students are able to formulate arguments which can be analyzed using the Toulmin scheme, even though basic elements are not mentioned through a lack of need for argumentation. Focusing on argumentative skills of mathematically gifted students, the relation is not explicitly clarified (Fritzlar, 2011). On the one hand, Fritzlar (2011) assumes that mainly other factors apart from giftedness influence argumentative skills and the need for argumentations. Other models see the potential of gifted students (inter alia through supportive characteristics, such as creativity) to develop outstanding argumentative skills, which can be forced through training (Bardy, 2013).

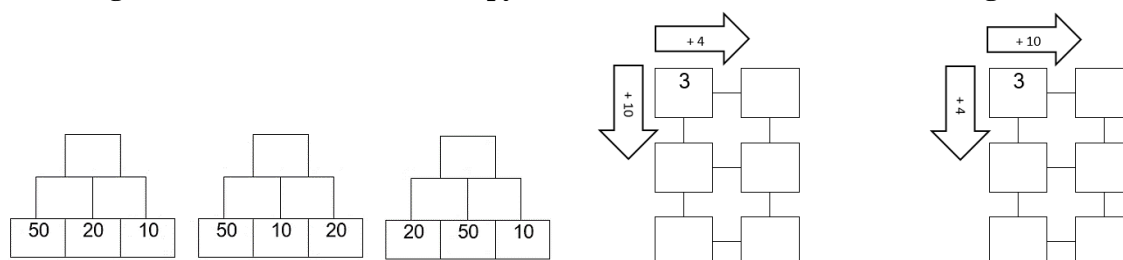
Research Questions and Methodology

Based on this theoretical outline and former research findings, the research questions of this paper is formulated as follows: *What are the characteristics of mathematically gifted and interested primary students' arguments? In how far do they differ from general research findings on the arguments of primary students with special regards to the Toulmin scheme and the need for arguments?*

The research question is answered in a qualitative study. The sample is taken from the participants of the extra-curricular enrichment program “Young Math Eagles Frankfurt”. The aim of the program is to support mathematical interested and gifted students and to increase their mathematical interest and joy on a regular and long-term level. In the school term 2017/2018, about 50 students between 8 and 10 years from 14 schools participate in the program after nomination of their teachers.

To focus on the characteristics of the oral arguments of the participants, task-based and problem-orientated interviews are created (Goldin, 2000). This interview form is characterized by the openness of answers while focusing on a special problem. The interview material includes tasks that emphasize arguments and ask primary students to formulate warrants through irritation or the task itself (Bezold, 2009). The task formats focus on number pyramids and numerical lattices with regard to special number relations. The formats can be expected to be known by the primary students and do not ask for difficult calculations. Therefore, the task formats' focus is basically on argumentation and it can be expected that the students are able to use their findings on number relations as basis for warrants that can be analyzed by means of the Toulmin scheme. In each interview, different components of the formulation of arguments are included. In particular, the students are asked to comment on a wrong assumption e.g. through giving a counter example, to argue on different mathematical relations and to generalize their detections.

Figure 2: Task formats number pyramid (left) and numerical lattices (right)



One task is to answer the question how the basic stones of the number pyramid/the arrow numbers of the numerical lattices have to be arranged in order to maximize the top of the pyramid/the result of the numerical lattices (see Figure 2). Every interview is done individually and includes four tasks which are done within 15 minutes. To make the interviews comparable, they are directed with help of a guideline. The interviews are transcribed on the basis of an audio recording and an observational protocol.

The task formats, as well as the guideline were tested during the pilot phase regarding their appropriateness in terms of the students' age, needed time and mathematical content. Further, they were analyzed according to the need for argumentations. The results show that the formats are adequate in the named categories. Most students were able to identify relevant mathematical characteristics and some were even able to generalize them to some extent. In all interviews, the students formulated arguments which could be reconstructed with the Toulmin scheme. Therefore, the piloting could confirm that the tasks emphasize arguments, which shows the suitability of the tasks for the research question. Nevertheless, many arguments were initiated by the interviewer. Through these initiation processes in the interview, it is therefore possible to analyze the independence of and need for arguments.

On the basis of the results in the piloting and the theoretical framework, a scheme for the analysis of the arguments within the interviews is created. The analyzed categories within this paper are

- (1) Structure of Arguments: Within this part, it is analyzed which of the elements *Data*, *Conclusion* and *Warrant* could be observed during the focus on the specific task.
- (2) Independency of Arguments: Here the focus is led to whether a warrant is initiated by the interviewer through questions/given after mediation, or given on an independent level.
- (3) Content of Arguments: This part involves a categorization of the warrant in terms of its mathematical content (inacceptable/wrong, substantial, analytic).

The use of the Toulmin scheme as a basis for the analysis can be legitimated as the results are to be compared to former research findings on arguments in the primary age and the Toulmin layout is frequently used in these studies. Through adapting the independency and the content, its limitations apart from a structural analysis for this setting are recognized.

During May and June 2018, 32 participants (all students with parental consent form and presence during the interview dates) were interviewed. Within the interviews, 128 argumentation tasks are analyzed. Beforehand, no special training in the formulation and awareness of arguments or an education on the formulation of analytic warrants took place. Argumentation and reasoning tasks were part of some lessons, but not to a greater extent than the focus on other competences, such as problem solving. First observations within the enrichment program did not seem to show obvious differences in the need for argumentations. Nevertheless, the detection of mathematical findings, might be an important influence on the arguments of gifted children. The following results from the analysis should specify the relation of argument and giftedness on a systematic level.

Results

Table 1 shows an example of an interview extract with a functional analysis according to the elements of the core of the Toulmin scheme. At the date of the interview, the student (S) was in ten years old. His interview bases on the task format *numerical lattices*. The sample analysis focuses on the task within the interview, in which the participant has to argue in which case the result is the highest. In advance, he does calculations with changed orders of the arrow numbers and concludes that the results change. Through initiation of the interviewer, he gives a substantial warrant for the relation of the basic elements and result. His argumentation bases on the order of the boxes of the numerical lattices and the switch of numbers comparing both tasks.

On the basis of this observation (which is coded as *Data* for the following task), he is able to identify the case in which the result is the highest, namely when the arrow with the higher number shows downwards (see Figure 2). Without initiation, he uses a substantial warrant and through initiation he supports a conclusion through an analytic warrant by transforming it to a general case.

Table 1: Extract from the transcript (translated from German by the author)

I: [...] In which case do you receive the highest result? You can use your former calculation.	Introduction of Task
S: When the higher number is taken plus here (<i>shows on arrow downwards of numerical lattices</i>).	Conclusion
Because here, there are three and here there are solely two (<i>shows on boxes of numerical lattices</i>). [...]	Warrant (substantial)
I: Do you think this is always the case or solely in our example?	Initiation
S: Well, I think this is in every example like this.	Conclusion
I: Can you tell me why this is the case or/?	Initiation
S: (<i>shows on arrows of numerical lattices</i>) Because when you calculate here the higher number plus at the arrow downwards, then there is one box more, where you can calculate plus and here on top, there is no more	Warrant (analytic)
and therefore I think, this is always like that.	Conclusion

With the former task as data, a conclusion is drawn and supported through a warrant. The task is therefore coded as *Data – Conclusion – Warrant*. For the warrant, the highest observable category before initiation is *substantial*, and after initiation *analytic*. The development of the warrant after initiation can be observed frequently, so correspondingly, two phases within each interview task are distinguished for the following analysis: Phase 1 describing the argument respectively the warrant on an independent level, and Phase 2 after an initiation of a warrant.

Table 2 shows the results from the structural analysis of all interview tasks through categorization according to the included elements on an independent level, namely without a question as initiation of reasoning (Phase 1). Without initiation processes, only 24.2% of all analyzed tasks include a warrant on an independent level. In the majority of the tasks, a conclusion is drawn, but ungrounded. Taking those tasks that are coded within the category *Conclusion or Data – Conclusion* on an independent level and in which a warrant was initiated afterwards, in 87%, a warrant could be formulated after being initiated. By combining the independently formulated and initiated arguments, within 98 of all 128 tasks the complete core according to the Toulmin scheme (*Data – Conclusion – Warrant*) could be reconstructed. Nevertheless, within 67 of these 98 tasks, the warrant was only formulated after being initiated.

Table 2: Structural categorization according to the Toulmin scheme on independent level

N=128	No Conclusion	Conclusion or Data – Conclusion	Data – Conclusion – Warrant
Argument on an independent level	6 (4.7%)	91 (71.1%)	31 (24.2%)

To focus more in detail on the mathematical content of the arguments, the tasks including a warrant are categorized as *not acceptable/wrong*, *substantial* and *analytic*. Again, the highest observable category is coded, e.g. a task including a substantial and an analytic warrant before initiation is coded as analytic. In Table 3, the independently given warrants from Table 2 are categorized (N=31). In addition to the former finding that in about 75% of all tasks no warrant is coded on an independent level, one can observe that two thirds of the independently given warrants are substantial.

Table 3: Content-related analysis of the tasks with an independently formulated warrant

N=31	Not acceptable	Substantial	Analytic
Independent Warrants	3 (9.7%)	21 (67.7%)	7 (22.6%)

In Table 4, a further categorization is made before and after initiation (Phase 1 and 2) for those tasks in which an initiation of a warrant is coded (N=95). Even though in 10.5% of initiated tasks still no warrant is coded, this number decreases significantly after initiation. In addition, it becomes obvious that the proportion of analytic warrants increases after a question on (further) reasoning is posed.

Table 4: Content-related analysis of the tasks with initiation

N=95	No warrant	Not acceptable	Substantial	Analytic
Before initiation	77 (81.0%)	2 (2.1%)	15 (15.8%)	1 (1.1%)
After initiation	10 (10.5%)	7 (7.4%)	42 (44.2%)	36 (37.9%)

Discussion and Conclusion

A comparison of the results with former research findings gives different insights into the arguments of mathematically gifted and interested primary students. On the one hand, the results can confirm the general conclusion by Schwarzkopf (2000) that arguments have to be initiated at least partly. In two thirds of all tasks including a warrant, the warrant was only mentioned after initiation of the interviewer. Therefore, one can assume that gifted primary students do not seem to have an exceptionally sensible need for reasoning and arguing. Without initiation processes, the research findings by Fetzner (2012) on the lack of a warrant in primary students' arguments seem to be accurate for this sample. Also the independently formulated warrants are mostly substantial what fits the findings by Fetzner (2012). On the other hand, it is nevertheless noticeable that nearly 40% of the analyzed tasks with initiation involve an analytic warrant after an initiation process took place. Further, only in 10.5%, still no warrant is mentioned after initiation. The focus on initiation emphasizes the hypothesis that a large group of the students is able to formulate (analytic) warrants after being forced to. Nevertheless, only the minority of the students formulates (analytic) warrants on an independent level.

With special regard to the research questions, one can formulate the hypothesis that mathematical giftedness might not influence the need for argumentations. Nevertheless, most of the gifted students are able to formulate warrants after initiation, some of them even analytic. Mathematical giftedness therefore might have an impact on the mathematical basis for the formulation of analytic warrants in order to support (creative) detections and findings. So, incomplete arguments without initiation do not seem to be caused by a lack of ability, but by the low significance of arguments' needs or the missing of structural knowledge of arguments. This finding is further supported by the fact that the students were not taught on arguments in advance. Hereby, former studies of mathematically gifted students' intuition (e.g. Käpnick, 2010) might be relevant in order to explain the low need of giving warrants. In the interpretation of these findings, one has to take some limitations of the study into consideration. The interviews were analyzed on a qualitative level without a control group as it was not possible to exclude many different influencing factors (e.g. differences in school, teacher, language skills and mother tongue). Therefore, the comparison was built on former research findings and studies by different researchers, which mostly show consent in the analyzed categories and therefore allow a certain generalization of the findings for different settings. The comparison nevertheless has to be seen under varying task formats and different settings.

We take up the findings as a basis for a longitudinal study on the changes of the arguments of the students in terms of the structure and content, as well as the need for arguments. During this, the focus will be laid on the individual changes in arguments of the children with special regards to their environmental background in terms of language and family. Moreover, the study focuses on the question whether mathematical giftedness as a potential for outstanding mathematical performance can further be a potential for arguments.

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