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► To cite this version:

Marisa Gregório, Hélia Oliveira. The justification of conjectures in the study of the congruence of triangles by 5th grade students. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02398098

HAL Id: hal-02398098

<https://hal.science/hal-02398098>

Submitted on 6 Dec 2019

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The justification of conjectures in the study of the congruence of triangles by 5th grade students

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Keywords: Geometry, mathematical reasoning, justification.

Introduction

Mathematical reasoning (MR) is recognized as one of the fundamental aspects of mathematics learning (NCTM, 2000), however, in the literature, sometimes this concept is not clearly defined. From a review of literature, Jeannotte and Kieran (2017) synthesized converging features of MR and developed a conceptual model for MR in school mathematics. According to these researchers, MR is a process of communication that allows to infer mathematical statements from other mathematical statements and that encompasses five processes regarding students' search for similarities and differences (generalising, conjecturing, identifying a pattern, comparing, and classifying) and three processes regarding students' aim for validation (justifying, proving, and formal proving). In this study, we aim to analyse 5th grade students' justifications of a conjecture on the properties of triangles, during a teaching intervention to promote students' MR.

Theoretical framework

For the purpose of this study, we will use the term *justification* as an argument that guarantees (or disproves) the truth of an statement and uses mathematical forms of reasoning accepted as universal in the classroom community. This way of defining justification is similar to the one by Staples, Bartlo and Thanheiser (2012) but to which we add the focus on the community. Therefore justification consists in the process of searching for data and guarantees that allows the change of the epistemic value of a narrative (Jeannotte & Kieran, 2017) and it is supported by the discourse of the community. However, according to these authors, that change does not occur “necessarily from likely to true” but “from likely to more likely” (p. 12) and, as such, this process does not require a deductive structure. In order to analyze students' justifications, we use in this study a framework based on Balacheff (1988) and Harel and Sowder (1998), with five levels hierarchically organized. At level 1 – *external authority* – the justification is based on an element considered as an authority, which can be the teacher, a colleague or the textbook. At level 2 – *empiricism naïf* – we consider two categories: *perceptual naïf empiricism*, when a justification is based on perceptual observations, showing a drawing or gesturing; and *inductive naïf empiricism*, when a justification is based on the verification of some examples. At the intermediate level – *crucial experience* – a justification is grounded on a carefully selected example, revealing intentionality in the choice. At level 4 – *generic empiricism* – operations are used, based on the properties of objects, for justification, however, the student does not identify or justify the applicability of the property used in the

operation. At the most sophisticated level – *mental empiricism* – the justification is based on the properties and relations between objects.

Methods

The study was carried out in a 5th grade class with 30 students, during a teaching intervention on geometry conducted by the first author. The focus of this poster are the students' justifications of a conjecture presented by one student (Mariana) who stated that "If two triangles have the same perimeter, they are equal", when they were working on the topic of congruence of triangles. Students were asked by the teacher to validate or refute the conjecture individually and then to discuss their ideas with whole class. The methodology of the study is qualitative with data coming from students' written productions and the collective discussion of their work. In this poster we use the 5-level justification framework referred above to analyse the justifications of four students, whose solutions were selected by the teacher to be collectively discussed in order to clarify the refutation of the conjecture and the importance of the counterexample used in this refutation.

Results

The analysis shows that students' justifications of the conjecture are at the second and third levels according with the adopted framework. At the second level, justifications are based on perceptual or intuitive observations. These students perceive other possibilities of different lengths for the sides of the triangle from which an equal perimeter can result, without the triangles being equal, however, the counterexample they present does not take into consideration the triangular inequality. At the third level, students use carefully selected examples, such as two triangles of different lengths and with the same perimeter, that they draw rigorously using the compass and the ruler, and which they present as a counterexample to refute the conjecture. Still, there is evidence that students are producing arguments that may lead to a more general conclusion by making important connections with triangular inequality to choose the counterexample. The analysis of this classroom episode also emphasizes the importance of the teacher's role in promoting the development of MR processes. The fact that the conjecture was based on a wrong assumption was seen by the teacher as an opportunity to trigger MR validation processes and, more generally, for promoting students' MR.

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