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# **Towards an interactional perspective on argumentation in school mathematics**

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*This paper discusses theoretical and methodological considerations which have emerged from reviewing the literature related to my PhD research project which adopts an interactional perspective on the development of the argumentation process in primary school mathematics. This led me to distinguish the factors involved in mathematics classroom interaction during the development of the argumentation process, as well as to examine the possible relations and interrelations of these factors. The outcome of this process was, first, to clarify theoretical aspects and, second, to create a preliminary model, called the “Mathematics Classroom Interactional Model” (MCIM), positing two levels of classroom interaction and the possible relations within each of them and between them.*

*Keywords: Socio-mathematical norms, participation, mathematical argumentation, mathematical communication, mathematics classroom interaction*

## **Introduction**

In the field of Mathematics Education, according to the related literature review, the notions of argument and of mathematical argument can be found in two different strands. The first strand relates to socio-mathematical norms and participation in the learning environment (e.g. Yackel & Cobb, 1996; Wood, 2002), mostly based on situated learning theory in the research (e.g. Lave & Wenger, 1991; Greeno, 1997; Boaler, 2000) and the second strand relates to mathematical argumentation and classroom interaction (e.g. Steinbring, 2005; Stylianides, 2007). However, so far, the literature does not seem to have made any clear attempt to develop a coherent theoretical framework and methodological tools emerging from the two strands. Thus, in my study, I focus on the development of the argumentation process in primary mathematics classrooms, considering the norms, the interactions and the role of the participants in learning and teaching practices. In this paper, I discuss aspects related to mathematical argument, argumentation and participation from the two strands of the literature to formulate the preliminary central research questions of the study, as well as to discuss the coherence –both compatibility and complementarity– of the two strands. On this basis, I develop a preliminary model, called the “Mathematics Classroom Interactional Model” (MCIM), of the concepts and their possible relations that will guide the scope of the study.

## **Socio-mathematical norms and participation in the learning environment**

Lave and Wenger (1991) describe learning as a social phenomenon that is constituted in the real world through a process of legitimate peripheral participation in communities of practice which are in development. This means that pupils are members of wider worlds that are socially and culturally formed, so developing links that cross the identities of ethnicity, gender, religion, etc., acts in classrooms, schools and communities and the practices that govern all these environments. The word “knowledge” has been replaced by the word “knowing” that declares an action. This fundamental

shift indicates that activities cannot be considered independent of the context. The “practice” is mainly characterized by the terms “discourse” and “communication” which implies that a pupil should be regarded as a person interested in participation in certain types of activities not only in the accumulation of knowledge (Sfard, 1998).

While the learning process in the current study is regarded in terms of the participation metaphor (Sfard, 1998), what is important is the person’s participation in activities influenced by the context. The underlying theory is that of Vygotsky’s (1978) socio-cultural approach, considering learning as an outcome of interaction with others, while theoretical approaches in situated learning are preeminent in contextualizing and describing classroom communities. These approaches, according to Lave and Wenger (1991) and Bransford, Zech, Schwarz, Barron and Vye (2000), have led to situated learning theories, in which knowledge is situated in particular forms of experience that arise in specific situations, and are understood in a relational way as something shared between people, activities and environments rather than as a fixed, individual characteristic (Boaler, 2000). Hence, mathematical knowledge, as a dynamic process of mathematization, is “still being open and not fixed in advance of the learning and acquisition processes” (Steinbring, 2005, p. 48).

According to Greeno (1997), many researchers based on situated perspectives study the development of classroom activities which involve pupils participating “in the discourse of the subject matter, including formulating and evaluating hypotheses, conjectures, arguments, evidence, examples, and conclusions” (Hatano & Lambert, 1990; Inagaki, 1991; Cobb et al., 1993; Cohen et al., 1993; Schoenfeld, 1994 as cited in Greeno, 1997, p. 99). A situated view suggests that activities of different practices are important, for instance involving pupils in classroom discussions is a way of pupils learning not only the content knowledge but also to participate in discourse practices (Greeno & MMAP, 1998 as cited in Boaler, 2000). Pupils learn not only methods and processes in the mathematics classrooms, but they are trained in mathematics, and the learning of content knowledge cannot be separated from the classroom interaction, as they are two reciprocal components (Boaler, 2000). One question posed in the situated learning theory is whether the pupil’s pattern of participation can be a potential obstacle to his/her membership of the classroom community.

Wood (2002) states that classroom culture consists of a set of social norms, a specific structure of participation, as well as characteristic forms of discourse that support both social norms and the structure of participation. A participatory structure refers to the specific characteristics of the classroom that affect pupils’ participation in the classroom: who is involved, when and how. Wood recognizes three types of culture that characterize a classroom of inquiry and can lead to different patterns of participation. The first relates to the development of alternative resolution strategies, the second relates to a culture of exploration of the strategies developed by their classmates, and the third to a culture of argumentation where social norms require pupils to justify or defend the methods of solution they choose. Wood, Williams and McNeal (2006) investigated primary mathematics classroom interactions and the development of mathematical thinking. One of the most important results was the finding that only in an inquiry/argument classroom culture were there opportunities for all children to be involved in meaning making and shared understanding. Nevertheless, as Klein (2001) points out, participation in cultures such as the above may be problematic for pupils who either have not conquered the tools of defending or challenging ideas through discussion (e.g. language or

norms such as what constitutes a different answer) or lack the self-confidence or self-image expected by an apprentice working in a collaborative learning environment.

Many researchers attach importance to the role of classroom culture, providing cooperative learning opportunities and, in particular, developing the intellectual autonomy of pupils. They focus on socio-mathematical norms and argumentative skills in the constitution of mathematical meaning in the classroom. Yackel and Cobb (1996), investigating the role of communication as a cultural tool, concluded that social norms (e.g. explanation and justification of a solution) directly affect the patterns of participation, a conclusion also supported by Sfard (1998), while socio-mathematical norms (e.g. which answer is considered mathematically different) provide equal opportunities to all pupils in that particular structure and regulate mathematical arguments. Finally, Kazemi and Stipek (2001) recognized, defined and described four categories of social norms and socio-mathematical norms respectively. The authors, in their discussion, emphasize the need for future research with longitudinal data that may reveal other norms, how socio-mathematical norms are created and sustained, and how they influence pupils' mathematical understanding.

The discussion so far led me to consider the connection between social and socio-mathematical norms relating to argumentation in the mathematics classroom. It seems that the mediator in this connection is the specific structure of the patterns of participation that allows (or not) the social norms to be transformed, created and sustained as socio-mathematical norms. Thus, the first preliminary central research question which emerged is: "what social and socio-mathematical norms relating to argumentation are established in the mathematics classroom, and how are these expressed in terms of patterns of participation?".

### **Mathematical argumentation and classroom interaction**

In the field of Mathematics Education, Krummheuer (2007, 2015) started off using Toulmin's (2003) argumentation scheme to analyse classroom-based mathematical arguments. However, Krummheuer (2007, 2015) used a reduced version (conclusion, data, warrants and backings) of Toulmin's full scheme of argumentation (conclusion, data, warrants, backings, modal qualifier and rebuttal). Many researchers (Yackel, 2001; Hoyles & Küchemann, 2002; Evens & Houssart, 2004; Cabassut, 2005; Pedemonte, 2005; Weber & Alcock, 2005) as cited in Inglis, Mejia-Ramos and Simpson (2007) appear to have followed Krummheuer in using the reduced scheme. While Inglis et al. (2007) concluded that without using Toulmin's full scheme of argumentation it may be difficult to accurately formulate the full range of mathematical arguments, on the other hand Mariotti, Durand-Guerrier and Stylianides (2018) mention that difficulties of pupils to organize arguments in a deductive chain in the form of proof cannot be fully explained by Toulmin's model.

Despite the widespread use and proven usefulness of Toulmin's scheme of argumentation over the last two decades, researchers in Mathematics Education do not use this scheme in a consistent way. Besides different emerging interpretations, limitations can be identified. Thus, I studied further the related literature to find a model created in the field of Mathematics Education that could serve both aspects of mathematical argumentation and classroom interaction.

Stylianides (2007) developed a theoretical framework about proof and proving in the context of K-12 mathematics. "Proof is a mathematical argument, a connected sequence of assertions against a

mathematical claim” (Stylianides, 2007, p. 191). Any given argument can be broken down into three major components: the set of accepted statements, the modes of argumentation and the modes of argument representation. The distinction between base arguments and ensuing arguments could provide the context in which instructional analysis and instructional interventions by teachers influence classroom interactions and vice versa. In terms of this distinction, it is worth to be mentioned that in everyday mathematics classrooms situations, where the learning process is considered in terms of a participation metaphor (Sfard, 1998), the participants could produce a range of arguments, for example, “relatively sophisticated arguments” or “explications of elements of an argument” (Krummheuer, 2015, p. 53). Thus, the basic assumption is that mathematical argumentation can only emerge through interaction and mathematical communication within the classroom culture. Steinbring (2005) developed an analytic framework to examine the relation between mathematical knowledge and mathematical communication. While, “language is the central medium for the creation of possible connections between communication and consciousness” (Steinbring, 2005 p. 53), proof is the communication medium of invisible mathematical objects and the mediator between communication and consciousness (Heintz, 2000 as cited in Steinbring, 2005; Steinbring, 2005).

Instructional school-mathematical interaction is expected to contribute to introducing individuals into mathematical communication practice, and thus to increase these individuals’ ability to participate in (mathematical) communication in the society. (Steinbring, 2005, p. 74)

This interaction could be understood by the term “situational”.

My perspective is situational, meaning here a concern for what one individual can be alive to at a particular moment, this often involving a few other particular individuals and not necessarily restricted to the mutually monitored arena of a face-to-face gathering. (Goffman, 1974, p. 8)

According to Krummheuer (2007), the term “situational” refers not only to a particular situation that could be characterized as “situated”, but to anything that can happen in the interaction between people. Thus, for example, if during a lesson the pupils solve an activity on their own it may be a “situated learning” process (Lave & Wenger, 1991) which is shaped by the pre-knowledge that allows them to face similar activities. The action changes into a “situational” process if the pupils take initiatives to act with their classmates. Levinson (1988) extended the ideas of Goffman (1981) and Krummheuer (2007, 2015) by adapting the concepts of participants’ (speakers’) roles in Mathematics Education: “author”, “relayer”, “ghostee” and “spokesman”. Although Krummheuer’s (2007, 2015) approach to participation in argumentation offers insights on the way that participation is performed in mathematical argumentation, the mechanism of being in one role or another is not obvious and cannot explain the obstacles in pupils’ participation in the developing of mathematical arguments, something also claimed by Cramer and Knipping (2018). Cramer and Knipping (2018) highlight the importance of participation in mathematics classroom argumentation, considering the discursive and social processes that affect argumentation. Thus, participation is not only a discourse but the practice of constructing arguments through the social order of participants’ interactions in the classroom which can be constructed, maintained and transformed. Cramer and Knipping (2018) mention an interesting case of pupils’ implicit participation (pupil’s initially spoken idea developed further in the classroom

discourse but the pupil's voice disappeared), to describe possible obstacles for participation in argumentation and possible interventions by a teacher.

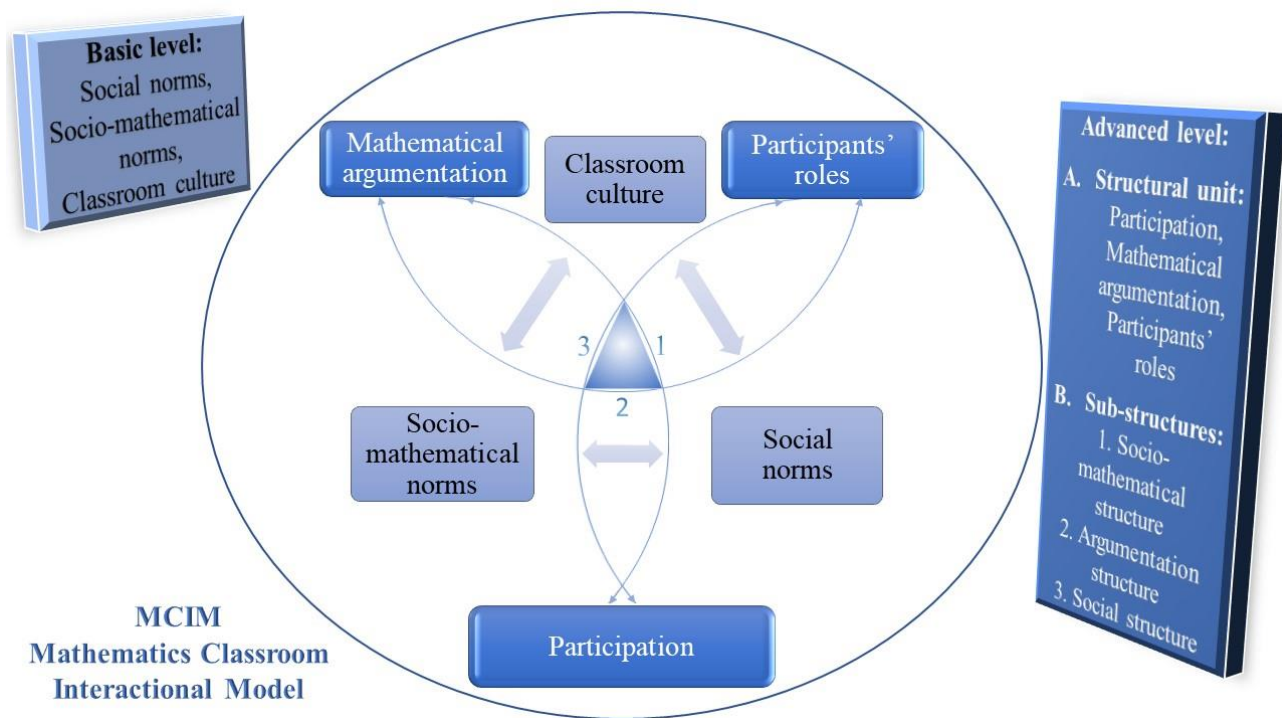
The discussion so far led me to consider the connection between mathematical argumentation and classroom interaction. It seems that the mediator in this connection is the specific role of the participant that regulates the participation in mathematical argumentation. Thus, the other two preliminary central research questions which emerged are: "how do pupils' interactions contribute to the development of the base arguments?" and "how do teachers' instructional interventions influence pupils' activity in the developing of ensuing arguments?".

## **Discussion**

The ideas about socio-mathematical norms and participation in the learning environment and those about mathematical argumentation and classroom interaction seem to be related. Socio-mathematical norms seem to be a major factor that regulates classroom interaction in developing arguments. Especially, pupils' and teachers' roles in developing base and ensuing arguments are related to socio-mathematical norms that could foster (or not) mathematical argumentation and participation in the classroom. Thus, I decided to create the preliminary MCIM model, of the basic concepts and their relations as considered in the previous two sections and expressed in terms of the three preliminary central research questions.

In this model, Figure 1, I posit two levels of classroom interaction. The first (basic) level of classroom interaction is defined by the relations among the social norms, the socio-mathematical norms and the classroom culture. These are the predominant factors that characterize classroom interaction and are defined fully through the review of the first strand of the literature. The socio-mathematical norms established in the classroom culture are affected by social norms. The second (advanced) level of classroom interaction is defined by the relations among the participation, the mathematical argumentation and the participants' roles. This level includes, A: the three factors as a structural unit (participation, mathematical argumentation, participants' roles) and B: three sub-structures: 1) socio-mathematical structure: participation-mathematical argumentation, 2) argumentation structure: mathematical argumentation-participants' roles, 3) social structure: participants' roles-participation. Each of these (sub)structures presupposes the connection of the factors at the basic level. Thus, the common ground of the structures at the advanced level is the connection among the three factors at the basic level. When a researcher or a teacher in the classroom would like to understand, investigate and further develop the advanced level, they should firstly understand, consider, and develop the connection of the social norms, socio-mathematical norms and the classroom culture, the factors at the basic level.

According to the related literature, the main connection between the two levels rests on social norms and socio-mathematical norms that regulate mathematical arguments, defining participation in the classroom culture (e.g. Yackel & Cobb, 1996; Wood, 2002; Wood et al., 2006). Therefore, in order to define this model with consistency, I consider theoretical and methodological frameworks to get access to mathematical argumentation and classroom interaction.



**Figure 1: Mathematics Classroom Interactional Model (MCIM)**

Stylianides (2007) framework could serve as an analytic tool in order to examine the development of mathematical argumentation and teachers' actions, through the processes of instructional analysis and instructional intervention. Especially, the distinction between base and ensuing arguments and the possible differentiation between them and in each of them could be related with the patterns of participation which emerge through the socio-mathematical norms. Nevertheless, it seems that the focus of the framework is from the teachers' perspectives rather than on pupils' and teachers' interactions and their roles in the classroom culture. Stylianides uses the notion of classroom community in the definition of proof with a perspective different from that usually found in the literature, and from the one I have taken in my study. He regards the pupils as the main members of the classroom community, giving the teacher a special membership status and distinct role, while in my study I focus on the dynamic of the classroom interactions and the power of the relationships that shape the participation in the classroom community. Thus, I elaborate Steinbring's (2005) framework of mathematical knowledge and communication where the interactions and communication among participants are presented as predominant in the classroom. In this context, the reference to the role of the language in mathematical communication and argumentation in the classroom is very interesting and this leads me to consider frameworks through which I could get insights on the utterance of the mathematical argumentation through the participants' roles. Levinson's (1988) categorization of speakers' roles seeks to offer insights on the patterns of participation. However, Krummheuer (2007, 2015) does not seem to undertake or investigate the limitations described by Levinson (1988), where multiplicity and alteration of the participants' roles in some utterance events, as well as new roles, could be recognized, defined and re-defined. Moreover, Cramer's and Knipping's (2018) evidence of implicit participation is an interesting aspect related to participants' roles in mathematical argumentation and could be investigated further, either as forms of participation

or non-participation. Considering the roles of the participants and interactions among pupils and teachers in the classroom culture, it could be possible to examine pupils' interactions in developing base arguments and explain teachers' actions in influencing pupils' activity to develop ensuing arguments.

Finally, the way that the socio-mathematical norms and the social order, related to the development of mathematical arguments and argumentation, are created, sustained and transformed could provide the context of an interactional perspective on argumentation in school mathematics through the MCIM model. However, this model has still to be considered and defined fully through empirical research to be conducted, developing the methodological context and the research protocols that fulfil the goal of the study.

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