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Understanding geometric proofs: scaffolding pre-service mathematics teacher students through dynamic geometry system (dgs) and flow-chart proof

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The objective of this paper is to discuss the pedagogic potential that is offered by the use of a flow-chart proof with open problems and a Dynamic Geometry System in understanding geometric proofs by pre-service mathematics student teachers at an Indonesian university. Based on a literature review, we discuss aspects and levels of understanding of geometric proof and how to assess students' understanding of the structure of deductive proofs, and how the use of a Digital Geometry System may support students' understanding of geometric terms and statements, including definitions, postulates, and theorems. The pedagogic focus consists of exploiting the semiotic potential of a DGS, especially the use of GeoGebra tools that may function as tools of semiotic mediation to understand the geometry statements and the scaffolding potential of flow-chart proof with open problems in identifying the structure of deductive geometry proofs.

Keywords: Understanding, geometry, proof.

Introduction

Proof plays essential roles in mathematics and mathematics education. Proofs help mathematicians understand the meaning of statements or theorems and their validity within a framework. Through the process of proving, mathematicians discover or create new results or meanings (Samkoff & Weber, 2015; Zaslavsky, Nickerson, Stylianides, Kidron, & Winicki-Landman, 2012). In the context of mathematics learning, understanding proof is an essential component of mathematics competence as it offers powerful ways of developing and expressing mathematical understanding (NCTM, 2000). However, research has demonstrated that students at all levels have difficulties to deal with understanding and constructing proofs (Doruk & Kaplan, 2015; Güler, 2016; Knapp, 2005; Weber, 2002).

This paper presents some findings of the literature review we conducted aiming at developing a theoretical framework, which will underpin the design of a learning intervention. This intervention will foster understanding of Euclidean geometry proof by mathematics preservice teacher students. The students are first year students at an Indonesian university and will become teachers for secondary school. In this stage, students start to learn formal mathematical proof for the first time. This paper describes the aspects and levels of understanding of geometric proof and elaborate the use of a digital geometry system (DGS) and flow-chart proof representation as scaffolds in order to understand the structure of proof based on our literature review findings. We focus on these issues because they are relevant for our research project regarding the topic (geometric proof in undergraduate level), context and characteristics of participants, Indonesian university students, who have a little mathematical proof background.

Method

This paper presents some parts of the outcomes of our literature study which focuses on the understanding of proof and the use of DGS and flow-chart proof to support students' understanding of proof. The methodological process of the literature study is *anasynthesis* (coined from the words analysis and synthesis) adapted from Legendre's method (as cited in Jeannotte & Kieran, 2017). First, we create a corpus from a search in three databases of research, namely ERIC, MathEduc, and Web of Science, and select articles/papers having mathematical proof as keywords or associated keywords by entering 'mathematic* proof' or 'mathematical prov*' and 'geometric* proof' or 'geometrical prov*' as keywords. It is followed by selection and un-doubling. To assure the quality of the sources, we characterized the papers regarding their relevance of method and findings for education, quality of author (the number of citations) and quality of journal (impact factor). By the end of the process, 32 English texts (books, chapters, articles, and research reports in proceedings) constituted the corpus (modified corpus).

Secondly, the resulting corpus is *analyzed* for relevant information, namely definitions of mathematical proof, structure of proof, learning difficulties, interventions, role of teacher, successful approaches. In order to analyze, we created a matrix of the resulting corpus consisting of four main aspects: source, research context, method, outcomes/conclusions. Thirdly, the information is then *synthesized* to check convergences, divergences, and to identify the theoretical gaps. Underpinned by this research, we developed a framework and prototype for a teaching intervention to foster prospective teacher's geometrical proof competence. We present this process of analysis and synthesis as linear, but the reader should consider that it is a cyclical process, see Figure 1.



Figure 1: Cyclic process of an Anasynthesis (adapted from Jeannotte & Kieran, 2017)

Framework underpinning understanding of geometrical proof

While the ability to understand, construct and validate proofs is central to mathematics, student difficulties with understanding of proof are well-recognized internationally. In order to help students to understand, teachers need to know levels of understanding to be achieved by their students, and also potential tools and tasks scaffolding their thinking. This information is helpful for the teachers/researchers in designing their learning goals and a sequence of learning activities fostering students' understanding of proofs. As a result of our literature review, we present, in this part, our framework regarding aspects and levels of understanding as frameworks to develop an assessment model to capture students' understanding of geometrical proof, and, next to that, potential tools and methods which can be implemented in the classroom intervention to scaffold students' thinking in constructing geometric figure, emerging the meaning of geometric concepts, axioms and theorems.

Understanding of geometrical proof

Based on literature on reading comprehension of geometry proofs (Yang & Lin, 2008), the assessment model of proof comprehension for undergraduate students (Mejia-Ramos, Fuller, Weber, Rhoads, & Samkoff, 2012), and students' understanding of the structure of deductive proof (Miyazaki, Fujita, & Jones, 2017), we distinguish three essential aspects in understanding proofs, namely a *functional aspect*, a *structural aspect*, and a *communicative aspect*. These three aspects are inspired by three main aspects of proof and proving proposed by Miyazaki et al. (2017): “*understanding a proof as a structural object, seeing proof as intellectual activity and the role, functions, and meaning of proof and proving*” (Miyazaki, Fujita, & Jones, 2017, page 225).

First of all, the *functional aspect* regards roles or functions of proofs. The ‘*function*’ of proof means the meaning, purpose and usefulness of a proof. In the context of learning, students should regard proof as a meaningful activity, experiencing the functionality (usefulness) of the activities, they are involved in. Second, the *structural aspect* refers to the deductive structure of proofs. Miyazaki et al. (2017) see the structure of a proof as a network of *singular* and *universal propositions* between *premises* and *conclusions*, connected by *universal instantiation* and *hypothetical syllogism*, see Figure 2. That means that the singular propositions are *universally instantiated* from universal proposition and then these singular propositions are connected by a *hypothetical syllogism*. Students need to know the meaning of these terms and statements (e.g. axioms, theorems) involved in a proof, recognize the status of the statements (e.g. premises, conclusions), to be able to justify the claim.

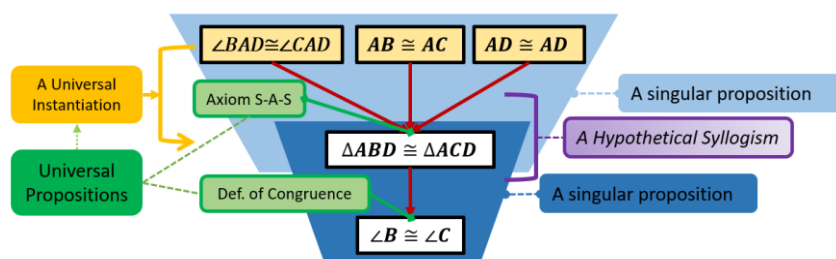


Figure 2: An example of flow-chart of proof visualizing the structure of proof

(adapted from Miyazaki, Fujita, & Jones, 2015)

The last aspect, the *communicative aspect* regards to students' intellectual activity to read and comprehend proofs, and to justify and persuade others about the validity of a proof. Through a *validating process*, students determine the truth of a proof by line-by-line checking or step-by-step checking of multi-step proofs using their understanding of the structure of deductive proofs. We interpret that this level of understanding relates to a holistic comprehension of proof where the proof is understood in terms of main ideas, methods and applications in other contexts.

Related to the level of understanding, we elaborate two models of understanding of proofs by Mejia-Ramos and colleagues (2012) and by Miyazaki and colleagues (2017). In their model of levels of structural proof understanding, Miyazaki and colleagues distinguish three levels of understanding of proof structure, namely ‘*Pre-structural*’ level, ‘*Partial-structural*’ level, and ‘*Holistic-structural*’ level. Then, they break down the second level into two sub-levels, ‘*Elemental*’

and '*Relational sub-level*'. At the first level (*Pre-structural*), students see a proof as a collection of meaningless symbolic objects. When students start to consider the components, they are at the second level, particularly the *Partial-structural Elemental* sub-level. At the *Partial-Structural Relational* sub-level, students understand hypothetical syllogisms and universal instantiations and are able to use theorems, axioms and definitions as supporting their reasoning. At the third level of understanding (*holistic-structural*), students understand the components, inter-relationships between those components and how to connect them. Then, they are able to reconstruct the proof and become aware of the hierarchical relationship between theorems and will be able to construct their own proofs.

Mejia-Ramos et al.(2012) have developed a model of assessment for proof comprehension in undergraduate mathematics. They distinguish two levels of understanding of proof, namely a *local* and a *holistic understanding*. The local understanding refers to knowing of basic terms and statements in the proof, knowing the logical status of statements in proof and the logical interrelationship between them and the statement which will be proved, and able to justify oh how the claims in the proof follows from the previous statements. Meanwhile, the holistic understanding regards being able to summarize the main idea of the proof, to identify the sub-proofs and the logical relationship between them, to adapt the idea and procedures of proof to solve other proving tasks, and to illustrate the proof regarding its relationship to specific examples.

A semiotic potential of the use of dynamic geometry system (dgs)

In terms of the structural aspects of understanding proof, several studies confirmed that the use of a Dynamic Geometry System (DGS) may help students not only solve construction problems but also helped them to understand geometrical postulates, definitions and theorems of Euclidean geometry, which are elements of the structure of proof (Jiang, 2002; Mariotti, 2012, 2013). Mariotti (2013) summarized that the semiotic potential of the features of DGS relates to specific mathematical meanings, namely "(1) the dragging test can be related to the theoretical validation of a geometric construction, (2) specific tools can be related to specific elements of the corresponding geometry theory: postulates, theorems; (3) actions concerning the management of the DGS's menu can be related to fundamental meta-theoretical actions concerning the construction of a theory, such as the introduction of a new theorem or a definition." (Mariotti, 2013, pp. 444-445). In the following paragraph, we elaborate two studies about the use of DGS in supporting students' ability in proving.

A study by Jiang (2002) investigates learning processes of two pre-service teachers, Lisa and Fred, in exploring geometry problems using the dynamic geometry software Geometer's Sketchpad (GSP) to develop mathematical reasoning and proof abilities. They use a constant comparison approach in order to analyze participants' pre-tests, post-tests, and teaching interviews indicating that the geometer's Sketchpad can not only encourage students to make conjectures but also enhances students' mathematical reasoning and proof abilities. Particularly, the use of a Dynamic Geometry System (DGS) improves students' level of geometric thinking in terms of van Hiele levels (e.g. Lisa's level increased from level 3 to level 4) and positively changes students' conceptions of mathematics and mathematics teaching. Jiang argues that pre-service teachers' experience in using DGS to foster their mathematical reasoning and proof abilities helps them to

recognize the need to improve students' knowledge of geometry, to develop their own mathematical power and their ability to develop teaching innovations.

A theoretical study by Mariotti (2012) discusses the potential offered by the use of DGS in supporting and fostering 9th and 10th grade student's proof competence in geometry. The theoretical framework used to support the use of DGS, Cabri Geometer, is her own Theory of Semiotic Mediation (TSM). In this context, students' personal meanings emerge and then evolve from personal meanings towards mathematical meanings, when students use an artefact for accomplishing a task through social interaction. Particularly, the specific DGS tools can also be related to geometrical axioms and theorems. Meanings emerging from the use of virtual drawing tools for solving geometry tasks can be related not only to the theoretical meaning of geometry construction but also the meaning of theorems.

Mariotti (2012) argues that the DGS could support not only the conjecturing process, but also mediate the mathematical meaning of conjectures, particularly premises/singular propositions in the context of geometry proofs. Particularly, the dragging feature provided by DGS (Cabri software) supports the emergence of different meanings related to the notion of conjecture as a conditional statement relating a premise and a conclusion. Mariotti also discusses findings by Baccaglioni-Frank (2010) focused on the analysis of the process of exploration that can be expected by using Maintaining Dragging (MD). Baccaglioni-Frank's teaching experiment involves students from three high schools (aged 15-18), which used Cabri in the classroom. Mariotti concludes that this teaching experiment indicates that the DGS tools and dragging activities help students to solve construction tasks and to understand the notion of theorems; particularly the mathematical meaning of conditional statements such as expressing the logical dependency between premises and conclusions.

GeoGebra as another DGS is an open-source well-developed tool with a stable interface familiar to many users and works in any operating system. The software has a number of features such as dynamic geometry which can help students in steps of problem solving towards a proof (Botana et al., 2015). Botana et al. (2015) conclude that GeoGebra tools provide some useful features. Firstly, GeoGebra could not only give yes/no answers but could also show step-by-step explanations. Secondly, GeoGebra could identify properties on the construction of geometric figures. Thirdly, GeoGebra could give a counterexample to check the truth of a statement. However, research findings by Doruk, Aktumen, and Aytakin (2013) show that some preservice teachers highlighted some limitations, such as difficulties in translating mathematical expressions into GeoGebra. Preservice teachers thought that GeoGebra is a complicated program and that it would take a long time and needs a big effort to become competent in GeoGebra.

A scaffolding potential of the use of flow-chart proof form with open problems

A flow-chart proof form is a means to visualize the deductive connections from premises to conclusion by identifying singular and universal propositions in the chart, see Figure 2. Flow-chart proofs show a storyline of the proof starting with premises from which the conclusion is deduced and includes the theorems or/and axioms being used, how the premises/hypotheses and conclusion are connected, and so on (Miyazaki et al., 2015). Gardiner (2004, cited in Miyazaki, Fujita, &

Jones, 2017) claims that this format is a good starting point to learn other formats of proofs such as narrative proofs and two-column proofs.

Miyazaki, Fujita, and Jones (2012) developed a learning progression based on flow-chart proving aimed at providing a basis for introducing the structure of proof in Grade 8 school geometry. Based on the theoretical underpinning of the design, researchers proposed three phases of learning progression: (1) constructing flow-chart proofs in an open problem, (2) constructing a formal proof by reference to a flow-chart proof in a closed situation, (3) refining formal proofs by placing them into a flow-chart proof format in a closed situation. The term '*open*' refers to a situation where students can construct more than one suitable proof. In the open problem task, students are given a conclusion of the proof in the form of flow-chart proof format, and they are asked to determine the suitable statements to fill in the blank boxes of the flow-chart so the proof is complete. Based on three phases of learning progression, the authors designed nine lessons considering open/closed situations, varying steps of deductive reasoning, and different problems and contexts. The results from a Math test of Japanese Survey Item shows that students who followed the nine lessons are more likely to plan and construct a proof in accordance with their plan. This is due to (a) their experience with open problems that encourage them to think backward and forward to identify assumptions and conclusions in proof, (b) they could grasp the structure of proofs better through using flow-chart proofs.

Another research by Miyazaki et al. (2015), as follow-up of their previous study (Miyazaki, Fujita, & Jones, 2012), showed the role of *flow-chart with open problem* as scaffolding to support Grade 8 students' learning about geometrical proofs. The results of data analysis of students' activities during a classroom intervention indicates that flow-chart proof with open problems as scaffolding enhances students to understand the structure of proof by providing a visualization of both the connection between singular propositions (via hypothetical syllogism) and the connections between a singular proposition and the necessary universal proposition in the form of universal instantiation.

Concluding Remarks

We distinguish two levels of proof understanding which are proposed by Mejia-Ramos et al.: *a local understanding* related to knowing the definition of basic concepts/terms, knowing the logical status of statements in proof, knowing how and why each statement connects to previous statements. and *a holistic understanding* when students are able to summarize the main idea of the proof, to identify the sub-proofs and how these relate to the proof structure, to transfer the idea of proof to others, and instantiate the proof with examples (Mejia-Ramos et al., 2012; Miyazaki et al., 2017). Classroom interventions, supporting students' activities in understanding the structure of proofs including the elements of proofs such as singular propositions (premises/geometric statements), universal proposition (geometric definition, axioms, theorems) and hypothetical syllogism (the inter-relationship of these elements), are needed to help students reach both levels of proof understanding.

Several studies confirmed that the use of a Dynamic Geometry System (DGS), such as Geometer's Sketchpad, Cabri and Geogebra, may help students not only solve construction problems but also may help them identify properties on the construction of geometric figures, understand geometrical

axioms, definitions and theorems of Euclidean geometry, which are elements of the structure of proof, and universal propositions (Botana et al., 2015; Jiang, 2002; Mariotti, 2012, 2013).

The use of the flow-chart proof form may provide opportunities for students to understand the structure of proof and may help students identify the components of proofs and the inter-relationship among the components (Miyazaki et al., 2015; Miyazaki et al., 2017). The use of open problems provides an opportunity for students to construct multiple solutions by deciding about the given statements and intermediate propositions necessary to deduce a given conclusion. This ability promotes student thinking forward and backward interactively when constructing a proof under the flow-chart proof format (Cheng & Lin, 2007; Heinze et al., 2008).

A study by Miyazaki et al. (2017) suggests that the level of understanding of the structure of proof as a part of local understanding should be taken into consideration in designing an effective learning intervention. Mariotti (2012) also recommends future research to better describe how the complex web of meanings emerging from activities with the DGS may be transformed into mathematical meanings such as geometric definitions, axioms and theorems. Hence, the findings presented in this paper indicate our future promising research direction for the study on designing a learning intervention which implements the use of DGS, GeoGebra, as semiotic tools and flow-chart proof format as a scaffolding tool to support a local and holistic understanding of prospective mathematics teachers who have little mathematical proof background.

References

- Baccaglini-Frank, A. (2010). *Conjecturing in dynamic geometry: A model for conjecture-generation through maintaining dragging*. Doctoral dissertation, University of New Hampshire.
- Botana, F., Hohenwarter, M., Janičić, P., Kovács, Z., Petrović, I., Recio, T., & Weitzhofer, S. (2015). Automated Theorem Proving in GeoGebra: Current Achievements. *Journal of Automated Reasoning*, 55(1), 39–59.
- Cheng, Y.-H., & Lin, F.-L. (2007). The Effectiveness and Limitation of Reading and Coloring Strategy in Learning Geometry Proof. *Proceedings of the 31 Conference of the International Group for the Psychology of Mathematics Education*, 2, 113–120.
- Doruk, B. K., Aktumen, M., & Aytakin, C. (2013). Pre-service elementary mathematics teachers' opinions about using GeoGebra in mathematics education with reference to "teaching practices." *Teaching Mathematics and Its Applications*, 32(3), 140–157.
- Doruk, M., & Kaplan, A. (2015). Prospective mathematics teachers' difficulties in doing proofs and causes of their struggle with proofs. *Bayburt University Journal of Education Faculty*, 10(2), 315–328.
- Güler, G. (2016). The Difficulties Experienced in Teaching Proof to Prospective Mathematics Teachers: Academician Views. *Higher Education Studies*, 6(1), 145–158.
- Heinze, A., Cheng, Y. H., Ufer, S., Lin, F. L., & Reiss, K. (2008). Strategies to foster students' competencies in constructing multi-steps geometric proofs: Teaching experiments in Taiwan and Germany. *ZDM - International Journal on Mathematics Education*, 40(3), 443–453.
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, 96(1), 1–16.

- Jiang, Z. (2002). Developing Preservice Teachers' Mathematical Reasoning and Proof Abilities in the Geometer's Sketchpad Environment. In D. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant & K. Nooney (Eds.), *Proceedings of the 24th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 717–729). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Knapp, J. (2005). *Learning To Prove in Order To Prove To Learn*. Retrieved from https://mathpost.asu.edu/~sjgm/issues/2005_spring/SJGM_knapp.pdf
- Mariotti, M. A. (2012). Proof and proving in the classroom: Dynamic Geometry Systems as tools of semiotic mediation. *Research in Mathematics Education*, 14(2), 163–185.
- Mariotti, M. A. (2013). Introducing students to geometric theorems: How the teacher can exploit the semiotic potential of a DGS. *ZDM - International Journal on Mathematics Education*, 45(3), 441–452.
- Mejia-Ramos, J. P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, 79(1), 3–18.
- Miyazaki, M., Fujita, T., & Jones, K. (2012). *Introducing the structure of proof in lower secondary school geometry: a learning progression based on flow-chart proving*. Paper presented in ICME 2012 (pp. 2858–2867). Seoul, Korea: ICMI.
- Miyazaki, M., Fujita, T., & Jones, K. (2015). Flow-chart proofs with open problems as scaffolds for learning about geometrical proofs. *ZDM - Mathematics Education*, 47(7), 1211–1224.
- Miyazaki, M., Fujita, T., & Jones, K. (2017). Students' understanding of the structure of deductive proof. *Educational Studies in Mathematics*, 94(2), 223–239.
- Miyazaki, M., Fujita, T., Jones, K., & Iwanaga, Y. (2017). Designing a web-based learning support system for flow-chart proving in school geometry. *Digital Experiences in Mathematics Education*, 3(3), 233–256.
- NCTM. (2000). Principles and Standards for School Mathematics. *School Science and Mathematics*, 47(8), 868–279. Retrieved from www.nctm.org
- Samkoff, A., & Weber, K. (2015). Lessons learned from an instructional intervention on proof comprehension. *Journal of Mathematical Behavior*, 39, 28–50.
- Weber, K. (2002). Student Difficulty in Constructing Proofs : The Need for Strategic Knowledge. *Educational Studies*, 48, 101–119.
- Yang, K. L., & Lin, F. L. (2008). A model of reading comprehension of geometry proof. *Educational Studies in Mathematics*, 67(1), 59–76.
- Baccaglini-Frank, A. (2010). *Conjecturing in dynamic geometry: A model for conjecture-generation through maintaining dragging*. Doctoral dissertation, University of New Hampshire.
- Zaslavsky, O., Nickerson, S. D., Stylianides, A.J., Kidron, I., & Winicki-Landman, G. (2012). The Need for Proof and Proving: Mathematical and Pedagogical Perspectives. In G. Hanna & M. de Villiers (Eds.), *Proof and Proving in Mathematics Education* (Vol.15, pp. 215-229). Netherlands: Springer.