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# Introduction to the papers of TWG01: Argumentation and Proof

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## Introduction

The role and importance assigned to argumentation and proof in the last decades internationally has led to a variety of approaches to research in this area, which is reflected in the growing number of submissions to Thematic Working Group 1 (TWG1) on "argumentation and proof". The 30 papers and 11 posters presented in TWG1 come from 16 countries and offer a wide spectrum of perspectives. These contributions intertwine educational issues with explicit references to mathematical, logical, historical, philosophical, epistemological, psychological, curricular, anthropological, and sociological viewpoints.

Taking into account this diversity, the paper contributions were presented and discussed in working sessions (some of which were parallel) organized under the following themes: (1) argumentation and proof at the school level, (2) argumentation and proof in teacher education, (3) tools for analysing argumentation and proof, (4) task design in argumentation and proof, (5) theoretical and philosophical issues of argumentation and proof, (6) assessment issues of argumentation and proof, and (7) intervention studies on argumentation and proof. Since the themes are intertwined, a paper could be assigned to multiple themes. Therefore, the assignment of papers to themes was guided by a "best fit" approach, as well as practical considerations.

In this introductory chapter, we organize our subsequent discussion across three broad topics that emerged from the discussions we had in our TWG and cut across the aforementioned themes. These topics are the following: (1) argumentation and proof in the society, (2) argumentation and proof in school, and (3) argumentation and proof in research. We will briefly discuss each topic separately. In our discussion, we will refer to a few papers that help illustrate broader points, but it was not our intention here to refer to or discuss all the papers. Rather, we hope that this discussion will spark the readers' interest to explore all the papers in the Proceedings under TWG1.

## Discussion of Papers

### Argumentation and proof in the society

Argumentation and proof can be considered within a purely mathematical realm, but such a view might be limited when one considers a larger purpose for *the role of argumentation, reasoning, and proof in the society.* These considerations would ultimately affect the teaching and learning of proof as a way to achieve certain societal goals. However, a first step in this direction is clarifying what

we mean by argumentation and proof. This critical question has been historically addressed by TWG1 members since the group was established back in 1998, however the focus of the discussion has shifted from trying to reach a consensus towards identifying key features and factors that affect our understanding of the concepts of argumentation and proof. Specifically, the group discussed the cultural origins of our perception of proof, including cultural differences in what types of arguments are considered as proof. In this regard, the role of proof by contradiction has been a focus of several group discussions fostered by the papers of Turiano and Boero, and of Hamanaka and Otaki.

Another key feature of argumentation and proof addressed by the group participants was the *relationship between logic and proof with connection to socio-cultural linguistic structures*. The different grammar and/or syntax of different (natural) languages may affect the transition between oral and written communication, influence how students understand statements, their proofs, as well as the relationships between definitions and proofs. These issues were reflected in the works of Hein, of Kempen, Tebaartz and Krieger, and of Dilberoglu, Haser, and Cakiroglu. Furthermore, the influence of culture and language on the perception of proof was considered broadly in the TWG1 discussions, as initiated by various papers, including those of Stubbemann and Knipping, of Asenova, and of Shinno, Miyakawa, Mizoguchi, Hamanaka, and Kunimune. Reference was made not only to the differences between countries and languages, but also to the differences between formal and informal uses of argumentation and proof, as well as to the use of natural logic in everyday life contrasted with academic settings that require greater formalism. Further distinctions were made with regard to its use in pure mathematical courses and in teacher preparation courses.

It is important to consider the relationships of argumentation and proof with society, in particular, when considering the goals of teaching proof to pre-university students who are the citizens of the future societies. Why foster proof? Is our goal mainly technical or do we, as a society, seek to influence students' thinking about how (mathematical) truths are established and, hence, do we mean to foster critical reasoning (for example, when making inferences about statistical data as illustrated in the study of Krummenauer and Kuntze)? Though the group seemed to agree that logical reasoning, argumentation, and proof might provide students with valuable tools for active citizenship, the question of appropriately developing those reasoning competencies to foster a harmonious transfer within and outside of mathematics remains open.

**Argumentation and proof in school**

The second broad topic, argumentation and proof in school, was a common theme in many of the papers and posters presented in the group, but from different perspectives. In particular, the following four perspectives were raised in the discussions across the presentations.

*Why foster proof?* Besides the above-mentioned societal considerations and the fact that proof is central to modern mathematics, the explanatory role of proof, as discussed in the paper of Müller-Hill, is also prominent in school mathematics. The reasoning behind teaching mathematical proof to all students, however, needs to be more clearly conceptualized and justified.

*What is proof?* There are different understandings of what proof is or can be, depending on personal experience, but also on factors such as specific area of mathematics considered and/or the educational level – primary school students' reasoning (see for example Jablonski and Ludwig's contribution) is very different from secondary school students' work on geometry. Proof requires

mathematical meta-knowledge (see, for example, Stubbemann and Knipping's paper), and our group discussed that it would be valuable to develop instructional approaches that promote learners' meta-mathematical knowledge in proving.

*Role of the teacher*. It is the teacher who, implicitly or explicitly, implements different socio-mathematical norms in a classroom, highlights key ideas in work with proof and can assign value to certain perspectives in mathematics. The studies of Lee, of Bersch and of Lekaus take up different aspects of teachers' views on the role of argumentation and proof in, respectively, Hong Kong, Germany and Norway, while Larsen and Østergaard discuss how teachers' questioning influences students' opportunities to reason. Moreover, the role of the teacher with respect to both the official and shadow education systems was considered in the study by Moutsios-Rentzos and Plyta. Despite the teachers' efforts, proving is difficult for students to learn and master, and, as a result, students often struggle with it and may not value work on proof in mathematics classrooms. Studies in TWG1, such as the one by Yan and Hanna, provide an insight into students' views about proofs.

*Task design – how can it foster proof?* There are several critical questions to be considered in designing of proof tasks: For whom? For what purpose? In what culture? Tasks need to be designed in order to foster the need for arguments, as discussed for example in the study presented by Cramer. Kempen, Tebaartz, and Krieger examine in their study how the phrasing of a proving task influences students' proof productions, while Komatsu, G. Stylianides, and A. Stylianides propose principles for the design of tasks that promote assumptions in mathematical activity. However, promoting reasoning and proving in school demands more than good tasks, as for example shown in the study of Buchbinder and McCrone. To support teachers' practices, there is a need for appropriately designed material that help teachers to enact the tasks in classroom.

**Argumentation and proof in research**

During the conference, there were many discussions on argumentation and proof in mathematics education research. A main issue concerns *methodological challenges* and the choice of *theoretical models* in approaching the proof and proving phenomena, including theorising and analysing. Toulmin's model has been used widely in mathematics education research projects on proof and proving (see paper by Jablonski and Ludwig). Due to its frequent application, the reasons for using this model in a given research are not often discussed. We collectively agreed on the importance of justifying methodological decisions for several reasons. First, the epistemic dimension of the mathematical subject area involved in the research and the role of data in the model are depending on the subject area (e.g. statistical versus geometrical reasoning). Finally, Toulmin's model has not been developed to be applied specifically for mathematics. While an advantage of this model lies in its opportunities to connect everyday argumentation to mathematical argumentation, in some cases (e.g. for looking at logical relations) other models more mathematically-oriented might be more suitable. These questions have been discussed in this TWG in previous CERMEs, as presented in Chapter 6 of the ERME anniversary book, *Developing research in mathematics education*, were the authors recall the proposal by Boero and his colleagues to articulate Toulmin's model with Habermas's rationality model (for the latter see Boero and Turiano's paper). Considering proof at the interface between mathematics and computer sciences raised also the need for appropriate

methodological tools such as that developed in the paper by Modeste, Beauvoir, Chappelon, Durand-Guerrier1, León, and Meyer.

When we move to *proof at elementary school level*, new questions arise, necessitating to reconsider the definition of proof and the possibility to identify students' practices that can be qualified as proof and proving. Epistemological considerations motivate the consideration of proof at the elementary school level: looking for what is invariant is at the core of doing mathematics and, thus, proof is important at all grades. Nevertheless, we need to conceptualize proof in a broader meaning to discuss its role and possibilities in elementary school mathematics. In this respect, the work by Balacheff on the Theory of Didactical Situations and Lakatos' work on Proofs and Refutations may offer an appropriate proof development from the empirical to the intellectual through the generic. There seem to be few studies recognizing and investigating opportunities for proof and proving in the elementary school (see, for example, the works of Arnesen, Enge, Rø, and Valenta, and of Datsogianni, Ufer, and Sodian).

## Conclusions and Future Directions for TWG1

We believe that the TWG on argumentation and proof has offered the participants the richness of diversity in this research domain and the opportunity for fruitful discussions. In the last session of the TWG, the participants engaged in a discussion to identify areas in which they would like, and hope, to see more research in future CERMEs. The following areas were identified:

*The teaching of proof and argumentation* in both school and university settings, including in teacher education with particular emphasis on argumentation and proof at the elementary school level. This area covered also developmental perspectives and learning trajectories, the study of the classroom implementation of tasks rich in argumentation and proof, and how teachers can be prepared to scaffold students' learning and to respond to unexpected student responses to help develop all students' learning of argumentation and proof.

*Issues of language in argumentation and proof*, including: the role of representations, structure, oral and written language, the relationship between mathematical language and natural language, as well as the relationship between the grammatical aspects of language and logic. Due to the complexity of this question, this could be studied in an international group emerging from CERME, in collaboration with colleagues involved on this topic in the TWG9 -Mathematics and Language, and with linguists.

*Argumentation and proof in policy documents and curriculum frameworks*, including the place and role of argumentation and proof in them, expectations and recommendations, and whether these are research-informed, etc.

The identification of these broad areas is aimed at describing the state of the art of the field, without suggesting prioritizing certain areas of research. The TWG1 is committed to representing the diversity of perspectives and research areas on argumentation and proof in future CERMEs.