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# Three-wave trapponic solitons for tunable high-repetition rate pulse train generation

Fabio Baronio, *Member, IEEE*, Matteo Conforti, Antonio Degasperis, and Stefan Wabnitz, *Member, IEEE*

**Abstract**—We demonstrate that the parametric mixing of an ultra-short Zakharov–Manakov (ZM) soliton pulse with a quasi-continuous wave (CW) signal in a weakly quadratic nonlinear and dispersive medium may lead to the generation of a three-wave soliton with variable speed, or *trappon*. We show that trappons may provide a novel mechanism of cavity-less ultra-short pulse train generation. The time width, the amplitude and the repetition rate of the generated pulse trains may be controlled in a stable manner by adjusting the intensity level of the CW signal and/or the ZM short soliton pulse.

**Index Terms**—Three wave interaction, Solitons, Pulse train generation.

## I. INTRODUCTION

EFFICIENT mechanisms for the generation of high-contrast, ultra-short pulse trains are of interest in a broad range of domains, such as time-resolved spectroscopy and microscopy, selective femtosecond chemistry, quantum coherent control in high-field physics, and optical communications. Relevant application examples include quantum-path control of high-harmonic generation [1]; multi-pulse excitation of atoms, molecules, and solids [2]; multiple-laser-pulse excitation of high-gradient plasma accelerators [3]; high-fluence THz wave-train generation for radar, microwave [4] and optical communication systems. Several techniques for generating trains of ultra-short pulses at repetition rates beyond those achievable by active laser mode locking or by means of electrically-controlled modulation have been explored in recent years. For example, linear pulse train generation techniques include the repetition-rate multiplication of a lower rate source by applying amplitude [5], [6] or phase-spectral filtering [7], [8], [9]. Moreover, different all-optical techniques that may result in up to THz pulse train rates have also been proposed by using both quadratic and cubic nonlinear media. Consider for example induced modulation instability [10], [11], multiple four-wave-mixing [12], [13], and backward quasi-phase-matched second harmonic generation [14], [15].

In this Paper, we present a novel nonlinear technique for the flexible generation of a train of ultra-short optical pulses

with light-controlled time width, amplitude and repetition rate. This technique originates from the interaction of three waves at resonance. Though this process is of interest in various physical contexts, our treatment here is phrased in the language of nonlinear optics where its potential applicability appears to be higher. The physical model describing three-wave interactions (TWI) is completely integrable by spectral methods [16], and it exhibits soliton solutions, i.e. particular waveforms that preserve their shapes upon collisions. The simplest of such soliton solutions of the TWI equations was discovered by Zakharov and Manakov (ZM), and it consists of hyperbolic secant pulses. We propose to exploit the parametric interaction of an ultra-short ZM soliton pulse [16] with a quasi-continuous wave signal, which collide in a weakly dispersive and nonlinear quadratic optical crystal. As we shall see, the phase-matched three-wave interaction of the ultra-short pulse with the CW signal may lead to the high-contrast modulation of the CW signal into a sequence of ultra-short pulses. Quite remarkably, the above pulse train generation process is analytically described in terms of the adiabatic reshaping of the initial single-wave ZM soliton into a three-wave locked soliton with variable speed, which is also known as the *trappon* [17]. By exploiting the degrees of freedom of such trapponic soliton solutions, we shall demonstrate here that the time width, amplitude and repetition rate of the generated pulse train may be controlled in a stable manner by simply adjusting the parameters of the two input pulses, namely the intensity level of the CW signal, and/or the intensity level and time width of the short ZM soliton pulse.

## II. THREE-WAVE INTERACTION EQUATIONS

The dimensionless equations that describe the quadratic resonant interaction of three waves in a nonlinear optical medium read as

$$\begin{aligned} \frac{\partial A_1}{\partial \xi} + \delta_1 \frac{\partial A_1}{\partial \tau} &= iA_2^* A_3, \\ \frac{\partial A_2}{\partial \xi} + \delta_2 \frac{\partial A_2}{\partial \tau} &= iA_1^* A_3, \\ \frac{\partial A_3}{\partial \xi} + \delta_3 \frac{\partial A_3}{\partial \tau} &= iA_1 A_2, \end{aligned} \quad (1)$$

with

$$A_j = \pi \chi^{(2)} \sqrt{\frac{n_j \omega_1 \omega_2 \omega_3}{n_1 n_2 n_3 \omega_j}} E_j. \quad (2)$$

Here  $\tau = t/t_0$ ,  $t_0$  is an arbitrary time scale;  $\xi = z/z_0$ ,  $z_0$  is an unit space-propagation parameter.  $E_j$  is the slowly varying

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electric field envelope of the wave at frequency  $\omega_j$ ,  $n_j$  is its refractive index,  $\chi^{(2)}$  is the quadratic nonlinear susceptibility,  $\delta_j = z_0/(v_j t_0)$  ( $v_j$  being the linear group velocity), and  $j = 1, 2, 3$ . We assume that the group velocity  $v_3$  of the wave with the highest frequency ( $\omega_3 = \omega_1 + \omega_2$ ) lies between the group velocities of the other waves, i.e.  $v_1 > v_3 > v_2$ . With no loss of generality, we shall write Eqs. (1) in a coordinate system such that  $\delta_1 = 0$ , which implies  $0 < \delta_3 < \delta_2$ . Eqs. (1) exhibit, among others, the conserved quantities

$$U_{13} = U_1 + U_3 = \frac{1}{2} \int_{-\infty}^{+\infty} (|A_1|^2 + |A_3|^2) d\tau, \quad (3)$$

$$U_{23} = U_2 + U_3 = \frac{1}{2} \int_{-\infty}^{+\infty} (|A_2|^2 + |A_3|^2) d\tau, \quad (4)$$

which is reported here for future reference, where  $U_1$ ,  $U_2$  and  $2U_3$  represent the energies of waves at the frequencies  $\omega_1$ ,  $\omega_2$  and  $\omega_3$ , respectively.

### III. SOLITON-BASED ULTRA-SHORT PULSE TRAIN GENERATION

Let us first present numerical evidence of the train generation process. Figure 1 illustrates the possibility of obtaining an efficient modulation of the input CW signal into a train of ultra-short pulses, as a result of its parametric mixing with an ultra-short ZM soliton pulse at a different carrier wavelength. In particular, in Fig. 1 we numerically solved Eqs. (1), where the initial excitation conditions at  $\xi = 0$  were provided by a quasi-CW signal at frequency  $\omega_1$  (a standing signal in a coordinate system where  $\delta_1 = 0$ ) and a short ZM soliton pulse at frequency  $\omega_2$  (with delay parameter  $\delta_2 > 0$ ). The two input pulses were pre-delayed, so that they could collide at some distance inside the crystal. Figure 1 shows that as soon as the faster ZM soliton pulse starts to overlap in time with the slower quasi-CW signal, their parametric mixing leads to the generation of an idler short pulse at frequency  $\omega_3$ . Additionally, Fig. 1 also shows that the TWI leads to a periodic oscillation with distance  $\xi$  of the intensity of the two short pulses at carrier frequencies  $\omega_2$  and  $\omega_3$ . As a result, it turns out [18] that the centre of mass of these two pulses periodically oscillates in time around an average value, which grows larger with distance according to a common average group velocity which is apparently (see Fig. 1) different from the group velocity of the incoming short pulse. As for the incoming CW signal, Fig. 1 also shows that a train of short pulses is carved on the CW background at frequency  $\omega_1$ . Finally, Fig. 1 also illustrates that at the end of the three-wave interaction process the quasi-CW background at  $\omega_1$  is modulated into a sequence of ultra-short pulses, the soliton pulse at  $\omega_2$  returns back to its original shape, and the generated sum-frequency pulse at  $\omega_3$  vanishes.

Quite remarkably, we shall demonstrate now that the entire three-wave interaction process which is displayed in Fig. 1 may be analytically represented in terms of particular analytical TWI soliton solutions. In the notation of Eqs. (1), the arbitrary input quasi CW signal at frequency  $\omega_1$  may be described as

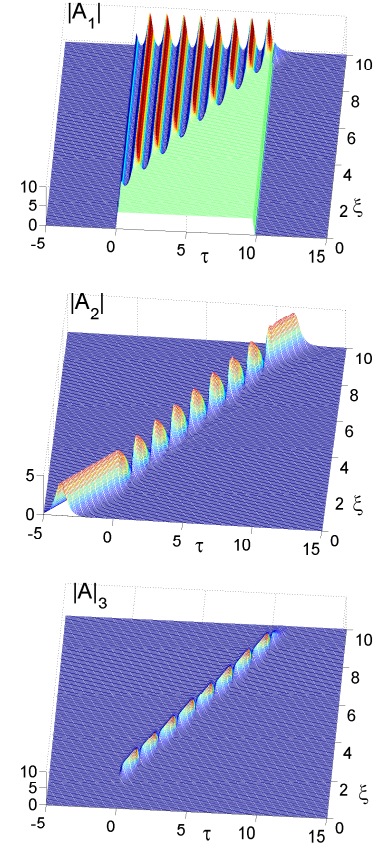


Fig. 1. Sum-frequency parametric interaction of a quasi-CW control at  $\omega_1$  and a short pulse at  $\omega_2$ . The characteristic delays are  $\delta_1 = 0$ ,  $\delta_2 = 2$ ,  $\delta_3 = 1$ .

$$\begin{aligned} A_1 &= \frac{Ce^{-i\gamma\tau}}{2} [\tanh(\frac{\tau - \tau_i}{\tau_r}) - \tanh(\frac{\tau - \tau_f}{\tau_r})], \\ A_2 &= 0, \\ A_3 &= 0, \end{aligned} \quad (5)$$

where  $C$  is the complex amplitude and  $\gamma$  is the frequency shift with respect to  $\omega_1$  of the quasi-CW signal;  $\tau_i$  ( $\tau_f$ ) and  $\tau_r$  are the switch-on (switch-off) time and the rise/fall time of the quasi-CW signal, respectively.

On the other hand, the input ZM single-wave soliton pulse [16] at frequency  $\omega_2$  reads as

$$\begin{aligned} A_1 &= 0, \\ A_2 &= 2P\sqrt{\varrho\delta_2\delta_3} \frac{e^{i\varrho\phi\tau_2}}{\cosh(2P\varrho\tau_2)}, \\ A_3 &= 0, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \varrho &= \delta_3/(\delta_2 - \delta_3), \\ \tau_2 &= -\tau + \delta_2\xi. \end{aligned} \quad (7)$$

For a given choice of the three linear group velocities, or characteristic delays  $\delta_j$ , the above ZM soliton is determined in terms of the two real parameters  $P > 0, \phi$ . The parameter  $P$  fixes both the soliton peak amplitude and its temporal width.

Whereas the parameter  $\phi$  corresponds to a phase shift which is linear in both  $\tau$  and  $\xi$ .

After the collision of the ZM pulse with the CW background, we surmise that their parametric interaction generates a three-wave trapponic soliton, which consists of a group-velocity locked bright-bright-dark triplet. Similar triplet solutions have been already considered in nonlinear optics in the form of simultons [19], [20] or boomerons [21]. Trapponic soliton solutions of the TWI Eqs. (1) may be found as discussed in the recent analysis of Ref. [18], [19]. TWI trappons, whose potential impact on nonlinear optics is suggested here for the first time, are characterized by a periodic (or trapped) spatio-temporal evolution of the energy distribution of the three waves, as well as of their locked propagation speed. As a result, the center of mass of a trappon also periodically oscillates within a limited time interval. As far as the potential applications are concerned, the most significant property of the trappon dynamics consists in the carving of a train of short pulses into the CW-background at frequency  $\omega_1$ . Indeed, as shown in the next section, the entire three-wave interaction dynamics after the collision of the short pulse with the CW background as shown by Fig. 1 may be analytically described in terms of the trappon solution

$$\begin{aligned} A_1^T &= ia\sqrt{\delta_2\delta_3}e^{iq_1\tau_1} - i\frac{\Delta}{4p}\left(\frac{\delta_2 - \delta_3}{\delta_2\delta_3}\right)A_2^*A_3, \\ A_2^T &= -\frac{2p\delta_3}{\Delta}\sqrt{\frac{2\delta_2}{\delta_2 - \delta_3}}e^{iq_2\tau_2}(H_+^* - H_-^*), \\ A_3^{T*} &= \frac{2p\delta_2}{\Delta}\sqrt{\frac{2\delta_3}{\delta_2 - \delta_3}}e^{iq_3\tau_3}(H_+e^{i\beta} - H_-e^{-i\beta}), \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Delta &= 1 + |H_+|^2 + |H_-|^2 - 2\cos(\beta)\mathcal{R}e(H_+H_-^*e^{i\beta}), \\ H_{\pm} &= e^{-(B \pm i\chi)\tau} e^{\frac{2\delta_2\delta_3}{\delta_2 - \delta_3}p\xi}, \\ \chi &= \sqrt{a^2 - p^2}, \\ B &= p\left(\frac{\delta_2 + \delta_3}{\delta_2 - \delta_3}\right), \quad \tan(\beta) = \sqrt{a^2 - p^2}/p, \\ \delta &= \frac{2\delta_2\delta_3}{\delta_2 + \delta_3}, \\ q_n &= q(\delta_{n+1} - \delta_{n+2}), \quad n = 1, 2, 3 \text{ mod}(3), \\ \tau_n &= -\tau + \delta_n\xi. \end{aligned} \quad (9)$$

For a given choice of the characteristic linear velocities in Eqs. (1), the trappon solution of Eqs. (8) is completely determined by just three independent real parameters, namely  $p, q, a$  (with the restrictions that  $p > 0$ ,  $a > 0$  and  $a^2 > p^2$ ). The parameter  $p$  is associated with the rescaling of the wave amplitudes, and of coordinates  $\tau$  and  $\xi$ . Whereas  $a$  measures the amplitude of the CW background in wave  $A_1^T$ . The parameter  $q$  adds a phase shift which is linear in both  $\tau$  and  $\xi$ .

The trappon solution (8) extends in time from  $\tau = -\infty$  to  $\tau = +\infty$  and at large distances (i.e., for  $\xi \rightarrow +\infty$ ) it exhibits a time-periodic behavior. Nevertheless, as we shall see in next section, Eqs. (8) exactly describe the mixing of the ZS soliton (6) with the quasi-CW (5) background signal over the finite interval  $[\tau_i, \tau_f]$  (which correspond to the times of turning on and off the CW wave at frequency  $\omega_1$ , respectively).

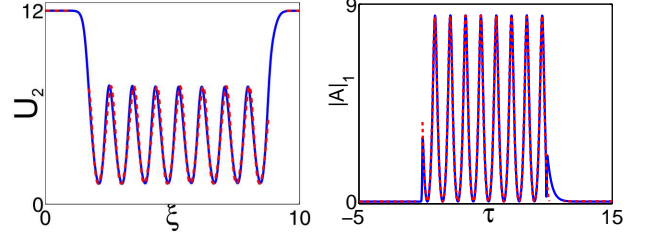


Fig. 2. Numerical evolution (continuous lines) and theoretical prediction from Eqs. (8) (dotted lines) of the energy at  $\omega_2$  and of the output pulse profile at  $\omega_1$  for the case of Fig. 1.

#### IV. THREE-WAVE ADIABATIC SOLITON SHAPING

Indeed, one may analytically predict the characteristic parameters  $p, q, a$  of the generated TWI trappon from the corresponding  $P, \phi$  parameters of the isolated input ZM soliton, and the complex amplitude  $C$  and the frequency shift  $\gamma$  of the CW signal. In order to demonstrate this, it is sufficient to suppose that the input ZM soliton adiabatically (i.e., without emission of radiation) reshapes into a trappon soliton after its collision with the quasi-CW background at a given point in time (say, at  $\tau = \tau_i$ ). In the spectral domain [16] this hypothesis is equivalent to imposing that the input ZM soliton, after the collision with the CW background, transfers its discrete eigenvalue to the trappon soliton. In the frame of this basic hypothesis, the conservative nature of the three-wave interaction permits us to suppose that: i) the energy  $U_{23}$  (4) of the input TWI soliton is conserved in the generated trappon soliton; ii) the phase of the  $\omega_2$  frequency components of the input TWI soliton and of the generated TWI soliton varies in a continuous manner across their time interface (i.e., at  $\tau = \tau_i$ ); iii) the amplitude and the phase of the CW background coincide with the corresponding values of the asymptotic plateau of the component at frequency  $\omega_1$  of the generated TWI trappon. The above three conditions, after some straightforward calculations, permit us to obtain the following equations that relate the parameters of the incident and of the transmitted solitons (i.e., before and after the collision, respectively)

$$P = p, \quad a = |C|/\sqrt{\delta_2\delta_3}, \quad q(\delta_2 - \delta_3) = \phi, \quad (10)$$

with the restrictions that  $|C|^2 > P^2\delta_2\delta_3$  and  $\gamma = \phi$ . The above conditions define the matching relations among the amplitudes and the phases of both the input ZM soliton and of the CW signal which should be satisfied in order to excite the trapponic soliton. As an example, consider the case of Fig. 1, where the CW background control is described by Eqs. (5) with  $C = 3$ ,  $\gamma = 1$ , and the input ZM soliton (6) is obtained from Eqs. (6) with  $P = 1.5, \phi = 1$ . After the collision with the CW background, Eqs. (10) predict that the generated trappon is obtained from Eqs. (8) with  $p = 1.5, q = 1$ , and  $a = 3/\sqrt{2}$ . We confirmed the accuracy of this prediction by means of extensive comparisons of the analytical expression (8) with numerical solutions of the TWI Eqs. (1). For example, Fig. 2 displays both numerically computed and analytical evolutions

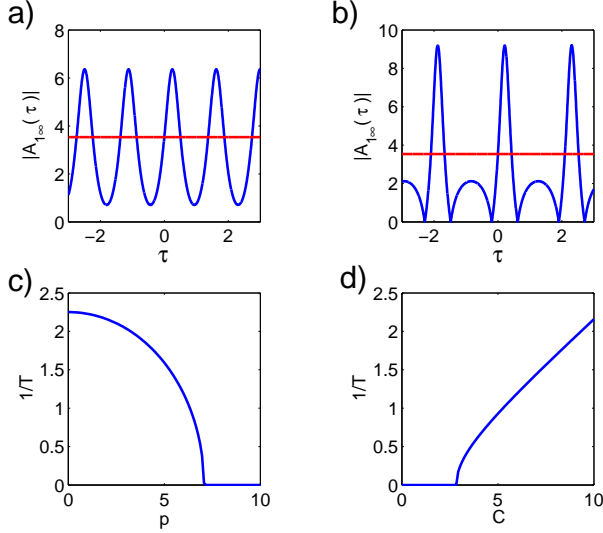


Fig. 3. Output pulse train (11) amplitude (blue line) and its time-average value (red line) for: a)  $C = 2.5\sqrt{2}$ ,  $P = 1$  and b)  $C = 2.5\sqrt{2}$ ,  $P = 2$ ; Pulse train frequency  $f = 1/T$  c) vs.  $P$  ( $C = 10$ ); and d) vs.  $C$  ( $P = 2$ ).

for the energy  $U_2$  (along the crystal length  $\xi$ ) and for the output transverse profile of wave  $A_1$  (along  $\tau$ ) (these correspond to the same interaction process previously shown in Fig. 1).

## V. CONTROLLED PULSE TRAIN GENERATION

The amplitude profile of the generated train of ultra-short pulses at frequency  $\omega_1$  may be exactly described by performing the limit for  $\xi \rightarrow +\infty$  of the first of Eqs. (8), which yields

$$|A_{1\infty}^T(\tau)| = |C| \left| 1 - 4 \cos(\beta) \frac{\sin(\pi\tau/T) \sin(\pi\tau/T - \beta)}{1 - \cos(\beta) \cos(2\pi\tau/T - \beta)} \right| \quad (11)$$

where  $|C| = a\sqrt{\delta_2\delta_3}$  (see (10)),  $\tan(\beta) = \sqrt{a^2 - p^2}/p$  (see (9)) and period  $T = \pi/\sqrt{a^2 - p^2}$ . Two different examples of the generated pulse trains as described by Eqs.(11) are shown in Fig. 3 a)-b).

Equation (11) provides relatively simple expressions that relate the parameters of the injected soliton pulse and CW background to the main features of the generated output pulse train. Indeed,  $|A_{1\infty}^T|$  is a time periodic signal (with period  $T$ ) with average value approximatively equal to  $|C|$ . Moreover, from Eqs.(11) one obtains that the peak amplitude of the generated pulses is  $M = |C| + 2\sqrt{\delta_2\delta_3}P$ ; whereas the minimum amplitude is  $m = |C| - 2\sqrt{\delta_2\delta_3}P$ . Hence the maximum intensity contrast for the generated train is obtained for  $|C| = 2P\sqrt{\delta_2\delta_3}$ ; in this case, the field amplitude at the minima vanishes and the peak amplitudes reads as  $M = 4\sqrt{\delta_2\delta_3}P$ .

In Figure 3 c)-d) we illustrate the analytically predicted dependence of the repetition rate of the ultra-short pulse train  $R = 1/T = \sqrt{|C|^2/(\delta_2\delta_3) - p^2}/\pi$ , as a function of the amplitudes of the input short pulse and of the CW signal, respectively. As it can be seen in Fig. 3 c), the repetition

rate is the largest whenever the amplitude of the input soliton vanishes. Whereas there is a critical input soliton amplitude, above which the period of the generated pulse train diverges to infinity. On the other hand, Fig. 3 d) shows that when varying the CW background amplitude, a finite repetition rate is only obtained for amplitudes  $|C|$  above a certain threshold level, and the rate grows larger with  $|C|$  in a linear fashion.

The above results show that the peak amplitude, the time width and the repetition rate of the pulse train which is generated at  $\omega_1$  may be controlled in a stable manner by simply adjusting the intensity level of the input CW signal  $|C|$  and/or the amplitude level (hence the time width)  $P$  of the input short soliton pulse.

It is worth noting the efficiency of the proposed method for generating pulse trains, since theoretically the entire energy of the pump pulse  $A_1$  is converted into the train. Indeed, from the energy conservation relation (3) and the fact that at the end of the interaction  $A_3 = 0$ , it follows that the energy of the quasi-CW pulse is conserved before and after the collision with the control pulse  $A_2$ .

It should be pointed out that the generation process we have so far described may be thought of as a way of "writing" a pulse train on a quasi CW signal. The opposite process of "erasing" such wave train to go back to a flat, quasi CW, signal is also possible. Indeed, it is easy to realize that this reverse process is well described in a similar way by the formation of the trapped soliton solution of the TWI Eqs. (1) which is obtained from the expression (8) via the invariance transformation  $\xi \rightarrow -\xi$ ,  $\tau \rightarrow -\tau$ ,  $A_1 \rightarrow A_1$ ,  $A_2 \rightarrow A_2$ ,  $A_3 \rightarrow -A_3$ .

We would like to point out that the observation of the above described three-wave phenomena appears to be readily achievable in nonlinear optical experiments. For instance, let us consider the *eee* interaction of three-waves with carrier wavelengths of  $\lambda_1 = 1.55\mu m$ ,  $\lambda_2 = 3.4\mu m$ ,  $\lambda_3 = 1.064\mu m$  in a 8cm long periodically poled bulk Lithium Tantalate crystal with 28μm periodicity. In this case, the parametric mixing of a 200fs incident pulse with a quasi-CW signal (say, with a 3ps time duration) leads to the generation of a train of 250fs pulses with a repetition rate of about 2THz; the corresponding peak power levels of the incident and of the generated fields are of the order of a few hundreds of MW/cm<sup>2</sup>. In practical terms, the upper limit to the maximum repetition rate of the generated pulse train which can be achieved by the present technique will be set by the limit of validity of Eqs. (1), which hold true in three-wave physical systems whenever the presence of group velocity dispersion within each wave can be neglected.

## VI. CONCLUSIONS

We revealed that the three-wave parametric interaction of an ultra-short ZM soliton pulse with a quasi-continuous wave (CW) signal in a weakly quadratic nonlinear and dispersive medium may lead to the generation of a trapped soliton, or trappion. Moreover, we predicted that the excitation of trapped solitons may lead to a new class of optically controlled, cavity-less sources of ultra-short pulse trains with up to multi-THz repetition rates.



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