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► **To cite this version:**

Christine Chevallereau, Yannick Aoustin. Self-stabilization of 3D walking via vertical oscillations of the hip. 2015 IEEE International Conference on Robotics and Automation (ICRA), May 2015, Seattle, United States. pp.5088-5093, <10.1109/ICRA.2015.7139907>. <hal-02397690>

**HAL Id: hal-02397690**

**<https://hal.science/hal-02397690v1>**

Submitted on 6 Dec 2019

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# Self-stabilization of 3D walking via vertical oscillations of the hip

Christine Chevallereau and Yannick Aoustin,

**Abstract**—Actual control of most humanoid robots is based on the 3D linear inverted pendulum and assumes an horizontal displacement of the center of mass of the robot while obviously the center of mass in human walking is characterized by vertical oscillations. The objective of the paper is to show that these oscillations have a crucial role for the high level control of the walk. Based on a controlled length inverted pendulum model of the walker, it will be shown that vertical oscillations induce a self stabilization of the walk while this self stabilization is not observed in the case of a horizontal motion of the center of mass. The results are essentially based on the evolution of the angular momentum throughout the walk. The decrease of the angular momentum during the change of support is determinant to introduce a dependence between the path of the center of mass and the walking velocity. For a large set of walking characteristics (stride, velocity ...) a self synchronization of the motion in the sagittal and frontal planes appears that allows a low level control to produce stable cyclic gaits.

## I. INTRODUCTION

Human walking is complex and still not well understood. Learning to walk is a long and arduous process in the beginning, but once learned, the act of walking across a level floor is second nature. Thus, it seems that human gait has self stabilisation property in order than high level control is not required. This characteristic is not current in humanoid control and is the object of this study.

The studies concerning walking of robot are very wide [6], from a passive walking on a slope [3], [11], and [17] to the complete control of a humanoid robot often based on 3D linear inverted pendulum (LIP3D), and [10], [12]. The first type of study is interesting due to its energy efficiency and also because it exploits the mechanical structure of the robot to avoid control. It is a nice illustration of the concept of computational morphology [13]. The importance of the morphology or more precisely of the characteristic of the walking gait for the self-stabilization is an element that will be investigated in this study.

Even if the humanoid robots are complex 3D system, a simplified model is useful to exhibit the main dynamic effects involved in the walking process. Two important points are the role of gravity and the limited torque available at ankle to avoid rotation of the foot. Thus the inverted pendulum model has been used since long time to study walking and running [1], [4]. The base of the pendulum corresponds to the punctual contact with the ground or to the ZMP (zero moment point or center of pressure) when a flat foot contact is modeled; the rotation with respect to

this point is free, no torques are applied. A concentrated mass is used to include the role of gravity. In bio-mechanical study an inverted pendulum with a constant length is often used while in robotics the LIP3D model proposed by Kajita [9] is popular. This model assumes a constant height of the CoM, consequently an analytical expression to define the CoM evolution exists and equations in sagittal and frontal planes are decoupled.

But in human motion, the height of the CoM is not constant [8], thus several approaches [5], [7], [14] have been proposed to extend the LIP to more human-like behavior. These studies include vertical oscillations of the concentrated mass and show self-stabilization. They are limited to a planar sagittal walking while the results presented here are extended to 3D walking. The control objective is to maintain the height of the CoM (or concentrated mass) on a prescribed sub-manifold defined as a function of the position of the CoM in the sagittal and frontal planes. A walking gait composed of single supports and changes of stance leg is considered. Based on an analyze of the evolution of the angular momentum around the contact point on the ground, analytical conditions on the self-stabilization for a periodic gait are deduced for a planar motion in sagittal plane or frontal plane. Then it will be numerically showed that self-stabilization of 3D walking appears for many gaits.

The article is structured as follows. In section II, the simplified inverted pendulum with vertical oscillations is presented for the 3D case. A normalized model is introduced. Then the effect of a vertical oscillation is studied, considering the sagittal plane only in Sect. III. Then, in place balancing in frontal plane is studied in Sect. IV. Then the synchronization of these two motions to obtain a 3D walking is addressed in Sect. V and the stability analysis is conducted. Lastly, this article ends with some concluding remarks and perspectives in Sect. VI.

## II. INVERTED PENDULUM WITH VERTICAL OSCILLATIONS

### A. The gait studied

The robot is modeled as an inverted pendulum with a concentrated mass. Its gait is composed of successive single supports and instantaneous changes of support as presented in Fig. 1. Since the legs are mass-less, the change of support is achieved without change of the velocity of the CoM. Periodic symmetric motions, essentially characterized by a step length denoted  $S$  and a step width  $D$ , are investigated. Due to symmetry, only the first single support is studied. The reference frame is placed at the stance leg tip as shown in Fig. 1. In the lateral plane the position of the CoM between both feet corresponds to  $y = \frac{D}{2}$ .

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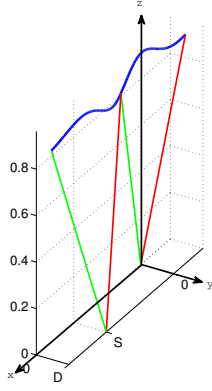


Fig. 1. The gait of the inverted pendulum is composed of successive single supports. The red (green) line corresponds to the pendulum at the beginning (end) of the single support.

### B. Proposed evolution of the mass center

In human walking the CoM oscillates in the sagittal plane. This oscillation can be represented as a sinusoidal function, the wavelength is obviously the step length. In the frontal plane, a sinusoidal function can also be used to describe the dependency between the height  $z$  and the lateral position of the CoM. For the sake of simplicity we can assume that motions along the sagittal and frontal planes are decoupled to express  $z$  as:

$$z = z_0 + f_x(x) + f_y(y) \quad (1)$$

The function  $f_x(x)$  is chosen as:

$$f_x(x) = -h_x \cos\left(\frac{2\pi(x - (S/2 + d_S))}{S} + \Phi\right); \quad (2)$$

where  $h_x$  is the amplitude of the oscillation,  $d_S$  an offset along  $x$  axis, and  $\Phi$  allows changing of support at a height different of the minimum height (see Fig. 3).

The function  $f_y(y)$  is chosen as:

$$f_y(y) = h_y \cos\left(\frac{2a_y\pi(y - D/2)}{D}\right); \quad (3)$$

where  $h_y$  is the amplitude of the oscillation and  $a_y$  defines the wavelength in frontal plane which is proportional to  $D$ . The maximal height is chosen for  $y = D/2$ . Note that in frontal plane  $y$  has only a small excursion around 0 and will never cross  $y = 0$  or  $y = D$ . The step starts at  $D/2 - d_D$  and finishes at  $D/2 + d_D$  as illustrated in Fig. 4. The constraint (1) defines a manifold which is presented in Fig. 2.

Due to our choice to use the length and width of the step to define the wavelength for  $f_x$  and  $f_y$ , we propose to introduce the dimensionless parameters  $S_{re} = \frac{d_S}{S}$ ,  $D_{re} = \frac{d_D}{D}$  and to use normalized variables  $X$  and  $Y$  to express the position of the CoM in frontal and sagittal planes:  $X = \frac{x}{S}$  and  $Y = \frac{y}{D}$ . Using these coordinates the vertical desired evolution of the CoM can be written, for any step characteristics  $S$  and  $D$  as:

$$\begin{aligned} z &= z_0 + f_X(X) + f_Y(Y) \\ f_X(X) &= h_x \cos(2\pi(X - S_{re}) + \Phi) \\ f_Y(Y) &= -h_y \cos(2a_y\pi Y). \end{aligned} \quad (4)$$

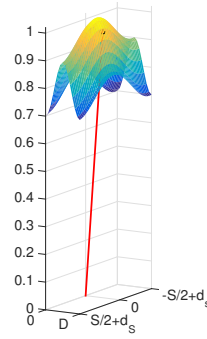


Fig. 2. Example of manifold corresponding to  $z$  defined by (1)-(3).

### C. The dynamic model

The inverted pendulum has a free rotation around the horizontal axes  $x$  and  $y$ . Under the assumption that the vertical evolution of the mass is given by (1), the evolution of the angular momentum is given by [9]:

$$\begin{cases} \dot{\sigma}_x = m\dot{z}y - mzy \\ \dot{\sigma}_y = -m\dot{z}x + mzx \\ \dot{\sigma}_x = -mgy \\ \dot{\sigma}_y = mgx \end{cases} \quad (5)$$

where  $\sigma_x$  and  $\sigma_y$  denote the angular momentums along  $x$  and  $y$  axes around the stance leg tip (see figure 1).

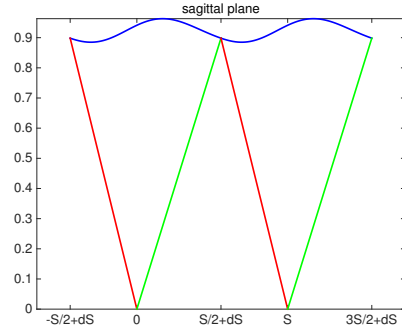


Fig. 3. Projection of the motion of the CoM for 2 steps in the sagittal plane.

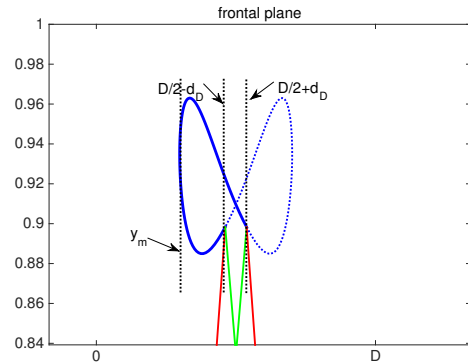


Fig. 4. Zoom on the projection of the motion of the CoM for 2 steps in the frontal plane. The combination of motion in the sagittal and frontal planes produces an eight shape motion of the CoM, the evolution of the CoM during the first single support is shown in solid line, while during the second step it is shown in dotted line.

Since we have chosen to normalized variables to express the constraints on  $z$ , it is pertinent to use also normalized variables for the angular momentum. Thus we introduce  $\epsilon_X = \frac{\sigma_x}{mD}$  and  $\epsilon_Y = \frac{\sigma_y}{mS}$ . Equation (5) can be rewritten as:

$$\begin{cases} \epsilon_X = Y \frac{\partial z}{\partial X} \dot{X} + (-z + Y \frac{\partial z}{\partial Y}) \dot{Y} \\ \epsilon_Y = -X \frac{\partial z}{\partial Y} \dot{Y} + (z - X \frac{\partial z}{\partial X}) \dot{X} \\ \dot{\epsilon}_X = -gY \\ \dot{\epsilon}_Y = gX \end{cases} \quad (6)$$

The change of support is characterized by a change of the point where the angular momentum is calculated, thus we have:

$$\begin{cases} \sigma_x^+ = \sigma_x^- - mD\dot{z}^- \\ \sigma_y^+ = \sigma_y^- + mS\dot{z}^- \end{cases} \quad \text{or} \quad \begin{cases} \epsilon_X^+ = \epsilon_X^- - \dot{z}^- \\ \epsilon_Y^+ = \epsilon_Y^- + \dot{z}^- \end{cases} \quad (7)$$

where the indices  $-$  and  $+$  denote the angular momentum before and after the change of support respectively. Equations (6) and (7), in normalized space  $X, Y, \epsilon_X$ , and  $\epsilon_Y$ , are invariant with respect to  $S, D$ , and  $m$ , thus all the following results will be independent of the step length, the step width and the mass of the robot. The model has been derived in the general case of 3D motion. To highlight the role of the vertical oscillation of the CoM, we will first study the motion in the sagittal plane only and then in the frontal plane only because analytical results for these one degree of underactuation cases can be obtained. Then the synchronization of the two motions to obtain the 3D case will be numerically studied.

### III. PLANAR WALKING IN SAGITTAL PLANE

When a planar motion in sagittal plane is studied, the robot motion along  $y$  axis is assumed to be zero. The objective of the control is to produce the vertical evolution of the CoM:

$$z = z_0 + f_X(X) \quad (8)$$

with  $f_X$  defined in (4), and to insure the change of support with the swing leg. The reference frame is attached to the stance foot, during the single support the CoM evolves from  $-1/2 + S_{re}$  to  $1/2 + S_{re}$  in normalized dimension and the swing leg touches the ground at 1 in front of the stance leg. During the single support, the evolution of the pendulum due to gravity is described by the following equation (from (6)):

$$\begin{cases} \epsilon_Y = (z - X \frac{\partial z}{\partial X}) \dot{X} \\ \dot{\epsilon}_Y = gX. \end{cases} \quad (9)$$

The two equations of system (9) can be combined into:

$$\epsilon_Y \frac{\partial \epsilon_Y}{\partial X} = g(zX - X^2 \frac{\partial z}{\partial X}) \quad (10)$$

and the integration of (10) gives the evolution of the square of the angular momentum along the single support phase as [2], [16]:

$$\begin{cases} \epsilon_Y^2(X) = \epsilon_Y^2(X^+) + V(X) \\ V(X) = 2g \int_{X^+}^X (z(\mu)\mu - \mu^2 \frac{\partial z(\mu)}{\partial \mu}) d\mu \end{cases} \quad (11)$$

When  $z(X)$  is given,  $V(X)$  can be numerically calculated and the change of angular momentum along one step (from  $X^+$  to  $X^-$ ) can be deduced:  $\epsilon_Y^2(X^-) = \epsilon_Y^2(X^+) + V(X^-)$ .

Let us consider the current step  $k$  and the previous step  $k-1$ . Combining (7) and the time derivative of (8), during the change of support, between step  $k-1$  and  $k$  the angular momentum varies as:

$$\begin{cases} \epsilon_Y^k(X^+) = \epsilon_Y^{k-1}(X^+) + \frac{\partial z(X^-)}{\partial X} \dot{X}^- = \delta_Y \epsilon_Y^{k-1}(X^+) \\ \delta_Y = \frac{\frac{\partial z(X^-)}{\partial X} (1 - X^-) + z(X^-)}{-\frac{\partial z(X^-)}{\partial X} X^- + z(X^-)} \end{cases} \quad (12)$$

with  $\delta_Y < 1$  if  $\frac{\partial z(X^-)}{\partial X} < 0$  since  $X^- > 0$  and  $z(X^-) > 0$ .

A cyclic motion will be obtained if and only if the angular momentum has the same value  $\epsilon_Y^*(X^+)$  at the beginning of following steps thus if:

$$\epsilon_Y^{*2}(X^+) = \delta_Y^2 (\epsilon_Y^{*2}(X^+) + V(X^-)) \quad (13)$$

or:

$$\epsilon_Y^{*2}(X^+) = \frac{\delta_Y^2}{1 - \delta_Y^2} V(X^-) \quad (14)$$

As a consequence, for a step defined by  $z(X)$  and  $S_{re}$  such that  $X^+ = -1/2 + S_{re}$ ,  $X^- = 1/2 + S_{re}$ , if a cyclic motion exists, it is defined by its angular momentum  $\epsilon_Y^*(X^+)$  given by (14). It can be shown [2], [16] that this cyclic motion is attractive or stable if  $\delta_Y < 1$  i.e.  $\frac{\partial z(X^-)}{\partial X} < 0$ . In this case, the angular momentum decreases at the change of support thus the angular momentum has to increase during the single support. It implies that  $V(X^-) > 0$  or roughly speaking that  $S_{re} > 0$ , the mean value of  $X$  during a single support is positive.

From the definition (4) of  $f_X(X)$  and since  $X^- = 1/2 + S_{re}$ ,  $\frac{\partial z(X^-)}{\partial X}$  depends on  $h_x$  and  $\Phi$  only but not on  $S_{re}$ . We choose  $\Phi = -0.2$ , to have  $\frac{\partial z(X^-)}{\partial X} < 0$  and thus to guaranty stability (see Fig. 3). A positive value of  $h_x$  produces a stable walk while for  $h_x=0$ , the walking is only critically stable.

It has to be noted that if the altitude  $z$  is constant  $h_x = 0$ , as in LIP, then there is no change of angular momentum at the change of support, thus  $\delta_Y = 1$ . As a consequence to have a cyclic motion, the angular momentum must also be conserved during the single support. It is conserved if:  $X^- = -X^+ = 1/2$ ,  $S_{re} = 0$ . The same path  $z(X) = 0$  can be followed at various velocity, no step duration is preferred or attractive.

The result presented in this section is a rewritten, with a pendulum model, of results previously obtained for the control of the robot Rabbit with virtual constraint [2], [16]. This type of control has shown its efficiency in experiments for several robots [15], [16].

### IV. IN PLACE BALANCING IN FRONTAL PLANE

The 3D walking involves a displacement in the sagittal plane but also a balancing in the frontal plane. The in place balancing in frontal plane is now studied, assuming that  $X$  is constant. Symmetric motions with respect to the sagittal plane are assumed when the stance leg is the right or left leg.

We consider here only the support on right leg placed at the origin of the reference frame (see Fig. 1). The objective of the control is to produce the vertical evolution of the CoM as function of the scaled variable  $Y$ :

$$z = z_0 + f_Y(Y) \quad (15)$$

and to insure the change of supports. As illustrated in Fig. 4, the support on the leg starts with the CoM at  $Y = 1/2 - D_{re}$  and finishes at  $Y = 1/2 + D_{re}$ .

During the single support, from (6), the evolution of the pendulum is:

$$\begin{cases} \epsilon_X = (-z + Y \frac{\partial z}{\partial Y}) \dot{Y} \\ \dot{\epsilon}_X = -gY \end{cases} \quad (16)$$

As in the sagittal case, these two equations can be combined and the integration of  $\epsilon_X \frac{\partial \epsilon_X}{\partial Y}$  gives the evolution of the square of the angular momentum along the single support phase:

$$\begin{cases} \epsilon_X^2(Y) = \epsilon_X^2(Y^+) + W(Y) \\ W(Y) = 2g \int_{Y^+}^Y (z(\mu)\mu - \mu^2 \frac{\partial z(\mu)}{\partial \mu}) d\mu \end{cases} \quad (17)$$

Considering a complete step, from  $Y^+ = 1/2 - D_{re}$  to  $Y^- = 1/2 + D_{re}$ , it can be noticed that  $Y$  is generally not monotonic since  $Y$  decreases until  $Y_m < Y^-$ . In the case of the in place balancing the motion obtained is shown in Fig. 5. Contrarily to Fig. 4 concerning 3D motion, a round-trip with exactly the same path can be observed in the frontal plane since  $X$  is fixed. It is remarkable that  $W(Y^-)$  does not depend on  $Y_m$ , thus starting from various initial velocities, the path followed by the CoM will have different excursions  $Y_m$  but if  $Y_m \geq 0$  then the total change of the angular momentum will be in any case given by:  $\epsilon_X^2(Y^-) = \epsilon_X^2(Y^+) + W(Y^-)$ . If  $Y_m < 0$ , the robot will fall sideward and the step cannot be achieved. Considering the current step  $k$  and previous

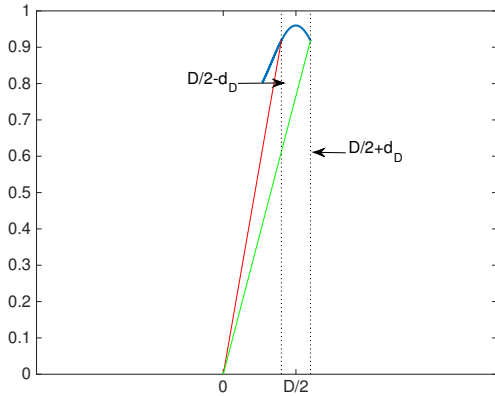


Fig. 5. Motion of the pendulum for two steps of in place balancing in the frontal plane,  $z = 0.86 + 0.1 \cos(3\pi Y)$  with  $D_{re} = 0.1$ . The red line corresponds to the pendulum at the beginning of the single support and the green line to the pendulum at the end of the single support.

$k-1$ , and combining (16) and (7), the transfer of the angular momentum at the change of support from 0 to  $D_{re}$  can be

written as:

$$\begin{cases} \epsilon_X^k(Y^+) = \epsilon_X^{k-1}(Y^+) - \frac{\partial z(Y^-)}{\partial Y} Y^- = \delta_X \epsilon_X^{k-1}(Y^+) \\ \delta_X = \frac{-z(Y^-) + \frac{\partial z(Y^-)}{\partial Y} (-1 + Y^-)}{-z(Y^-) + \frac{\partial z(Y^-)}{\partial Y} Y^-} \end{cases} \quad (18)$$

with  $\delta_X < 1$  if  $\frac{\partial z(Y^-)}{\partial Y} < 0$  and  $Y^- > 0$  since  $z(Y^-) > 0$ .

Due to symmetry consideration, a cyclic motion will be obtained if and only if the angular momentum is the same in norm at the beginning of the following step, thus if:

$$\epsilon_X^{*2}(Y^+) = \frac{\delta_X^2}{1 - \delta_X^2} W(Y^-) \quad (19)$$

As a consequence, for a step defined by  $z(Y)$  and  $Y^-$ , if a cyclic motion exists it is defined by its initial angular momentum  $\epsilon_X^*(Y^+)$  given by (19). This cyclic motion is attractive or stable if  $\delta_X < 1$  i.e.  $\frac{\partial z(Y^-)}{\partial Y} < 0$ . In this case, the angular momentum decreases at the change of support thus the angular momentum has to increase during the single support. It implies that  $W(Y^-) > 0$  or roughly speaking that  $D_{re} > 0$ . During the single support, the pendulum goes away from the stance leg. The initial angular momentum must be low enough to avoid falling down over the stance leg.

While in some way similar to the result obtained in the sagittal plane, to the best of our knowledge, these results are new. The originality is that the path of the CoM is given by the function  $z(Y)$  and the initial and final values of  $Y$  namely  $Y^+ = 1/2 - D_{re}$  and  $Y^- = 1/2 + D_{re}$  but  $Y$  is not monotonic and the limit value  $Y_m$  shown in Fig. 4 is not prescribed. The influences of  $h_y$  and  $D_{re}$  on the in place balancing are presented on Figure 6. For the case studied  $a_y = 1.5$ ,  $D_{re} > 0$ , minimal amplitude  $h_y > 0.05$  is required to induce cyclic motions, all the cyclic motions are stable.

If the altitude  $z$  is constant, as with the LIP model, then there is no change of angular momentum at the change of support, thus  $\delta_X = 1$ . As a consequence to have a cyclic motion, the angular momentum must also be conserved during the single support. Any low enough angular momentum can be chosen, it is conserved as long as:  $Y^+ = Y^- = 1/2$ . No step duration is preferred or attractive.

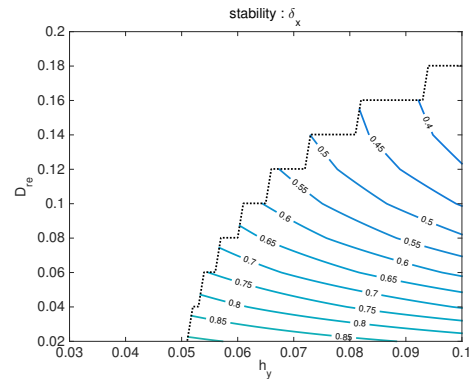


Fig. 6. Stability of the walk for several amplitudes of oscillation  $h_y$  and different  $D_{re}$ ,  $\delta_X = (h_y, D_{re})$  with  $z = 0.96 - h_y + h_y \cos(3\pi Y)$ . The black dotted line limits the set of values that corresponds to cyclic motions.

## V. SYNCHRONIZATION IN 3D MOTION

The 3D walking is a combination of a progression in the sagittal plane and an oscillation in the frontal plane. In case of walk, with a constant altitude, the motion in the sagittal and frontal planes are decoupled, as a consequence the initial velocity in the two planes can be independently adjusted to achieve the desired time duration. Thus the calculation of a cyclic motion is easy. But another consequence is that if a perturbation occurs that modifies the velocity along one direction, the motions between both planes are desynchronized and periodic walking is lost, thus 3D walking is not stable. In the presence of vertical oscillation,  $z = f(X, Y) \neq 0$  the cycling walking velocity is unique (see (14), (19)). Does this geometrical coordination via the choice of  $z = f(X, Y)$  imply a natural self-stabilisation of walking?

### A. Existence of cyclic motion

The separate studies of motions in the sagittal and frontal planes show that for a given vertical evolution of the CoM (equation (4)), the duration of one step can be adjusted by the choice of the change of support via  $S_{re}$  and  $D_{re}$ . Thus the definition of 3D walk will be stated as the choice of  $D_{re}$  and  $S_{re}$  that gives the expected step duration and solves through an optimization technique. The coupling of the dynamic model in the sagittal and frontal planes (equations (6), (7)) prevent to obtain an analytic expression for a cyclic motion contrarily to planar case. Due to the invariance of the equation with respect to  $S$  and  $D$ , when a normalized cyclic step is defined, the corresponding cyclic steps for any step length and step width can be deduced. Since all these steps correspond to the same duration, the walking velocity is accordingly adapted.

For a given vertical evolution of the CoM: (4) with  $z_0 = 0.86$  m,  $h_y = 0.1$  m,  $a_y = 1$ ,  $h_x = 0.06$  m,  $\Phi = -0.2$ , the values of  $D_{re}$  and  $S_{re}$  corresponding to cyclic motion are presented as function of the step duration on Fig. 7,  $D_{re}$  increases almost linearly while  $S_{re}$  decreases like hyperbola. The evolution of the CoM in the sagittal and frontal planes are illustrated for several step durations in Fig. 8 in the sagittal and frontal planes and by a top view.

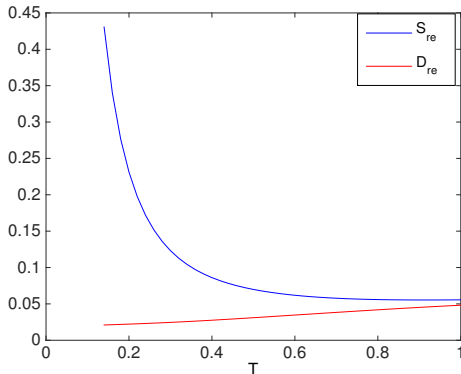


Fig. 7. For  $z = z_0 + h_x \cos(2\pi(X - S_{re}) + \Phi) - h_y \cos(2\pi Y)$  and  $z_0 = 0.86$  m,  $a_y = 1$ ,  $h_y = 0.1$  m,  $h_x = 0.06$  m,  $\Phi = -0.2$ , the values of  $D_{re}$  and  $S_{re}$  corresponding to cyclic 3D motion are presented as function of the step duration :  $D_{re}(T)$  and  $S_{re}(T)$ .

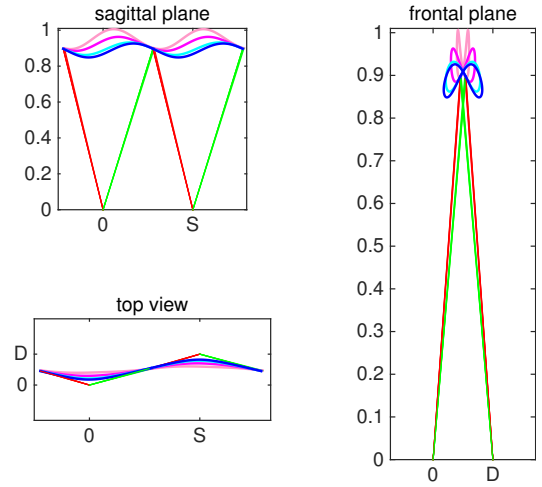


Fig. 8. Projection of the motion of the CoM for two steps in the sagittal and frontal planes and with a top view. The case  $T = 1$  s,  $T = 0.9$  s,  $T = 0.7$  s and  $T = 0.5$  s are respectively drawn with blue, cyan, magenta and pink lines.

### B. Stability of walking

The stability of the walk is studied using the Poincaré tools [16], the Poincaré section is defined just after the change the support that occurs when  $X$  reaches the value  $X^-$ . The swing foot is placed on the ground at a specified position (1, 1) wrt the stance foot. Equation (4) is invariant with respect to the equation of transition chosen. Thus after change of stance leg equation (4) and its derivative are satisfied.

The largest modulus of the eigenvalue of the Jacobian of the Poincaré return map is denoted  $\delta_{max}$ . The condition of stability is that  $\delta_{max}$  is less than 1. The effect of the vertical oscillations on the stability of the walking is illustrated in Fig. 9 via the numerically study of  $\delta_{max}$  as function of the step duration and of  $h_x$  when  $h_y = 0.1$  m,  $a_y = 1$ ,  $z_0 = 0.86$  m and  $\Phi = -0.2$ . Stability is obtained for a step duration less than 0.8 s approximately for  $h_x > 0$ . When  $h_x$  increases, a slower step can be achieved.

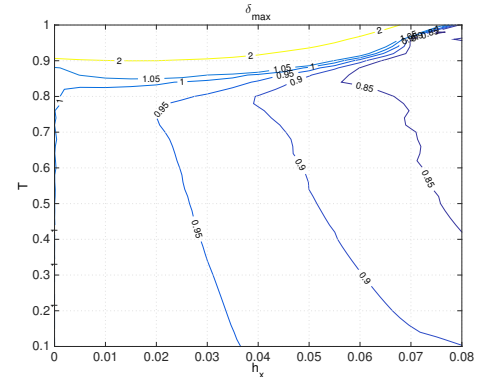


Fig. 9. The largest modulus of the eigenvalue of the Jacobian of the Poincaré Map as function of  $h_x$  and the duration of the step  $T$ ,  $\delta_{max}(T, h_x)$ , when  $h_y = 0.1$  m and  $z_0 = 0.86$  m,  $a_y = 1$ ,  $\Phi = -0.1$ .

Moreover the set of value  $(h_x, h_y)$  providing stability can be designed for several time duration of the step. The results

are presented on Fig. 10. It appears on this figure that  $h_y$  must have a high value around 0.1 m in order that a stable walk appears. For a fast walking since the lateral excursion is small, this value does not implies that the vertical amplitude of the CoM is high. For a slow walking, there is a compensation between  $h_y$  and  $h_x$  that can reduce the vertical amplitude of the vertical oscillations (see Fig. 8).

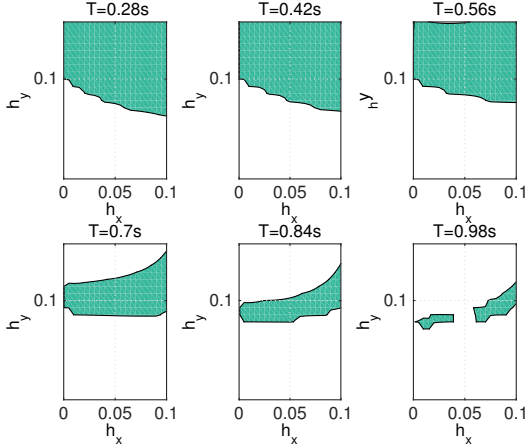


Fig. 10. Domain of stability ( $\delta_{max} < 1$ ) in green as function of  $h_x$  and  $h_y$  for six values of step duration  $T$  when  $z_0 = 0.86$  m,  $\alpha_y = 1$ ,  $\Phi = -0.2$ .

To emphasize the difference of stability of a walking with and without vertical oscillations,  $\delta_{max}$  are drawn as function of the time duration  $T$  for  $h_x = h_y = 0$  and for  $h_y = 0.1$  m,  $h_x = 0.06$  m on Fig. 11. In the second case walking gaits are stable until  $T = 0.9$  s while gaits are unstable for any step duration in the first case.

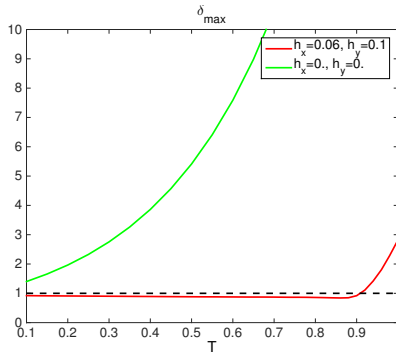


Fig. 11. Comparison of  $\delta_{max}(T)$  for a motion with a horizontal evolution of the CoM and with vertical oscillations ( $h_y = 0.1$  m,  $h_x = 0.06$  m).

## VI. CONCLUSION

Using a model of inverted pendulum with variable length, it has been shown that the vertical oscillations of the center of mass may have a crucial role in the self-stabilization of the walking on flat floor. The role of the control is simply a coordination of the legs to insure that the vertical position of the center of mass belongs to a predefined manifold, (the vertical position of the CoM depends on its position along the advance axis and lateral axis only, no temporal

evolution are imposed) and the change of supporting foot. This result appears coherent with the existence of vertical oscillations in human walk, even if the simplicity of the model does not allow to integrate the stabilizing effect of the double support phase, and the rotation of the pelvis. The normalization by the step length and step width has already permitted to consider simultaneously various walking gait. This preliminary study can be continued in different directions. The choice of functions  $f_X$  and  $f_Y$  to define the vertical oscillation may be improved. The extension to a realistic model of humanoid can probably be done using a virtual constraint as function of  $X$  and  $Y$  [16].

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