

Erratum: Landau phonon-roton theory revisited for superfluid helium 4 and Fermi gases [Phys. Rev. Lett. 119, 260402 (2017)]

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The results of the Letter on the effective ϕ - γ scattering amplitude and associated damping rate, based on the roton Hamiltonian [Eqs. (1) and (4) of the Letter], do not account for the phonon-phonon interaction, which is described, to lowest order in q , by the Hamiltonian [1]:

$$\hat{H}_{\phi-\phi}^{(3)} = \sum_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3} \delta_{\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q}_3} \frac{\mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3)}{\gamma^{1/2}} \left(\hat{b}_{\mathbf{q}_1}^\dagger \hat{b}_{\mathbf{q}_2}^\dagger \hat{b}_{\mathbf{q}_3} + \text{h.c.} \right) \quad (1)$$

with the $\phi + \phi \leftrightarrow \phi$ amplitude [2]:

$$\mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}_3) = \frac{mc^2}{\rho^{1/2}} \sqrt{\frac{\hbar^3 q_1 q_2 q_3}{32m^3 c^3}} \left(2 \frac{d \ln c}{d \ln \rho} - 1 + \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1 q_2} + \frac{\mathbf{q}_1 \cdot \mathbf{q}_3}{q_1 q_3} + \frac{\mathbf{q}_2 \cdot \mathbf{q}_3}{q_2 q_3} \right) \quad (2)$$

We have used here the Grüneisen parameter $\frac{d \ln c}{d \ln \rho} = (\rho \mu'' / \mu' + 1) / 2$. To the leading order, this adds two diagrams mediated by $\hat{H}_{\phi-\phi}^{(3)}$ to Eq. (7) of the Letter:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') &= \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5} \\ &= \mathcal{A}_2(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') + \frac{\mathcal{A}_1(\mathbf{k}, \mathbf{q}; \mathbf{k} + \mathbf{q}) \mathcal{A}_1(\mathbf{k}', \mathbf{q}'; \mathbf{k}' + \mathbf{q}')}{\hbar \omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k} + \mathbf{q}}} + \frac{\mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}'; \mathbf{k}) \mathcal{A}_1(\mathbf{k} - \mathbf{q}', \mathbf{q}; \mathbf{k}')}{\epsilon_{\mathbf{k}} - \hbar \omega_{\mathbf{q}'} - \epsilon_{\mathbf{k} - \mathbf{q}'}} \\ &\quad + \frac{2 \mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}', \mathbf{q} - \mathbf{q}'; \mathbf{q}) \mathcal{A}_1(\mathbf{k}, \mathbf{q} - \mathbf{q}'; \mathbf{k}')}{\hbar(\omega_{\mathbf{q}} - \omega_{\mathbf{q} - \mathbf{q}'} - \omega_{\mathbf{q}'})} + \frac{2 \mathcal{A}_{\phi-\phi}^{2\leftrightarrow 1}(\mathbf{q}, \mathbf{q}' - \mathbf{q}; \mathbf{q}') \mathcal{A}_1(\mathbf{k}', \mathbf{q}' - \mathbf{q}; \mathbf{k})}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'} - \hbar \omega_{\mathbf{q} - \mathbf{q}'}} \end{aligned} \quad (3)$$

The energy denominators of the two new diagrams are equal to $-\hbar \omega_{\mathbf{q} - \mathbf{q}'}$ to leading order (see note [12] of the Letter). One should thus add $-(\hbar q / mc \rho)(\rho \Delta' / 2)(2 d \ln c / d \ln \rho - 1 + w)$ to Eq. (10) of the Letter:

$$\begin{aligned} \mathcal{A}_2^{\text{eff}}(\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}') &\underset{T \rightarrow 0}{\sim} \frac{\hbar q}{mc \rho} \left\{ \frac{1}{2} \rho^2 \Delta'' - \frac{\rho \Delta'}{2} \left(2 \frac{d \ln c}{d \ln \rho} - 1 \right) + \frac{(\hbar \rho k'_0)^2}{2m_*} + \frac{\hbar^2 k_0^2}{2m_*} \left\{ \left(\frac{\rho \Delta'}{\hbar c k_0} \right)^2 uu' \right. \right. \\ &\quad \left. \left. + \frac{\rho \Delta'}{\hbar c k_0} \left[(u + u') \left(uu' - \frac{\rho k'_0}{k_0} \right) + \frac{m_* c}{\hbar k_0} w \right] + \frac{m_* c}{\hbar k_0} (u + u') w + u^2 u'^2 - \frac{\rho k'_0}{k_0} (u^2 + u'^2) \right\} \right\} \end{aligned} \quad (4)$$

This also changes the angular integral given in note [14] of the Letter: $I / (4\pi)^2 = \left(\frac{\hbar^2 k_0^2}{2m_* mc^2} \right)^2 \left[\frac{1}{25} - \frac{4\alpha}{15} + \frac{28}{45} \alpha^2 + \frac{2\beta^2}{9} + A \left(\frac{2}{9} - \frac{4\alpha}{3} \right) + A^2 + 4\beta B \left(\frac{1}{15} - \frac{\alpha}{9} \right) + B^2 \left(\frac{2}{15} - \frac{4\alpha}{9} + \frac{2\alpha^2}{3} + \frac{\beta^2}{3} \right) + \frac{2\beta}{9} B^3 + \frac{B^4}{9} \right]$, and the definition of $A = \frac{m_*}{(\hbar k_0)^2} [\rho^2 \Delta'' + \rho \Delta' (1 - 2 d \ln c / d \ln \rho)] + \alpha^2$. Accordingly, the black dashed curves in Figs. 1, 2(a), and 2(b) are multiplied, respectively, by 0.46, 0.78 and 0.85. Note that the Grüneisen parameter also appears in Ref. [3]; our coefficients of the uu' and w monomials of $\mathcal{A}_2^{\text{eff}}$ are however still in disagreement with that reference.

[1] L. Landau, I. Khalatnikov, “Teoriya vyzkostki Geliya-II”, Zh. Eksp. Teor. Fiz. **19**, 637 (1949).

[2] H. Kurkjian, Y. Castin, A. Sinatra, “Three-phonon and four phonon interaction processes in a pair-condensed Fermi gas”, Ann. Phys. (Berlin) **529**, 1600352 (2017).

[3] A. Nicolis, R. Penco, “Mutual interactions of phonons, rotons, and gravity”, Phys. Rev. B **97**, 134516 (2018).