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# STORNS: Stochastic Radio Access Network Slicing

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**Abstract**—Recently released 5G networks empower the novel *Network Slicing* concept. Network slicing introduces new business models such as allowing telecom providers to lease a virtualized slice of their infrastructure to *tenants* such as industry verticals, e.g. automotive, e-health, factories, etc. However, this new paradigm poses a major challenge when applied to Radio Access Networks (RAN): *how to achieve revenue maximization while meeting the diverse service level agreements (SLAs) requested by the infrastructure tenants?*

In this paper, we propose a new analytical framework, based on stochastic geometry theory, to model realistic RANs that leverage the business opportunities offered by network slicing. We mathematically prove the benefits of slicing radio access networks as compared to non-sliced infrastructures. Based on this, we design a new admission control functional block, STORNS, which takes decisions considering per slice SLA guaranteed average experienced throughput. A radio resource allocation strategy is introduced to optimally allocate transmit power and bandwidth (i.e., a slice of radio access resources) to the users of each infrastructure tenant. Numerical results are illustrated to validate our proposed solution in terms of potential spectral efficiency, and compare it against a non-slicing benchmark.

## I. INTRODUCTION

Upcoming service requirements from vertical industries call for a novel design of mobile networks, namely 5G. Key-enablers have been identified as network programmability and virtualization: the former brings the benefits of automation and reactivity of software modules, allowing to (re)configure mobile networks dynamically while in operation; the latter overcomes the limitations of monolithic network infrastructures by abstracting the concept of “network function” and providing flexibility in composing, placing and managing these functions. In this context, the novel definition of *network slicing* [1] encompasses such new requirements and constitutes an enabler for potential economical benefits. New vertical industries, e.g., automotive, e-health, factories etc., are entering into the telecom market and are disrupting the traditional business models of telecom operators. They are forcing infrastructure providers to open their networks to tenants, a solution that provides incentives for monetizing the availability of isolated and secure (virtualized) network slices [2], [3].

This new disruptive concept has spurred research interest in both academic and industrial communities. Its realization, however, requires the solution of a number of technical challenges that, for the time being, are not completely resolved [4]. In the future, telecom providers envision an increasing demand for end-to-end network slices, which involve heterogeneous service level agreements (SLAs) comprising different key performance indicators (KPIs), such as throughput, latency and reliability [5]. However, this requires appropriate automated admission control and resource allocation protocols for designing efficient network management systems [6]. In particular,

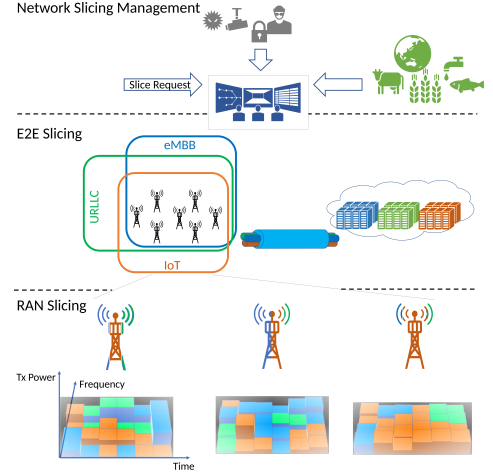


Fig. 1: Illustration of the Network Slicing Concept

the resource management of the radio access network (RAN) is one of the most challenging aspects that needs to be dealt with. In [7], a first network slicing brokering solution has been presented for automated network slicing admission control decisions. We build on that architecture to illustrate in Fig. 1 some of the challenges that need to be solved for efficiently slicing the RAN. The available resources of the air interface can be sliced at multiple levels: in frequency, time, and power domains. Slicing at this level of granularity requires to account for the cellular network topology, the other-cell interference and the radio channel conditions experienced by the users of every single tenant [8]. To solve this never-addressed and challenging issue, we leverage the mathematical tool of stochastic geometry and the theory of point processes [9]. To the best of our knowledge, we pioneer the design of an automated RAN slicing admission control and resource allocation scheme that provides throughput guarantees in the RAN, where the cellular network topology and the other-cell interference are taken into account.

Our main research contributions can be summarized as follows: *i)* we derive a new formulation of the network spectral efficiency with the aid of stochastic geometry tools explicitly accounting for the interplay among the transmit power of the cellular base stations (BSs), the available spectrum and the deployment densities of cellular BSs and mobile terminals (MTs) of each tenant, *ii)* we study the analytical properties of the newly proposed utility function for slicing the RAN and mathematically prove its convexity, and *iii)* we design a RAN admission control and a novel STOchastic RaN Slicing mechanisms (STORNS). STORNS accounts for slice SLAs in terms of average throughput and assigns, in an automated fashion, time/frequency resources and transmit power levels to each slice of the tenants admitted into the sliced RAN.

## II. SYSTEM MODEL

We explicitly account for the topology of cellular networks by using the mathematical tools of stochastic geometry and point processes [10]. Under a stochastic geometry framework, in particular, the locations of BSs and MTs are modeled as points of a point process with some specific spatial properties, due to its tractability.

We consider a RAN with multiple access points (i.e., the BSs), such as long-term evolution (LTE) eNBs, femto-cells, mm-wave access points. Multiple tenants are available in the network. The generic infrastructure tenant,  $i \in \mathcal{I}$ , is willing to pay for managing a “slice” of the resources of the RAN, provided that a certain slice SLA is guaranteed to it, e.g., a minimum average throughput requirement. Users  $u \in \mathcal{U}_i$  (i.e., the MTs) belong to a particular tenant  $i$  and are assumed to have density  $\lambda_{Ti}$ . We consider a single network operator making available its physical resources to the multiple tenants. In particular, each tenant accesses the RAN resources so that its users share common resources of the (same) air interface with specific privileges. Specifically, the BSs of the cellular network operate in an open access mode for all the users of each tenant. However, each BS serves different tenants in a non-overlapping frequency band and by using part of its available total transmit power. As a consequence, each tenant owns a dedicated part of the spectrum and of the transmit power, which constitute the “isolated” slice of the physical resources requested to the network operator. It is worth noting that the requests of the tenants are not related to the specific spectrum share or transmit power that are eventually assigned to them by the network operator. The tenants are, on the other hand, interested in getting a minimum required spectral efficiency (expressed in bit/sec/m<sup>2</sup>), which allows them to satisfy the specific service requirements of their own users, regardless of the presence of the other tenants in the network. In this paper, we formulate the minimum spectral efficiency requested by each tenant as a percentage of the spectral efficiency of the network without slicing the RAN, i.e., when tenants do not request any guaranteed service to the network operator. Based on these assumptions, we formulate an optimization problem and identify the optimal transmit power and spectrum to be assigned to each tenant so as to obtain the requested spectral efficiency. The solution of this system-level optimization problem provides insights on the advantages of a sliced network, and sheds light on the feasible set of spectral efficiencies that each tenant can request as a function of the network throughput without applying network slicing. In the sequel, for ease of description, the system model is introduced by considering a two-tenant scenario. The two tenants are denoted by T1 and T2. The generalization to more than two tenants is detailed in Section III-C.

### A. Cellular Network Modeling

The BSs are modeled as points of a homogeneous Poisson point process (PPP), denoted by  $\Psi_{BS}$ , of density  $\lambda_{BS}$ . The MTs of each tenant are modeled following a different homogeneous PPP, denoted by  $\Psi_{Ti}$ , of density  $\lambda_{Ti}$  for  $i = 1, 2$ .  $\Psi_{BS}$ ,  $\Psi_{T1}$  and  $\Psi_{T2}$  are assumed to be independent. The MTs are served by the BS providing the best average received

power on the downlink channel. All the other BSs transmitting over the same frequency spectrum act as interfering BSs (i.e., full-frequency reuse is considered). Each BS transmits with constant power.  $P_{tot}$  denotes the total power budget of each BS. Each BS transmits in a spectrum of total bandwidth  $B_{tot}$ . The percentage of transmit power and bandwidth used by  $T_i$  are denoted by  $P_{Ti}$  and  $B_{Ti}$  for  $i = 1, 2$ , respectively, such that  $P_{T1} + P_{T2} \leq P_{tot}$  and  $B_{T1} + B_{T2} \leq B_{tot}$ . The spectrum bands used by T1 and T2 are non-overlapping and, thus, no inter-tenant interference is available.

Let us consider a generic BS of the network. All the MTs of T1 and T2 served by this BS equally share the available transmit power and bandwidth, i.e., power and spectrum are viewed as continuous resources by the BS’s scheduler and, thus, no intra-cell interference is available<sup>1</sup>. As a result, the transmit power spectral density of the BSs of tenant  $T_i$  is  $P_{Ti}/B_{Ti}$  for  $i = 1, 2$ . This implies that a BS is off only if there are no MTs, either from T1 or T2, within its corresponding coverage region. If  $N_i$  MTs belong to tenant  $T_i$  for  $i = 1, 2$ , this implies that each MT uses bandwidth  $B_{Ti}/N_{Ti}$  and that the MTs do not interfere with each other. The other-cell interference (among BSs of the same tenant transmitting over the same spectrum) is, on the other hand, taken into account.

Based on this system model, in the next section we formulate the potential spectral efficiency (PSE), i.e., the average network throughput, in bit/sec/m<sup>2</sup> for each tenant of the network, by either using or not network slicing. In the latter case, the tenants equally share the resources of the network operator without any constraints on their minimum service requirements. In this case, in other words, tenants T1 and T2 equally share the transmit power  $P_{tot}$  and bandwidth  $B_{tot}$ .

### B. Potential Spectral Efficiency

For ease of notation, we formulate the PSE for a generic tenant whose MTs constitute a PPP of density  $\lambda_T$  and whose BSs allocate transmit power  $P$  and bandwidth  $B$ . The PSE can be formulated as follows:

$$\text{PSE}(P, B, \lambda_T) = \sum_{n=0}^{+\infty} \text{PSE}(P, B, \lambda_T | n+1) P_r\{n, \lambda_T\} \quad (1)$$

where  $\text{PSE}(P, B, \lambda_T | n+1)$  is the PSE by conditioning on the number,  $n+1$ , of MTs in a generic cell and  $P_r\{n, \lambda_T\}$  is the probability that, given a MT in a cell, there are other  $n$  MTs in it.

Let  $\gamma_I$  be the reliability threshold for successfully decoding a data packet and  $\gamma_A$  be the reliability threshold for detecting the presence of the serving BS during the cell association phase. With the aid of stochastic geometry ([10] and [11]),  $\text{PSE}(P, B, \lambda_T | n+1)$  and  $P_r\{n, \lambda_T\}$  can be formulated as follows:

$$\begin{aligned} \text{PSE}(P, B, \lambda_T | n+1) &= \lambda_T \frac{B}{n+1} \log_2(1 + \gamma_I) \\ &\times P_r\{\text{SIR}(n+1) \geq \gamma_I, \overline{\text{SNR}}(n+1) \geq \gamma_A\} \end{aligned} \quad (2)$$

<sup>1</sup>Considering continuous resources (e.g., bandwidth) makes our analysis tractable and easy to explain. However, this assumption can be relaxed by accounting for discrete resources (e.g., physical resource blocks (PRBs)) [11].

$$P_r \{n, \lambda_T\} = \frac{3.5^{3.5} \Gamma(n+4.5) (\lambda_T/\lambda_{BS})^n}{\Gamma(3.5) \Gamma(n+1) (3.5 + \lambda_T/\lambda_{BS})^{n+4.5}} \quad (3)$$

where  $\Gamma(\cdot)$  is the gamma function,  $\text{SIR}(n+1)$  and  $\overline{\text{SNR}}(n+1)$  are the signal-to-interference-ratio (SIR) and the average signal-to-noise-ratio (SNR), given the number of MTs,  $n+1$ , in a generic cell, during the information decoding and the cell association phases, respectively. They are defined as follows:

$$\text{SIR}(n+1) = \frac{(P/(n+1)) h_0/L_0}{\sum_{k \in \Psi_{BS}} (P/(n+1)) h_k/L_k \mathbf{1}_{(L_k > L_0)}} \quad (4)$$

$$\overline{\text{SNR}}((n+1)) = \frac{(P/(n+1))/L_0}{N_0(B/(n+1))} \quad (5)$$

where  $h_0$  and  $h_k$  are the fading power gains of the serving and interfering BSs of a generic MT, respectively, due to the wireless channels, which are assumed to be independent and identically distributed exponential random variables with unit mean,  $L_0 = \kappa r_0^\beta$  and  $L_k = \kappa r_k^\beta$  are the path-losses of serving and interfering BSs, respectively, where  $\kappa$  is the path-loss propagation constant,  $\beta > 2$  is the path-loss exponent, and  $r_0$  and  $r_k$  are the distances of serving and interfering BSs, and  $N_0$  is the noise-power spectral density. The indicator function  $\mathbf{1}_{(L_k > L_0)}$  accounts for the cell association strategy and implies that the path-loss of the serving BS, averaged over the fast fading, is smaller than the path-losses of the interfering BSs. It is worth mentioning that  $\overline{\text{SNR}}(n+1)$  is averaged with respect to the fast fading in order to avoid frequent handovers due to the channel variations, as usual in cellular networks.

By comparing (2) with the typical definition of PSE in [10] and [11], we note that our definition is more realistic since it accounts for non-zero values of  $\gamma_A$  and, thus, for the fact that the MTs cannot detect an arbitrary weak signal. This is a fundamental change of our modeling, which allows us to obtain an expression of the PSE that explicitly depends on the transmit power of the BSs. The PSE is reported in the following proposition.

**Proposition 1.** *The exact mathematical expression of the PSE is given in Eq. (6) below:*

$$\begin{aligned} \text{PSE}(P, B, \lambda_T) = & B \log_2(1 + \gamma_I) \frac{\lambda_{BS} L \left( \frac{\lambda_T}{\lambda_{BS}} \right)}{1 + L \left( \frac{\lambda_T}{\lambda_{BS}} \right) \Upsilon(\gamma_I, \beta)} \\ & \times \left[ 1 - \exp \left( -\pi \lambda_{BS} \left( \tau_A \frac{P}{B} \right)^{2/\beta} \left( 1 + L \left( \frac{\lambda_T}{\lambda_{BS}} \right) \Upsilon(\gamma_I, \beta) \right) \right) \right] \end{aligned} \quad (6)$$

where  $\tau_A = (\kappa \gamma_A N_0)^{-1}$  and:

$$L \left( \frac{\lambda_T}{\lambda_{BS}} \right) = 1 - \left( 1 + \frac{1}{3.5} \frac{\lambda_T}{\lambda_{BS}} \right)^{-3.5} \geq 0 \quad (7)$$

$$\Upsilon(\gamma_I, \beta) = {}_2F_1 \left( -\frac{2}{\beta}, 1, 1 - \frac{2}{\beta}, -\gamma_I \right) - 1 \geq 0 \quad (8)$$

with  ${}_2F_1(\cdot)$  denoting the Gauss hypergeometric function. *Sketch of Proof:* Eq. (6) is obtained from the definition of PSE in [11, Eq. (15)], by computing the coverage probability

with the aid of mathematical steps similar to those in [10, Theorem 1]. The difference with respect to [10, Theorem 1] lies in the non-zero value of  $\gamma_A$ , which modifies the upper-limit in [10, Eq. (2)] from  $L_0 \rightarrow \infty$  to  $\frac{1}{L_0} \leq N_0 B \gamma_A / P$ . The proof follows by taking into account the definition of  $L_0$ , i.e.,  $L_0 = \kappa r_0^\beta$ , and by solving the integral in closed-form.  $\square$

If  $\gamma_A = 0$ , it is worth noting that the PSE is independent of the transmit-power,  $P$ , of the BSs and the proposed framework simplifies to previously reported formulas in [10] and [11]. In addition, the PSE would linearly depend on the transmission bandwidth  $B$ . By considering that the BSs have a finite sensitivity for detecting the presence of the BSs, i.e.,  $\gamma_A \neq 0$ , on the other hand, we obtain a more accurate mathematical framework where  $P$  and  $B$  play a fundamental role for system optimization in the context of a multi-tenant cellular network with network slicing capabilities.

### III. SYSTEM-LEVEL OPTIMIZATION

Based on the mathematical formulation of the PSE in (6), the PSE of tenant  $T_i$  in the presence of network slicing is  $\text{PSE}_{T_i} = \text{PSE}(P_{T_i}, B_{T_i}, \lambda_{T_i})$  for  $i = 1, 2$ . Conversely, the PSE without a network slicing—MTs of T1 and T2 share the available resources of the RAN without any service requirement constraints—is  $\text{PSE}_{\text{NoSlicing}} = \text{PSE}(P_{\text{tot}}, B_{\text{tot}}, \lambda_{\text{tot}} = \lambda_{T1} + \lambda_{T2})$ .

Let us now consider a toy scenario. We assume a homogeneous network deployment, e.g., all the BSs have the same bandwidth  $B_{\text{tot}}$  and maximum transmission power  $P_{\text{tot}}$ . The tenant SLAs are formulated in terms of average PSE<sup>2</sup>, i.e.,  $\text{PSE}_{T_i} = \alpha_{T_i} \text{PSE}_{\text{NoSlicing}}$  where  $\alpha_{T_i} \geq 0, \forall i$ , which constitute the minimum spectral efficiency requirements of tenant T1 and T2, respectively. The SLAs, in particular, are expressed as a fraction of the spectral efficiency without performing network slicing, i.e., the baseline working operation of current cellular networks.

Let us introduce the short-hand notation:  $k_1^{(T_i)} = \log_2(1 + \gamma_I) \frac{\lambda_{BS} L \left( \frac{\lambda_{T_i}}{\lambda_{BS}} \right)}{1 + L \left( \frac{\lambda_{T_i}}{\lambda_{BS}} \right) \Upsilon(\gamma_I, \beta)}$  and  $k_2^{(T_i)} = \pi \lambda_{BS} (\tau_A)^{2/\beta} \left( 1 + L \left( \frac{\lambda_{T_i}}{\lambda_{BS}} \right) \Upsilon(\gamma_I, \beta) \right)$ . The following optimization problem can be formulated.

**Problem Bi-Sharing:**

$$\begin{aligned} & \text{minimize } \mathbb{1} \\ & \text{subject to } k_1^{(T1)} B_{T1} \left( 1 - e^{-\left( \frac{P_{T1}}{B_{T1}} \right)^{(2/\beta)} k_2^{(T1)}} \right) \geq \alpha_{T1} \text{PSE}_{\text{NoSlicing}}; \\ & \quad k_1^{(T2)} B_{T2} \left( 1 - e^{-\left( \frac{P_{T2}}{B_{T2}} \right)^{(2/\beta)} k_2^{(T2)}} \right) \geq \alpha_{T2} \text{PSE}_{\text{NoSlicing}}; \\ & \quad B_{T1} + B_{T2} \leq B_{\text{tot}}; \\ & \quad P_{T1} + P_{T2} \leq P_{\text{tot}}; \\ & \quad B_{T1}, B_{T2}, P_{T1}, P_{T2} \in \mathbb{R}_+; \end{aligned}$$

where the total throughput without slicing is  $\text{PSE}_{\text{NoSlicing}} = k_1^{(\text{tot})} B_{\text{tot}} \left( 1 - e^{-\left( \frac{P_{\text{tot}}}{B_{\text{tot}}} \right)^{(2/\beta)} k_2^{(\text{tot})}} \right)$ . Generally speaking, Problem Bi-Sharing provides the optimal set of values  $\mathbf{b} = \{B_i\}$  and  $\mathbf{p} = \{P_i\}$  given the tenants SLAs.

<sup>2</sup>The PSE is interpreted as the average network throughput experienced by the users of the tenant. This is a reasonable assumption when considering tenant SLAs in terms of cell throughput.

### A. The relevance of a sliced RAN

The previous example provides the baseline scenario for our analysis. It unveils important insights when applying the network slicing concept to the RAN of cellular networks.

**Lemma 1.** *The probability that the sum-PSE experienced by all the tenants is greater than the PSE experienced by a monolithic non-sliced network is greater than zero, i.e.,  $Pr\{(PSE_{T1} + PSE_{T2}) \geq PSE_{NoSlicing}\} \geq \epsilon, \forall \epsilon > 0$ .*

This lemma relies on the convexity property of the multi-variable PSE function shown in Eq. (6)<sup>3</sup>, showing the potential benefits of applying network slicing to the RAN. In some cases, slicing the RAN may increase the total experienced spectral efficiency, which, in turn, translates into higher operator's revenues. Tighter conditions on the case studies when a sliced network outperforms a monolithic network structure are formulated as follows.

**Lemma 2.** *The sum-PSE of two tenants with network slicing is always greater than the sum of the PSEs of the tenants in a non-sliced network where the tenants equally split the total transmit power and available bandwidth, i.e.,  $PSE_{T1} + PSE_{T2} > 2PSE_{NoSlicing}(P_{tot}/2, B_{tot}/2)$ , with  $B_{T1} + B_{T2} = B_{tot}, B_{T1} \neq B_{T2}$  or  $P_{T1} + P_{T2} = P_{tot}, P_{T1} \neq P_{T2}$ . If a uniform (equal) distribution of transmit power and bandwidth among the tenants is assumed, i.e.,  $PSE_{T1} = PSE_{T2}$ , then  $PSE_{T1} + PSE_{T2} = 2PSE_{NoSlicing}(P_{tot}/2, B_{tot}/2)$ .*

*Sketch of Proof:* Assuming the same MT densities, i.e.,  $\lambda_{T1} = \lambda_{T2}$  and relying on the convexity property, it yields the following:

$$PSE_{NoSlicing}\left(\left(\frac{B_{T1}}{2} + \frac{B_{T2}}{2}\right), \left(\frac{P_{T1}}{2} + \frac{P_{T2}}{2}\right)\right) \leq \frac{PSE_{T1}}{2} + \frac{PSE_{T2}}{2};$$

$$PSE_{NoSlicing}\left(\frac{B_{tot}}{2}, \frac{P_{tot}}{2}\right) \leq \frac{PSE_{T1} + PSE_{T2}}{2}. \quad (9)$$

If  $B_{T1} = B_{T2}$  and  $P_{T1} = P_{T2}$ , then  $B_{T1} = B_{tot}/2, P_{T1} = P_{tot}/2, PSE_{T1} = PSE_{T2}$ , and we have:

$$PSE_{NoSlicing}\left(\frac{B_{tot}}{2}, \frac{P_{tot}}{2}\right) \leq PSE_{T1} = PSE_{T2}\left(\frac{B_{tot}}{2}, \frac{P_{tot}}{2}\right), \quad (10)$$

where  $PSE_{T1}\left(\frac{B_{tot}}{2}, \frac{P_{tot}}{2}\right) = PSE_{NoSlicing}\left(\frac{B_{tot}}{2}, \frac{P_{tot}}{2}\right)$ . This concludes our proof.  $\square$

With the aid of this lemma, we can improve the system performance, by designing an admission control scheme for cellular networks that exploits network slicing and opportunistically admits subsets of tenants that maximize the PSE. This is discussed in Section III-C. In particular, the following important proposition holds.

**Proposition 2.** *Problem Bi-Sharing admits a feasible solution even if  $\alpha_{T1} + \alpha_{T2} \geq 1$ .*

*Sketch of Proof:* The proof is obtained by combining Lemma 1 and Lemma 2. Let us consider two tenants sharing the total available bandwidth and transmit power. From Lemma 1, there is a non-negligible probability that  $PSE_{T1} + PSE_{T2} =$

<sup>3</sup>Note that the Hessian condition for that function is not fully satisfied. Therefore, as shown in [12], it is needed to check that  $g(tx_1 + (1-t)x_2, ty_1 + (1-t)y_2) \leq tg(x_1, y_1) + (1-t)g(x_2, y_2)$ .

$\alpha_{NoSlicing}PSE_{NoSlicing}$ , where  $\alpha_{NoSlicing} > 1$ . From Problem Bi-Sharing, we obtain  $PSE_{Ti} = \alpha_{Ti}PSE_{NoSlicing}$ , and thus,  $\alpha_{T1} + \alpha_{T2} = \alpha_{NoSlicing} \geq 1$ . Therefore, Problem Bi-Sharing admits a feasible solution for  $\alpha_{T1} + \alpha_{T2} \geq 1$  if  $B_{T1} + B_{T2} \leq B_{tot}$  and  $P_{T1} + P_{T2} \leq P_{tot}$ .  $\square$

This proposition is a key-finding of this work: telecom operators can slice their radio access resources among the tenants and achieve a sum-throughput higher than that achieved without slicing the RAN, i.e., by sharing the available resources among the tenants without performance guarantees. In the next section, we discuss the solution of Problem Bi-Sharing.

### B. Lagrange Decomposition

In order to solve Problem Bi-Sharing, we propose to apply the Lagrange duality theorem [12]. Let us define the Lagrangian  $\mathcal{L} : R^m \times R^n \rightarrow R$ , where  $m = 6$  is the number of decision variables and  $n = 2$  is the number of constraints, as follows:

$$\begin{aligned} \mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1}, B_{T2}, P_{T2}) = & \\ 1 - \mu_{T1} \left( \alpha_1 PSE_{NoSlicing} - K_1 B_{T1} \left( 1 - e^{-\frac{P_{T1}}{B_{T1}} (2/\beta) k_2} \right) \right) - & \\ \mu_{T2} \left( \alpha_2 PSE_{NoSlicing} - K_1 B_{T2} \left( 1 - e^{-\frac{P_{T2}}{B_{T2}} (2/\beta) k_2} \right) \right). & \end{aligned} \quad (11)$$

We can derive the Lagrange dual function  $g(\mu_{T1}, \mu_{T2}) = \inf_{B_{T1}, B_{T2}, P_{T1}, P_{T2}} \mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1}, B_{T2}, P_{T2})$  that satisfies the constraints  $B_{T1} + B_{T2} \leq B_{tot}$  and  $P_{T1} + P_{T2} \leq P_{tot}$ . The unconstrained dual problem can be formulated as follows.

**Problem Bi-Sharing (DUAL):**

$$\begin{aligned} & \text{maximize} \quad g(\mu_{T1}, \mu_{T2}) \\ & \text{subject to} \quad \mu_{T1}, \mu_{T2} \geq 0. \end{aligned}$$

This problem can be solved by using the iterative sub-gradient update method to optimize the Lagrange multipliers  $\mu_{T1}$  and  $\mu_{T2}$ :

$$\begin{aligned} \mu_{T1}^{(k+1)} &= \left[ \mu_{T1}^{(k)} + \zeta^{(k)} \left( \frac{\partial \mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1}, B_{T2}, P_{T2})^{(k)}}{\partial \mu_{T1}} \right) \right]^+ \\ \mu_{T2}^{(k+1)} &= \left[ \mu_{T2}^{(k)} + \zeta^{(k)} \left( \frac{\partial \mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1}, B_{T2}, P_{T2})^{(k)}}{\partial \mu_{T2}} \right) \right]^+ \end{aligned} \quad (12)$$

where  $\zeta^{(k)}$  is defined as the step-size and can be chosen as follows ([13]):

$$\zeta^{(k)} = \frac{\nu^{(k)}}{\left\| \frac{\partial \mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1}, B_{T2}, P_{T2})^{(k)}}{\partial \mu} \right\|_2} \quad (13)$$

where  $\nu^{(k)} = \|\mu^{(k)} - \mu^{(k-1)}\|_2$  with  $\|\cdot\|_2$  denoting the norm-2 operation whereas  $\zeta^{(k)}$  must be greater than zero. The iterative process stops when the convergence is reached and the optimal Lagrange multipliers  $\mu_{T1}^{(*)}, \mu_{T2}^{(*)}$  are found, i.e., when  $\mu_{T1}^{(k+1)} = \mu_{T1}^{(k)}$  and  $\mu_{T2}^{(k+1)} = \mu_{T2}^{(k)}$ .

If two tenants are considered, the Lagrange multipliers can be obtained without using this recursive approach. This can be done as follows. Let us generalize the integrity constraint as  $B_{T1} + B_{T2} = B_{tot}$  and  $P_{T1} + P_{T2} = P_{tot}$ . This allows us to derive  $\mathcal{L}(\mu_{T1}, \mu_{T2}, B_{T1}, P_{T1})$ , where  $B_{T2}$  and  $P_{T2}$  can be obtained from the equalities  $B_{T1} + B_{T2} = B_{tot}$  and  $P_{T1} +$

$P_{T2} = P_{\text{tot}}$ . Therefore, we can rewrite Eq. (11) and obtain the optimal  $\mu_{T1}^{(*)}$  and  $\mu_{T2}^{(*)}$  for given  $B_{T1}$  and  $P_{T1}$ . The following is obtained:

$$\begin{aligned}\mu_{T1}^{(*)} &= \alpha_{T1}^{-1} \\ \mu_{T2}^{(*)} &= \alpha_{T2}^{-1}.\end{aligned}\quad (14)$$

Based on this finding, Problem Bi-Sharing can be reduced to an unconstrained feasibility problem as follows.

**Problem Bi-Sharing (UNCONSTRAINED):**

$$\begin{aligned}\text{minimize} \quad & 1 - 2\text{PSE}_{\text{NoSlicing}} - \frac{k_1^{(T1)}}{\alpha_{T1}} B_{T1} \left( 1 - e^{-\frac{P_{T1}}{B_{T1}} (2/\beta) k_2^{(T1)}} \right) \\ & - \frac{k_1^{(T2)}}{\alpha_{T2}} (B_{\text{tot}} - B_{T1}) \left( 1 - e^{-\frac{P_{\text{tot}} - P_{T1}}{B_{\text{tot}} - B_{T1}} (2/\beta) k_2^{(T2)}} \right) \\ \text{subject to} \quad & 0 \leq B_{T1} \leq B_{\text{tot}}; \\ & 0 \leq P_{T1} \leq P_{\text{tot}}.\end{aligned}$$

Based on this result, Problem Bi-Sharing can be solved with the aid of conventional numerical methods.

In the next section, we provide a generalized formulation of the problem for a multi-tenant system where  $\sum_i \alpha_i \geq 1$ .

### C. Generalized Problem Formulation

Let us consider a set of tenants  $i \in \mathcal{I}$  each of them requesting a network slice. The objective is to efficiently split the resources of the RAN among them. This encompasses the allocation of adequate transmit powers and spectrum bandwidths to each tenant, i.e.,  $\mathbf{p} = \{P_{Ti}\}$  and  $\mathbf{b} = \{B_{Ti}\}$ . Based on Problem Bi-Sharing, we can formulate a general optimization problem as follows.

**Problem MultiTenant-Optimizer:**

$$\begin{aligned}\text{minimize} \quad & \mathbb{1} \\ \text{subject to} \quad & \text{PSE}(\mathbf{b}, \mathbf{p}, \boldsymbol{\lambda}) \geq \alpha \text{PSE}_{\text{NoSlicing}} \\ & \|\mathbf{b}\|_1 \leq B_{\text{tot}}; \\ & \|\mathbf{p}\|_1 \leq P_{\text{tot}}.\end{aligned}$$

where  $\|\cdot\|_1$  is the norm-1 operator,  $\boldsymbol{\lambda} = \{\lambda_{Ti}\}$  is the set of user densities of the tenants and  $\alpha$  is the vector of SLA requirements of the tenants, which are formulated in terms of percentages of the PSE without performing network slicing, i.e.,  $\text{PSE}_{\text{NoSlicing}}$ . We can formulate the dual problem of Problem MultiTenant-Optimizer and calculate the optimal Lagrange multipliers  $\boldsymbol{\mu} = \{\mu_{Ti}\}$  by using the iterative function as follows:

$$\mu_{Ti}^{(k+1)} = \left[ \mu_{Ti}^{(k)} + \zeta^{(k)} \left( \frac{\partial \mathcal{L}(\boldsymbol{\mu}, \mathbf{b}, \mathbf{p})}{\partial \mu_{Ti}} \right) \right]^+. \quad (15)$$

This leads to a problem similar to Problem MultiTenant-Optimizer (UNCONSTRAINED) studied in Section III-B.

If the solution of the optimization problem consists of small values of transmit power or bandwidth, i.e.,  $P_{Ti} \approx 0$  or  $B_{Ti} \approx 0$ , the system discards the request of the slice that originates from tenant  $i$ : this is the essence of the proposed admission control protocol. To speed up the admission control phase, we propose to pre-filter the requests of RAN slices beforehand. In particular, we propose to process all the slices that need less than half of the non-sliced spectral efficiency, i.e., the

### Algorithm 1 Stochastic RAN Slicer (STORNS)

- 1) Initialise set  $\boldsymbol{\mu}$  and  $\gamma$ .
- 2) Initialise sets  $\mathbf{b} \leftarrow 0$ ,  $\mathbf{p} \leftarrow 0$  and value  $k \leftarrow 0$ .
- 3) Solve Problem MultiTenant-Optimizer (DUAL) (with INPUT  $\mathbf{b}^{(k)}$  and  $\mathbf{p}^{(k)}$ ) and get  $\boldsymbol{\mu}^{(k)}$ .
- 4) Calculate  $\boldsymbol{\mu}^{(k+1)}$  based on Eq. (15).
- 5) Update  $\zeta^{(k+1)}$  based on Eq. (13).
- 6) Solve Problem MultiTenant-Optimizer (UNCONSTRAINED) (with INPUT  $\boldsymbol{\mu}^{(k+1)}$ ) and get  $\mathbf{b}^{(k+1)}$  and  $\mathbf{p}^{(k+1)}$ .
- 7) If  $(\boldsymbol{\mu}^{(k+1)} \neq \boldsymbol{\mu}^{(k)})$ , then increase  $k = k + 1$  and Go to step (3).
- 8) Mark  $\boldsymbol{\mu}^{(*)} = \boldsymbol{\mu}^{(k+1)}$  as the optimal Lagrange multipliers.
- 9) Solve Problem MultiTenant-Optimizer (UNCONSTRAINED) (with INPUT  $\boldsymbol{\mu}^{(*)}$ ) and get the optimal solution of  $\mathbf{b}^{(*)}$  and  $\mathbf{p}^{(*)}$ .

slices with  $\alpha_{Ti} \leq 0.5$  are processed for optimal resource allocation while the others are discarded. This approach is motivated by the finding in Lemma 2. This leads the sliced RAN to have a sum-throughput that is in general close and may be higher than its non-sliced counterpart, as shown in Sec. IV-C. More precisely, our proposed admission control scheme works as follows: if, after the first assignment round, the sum-throughput of the admitted tenants is below the non-sliced PSE, then other slice requests from other tenants are considered in ascending order until the best sum-throughput is found. This is the essence of STORNS.

In Alg. 1, we provide the pseudo-code description of STORNS, which yields the set bandwidths and transmit powers for each admitted slice. Steps 3 – 7 are repeated until the Lagrange multipliers reach convergence. The speed and accuracy of the proposed algorithm are determined by the step-size  $\zeta^{(k)}$ , defined in Eq. (13). An empirical analysis of the convergence of the algorithm is provided in Sec. IV-D.

## IV. PERFORMANCE EVALUATION

We have carried out an extensive simulation study to *i)* validate the analytical framework based on stochastic geometry theory, *ii)* show the near-optimality of the proposed low-complex algorithm (STORNS) and *iii)* prove the finding that slicing the RAN may provide a higher sum-PSE compared to its non-sliced counterpart. We have implemented frameworks and algorithms by using commercial mathematical tools, such as MATLAB and MATHEMATICA. We consider different random instances of cellular network deployments based on the PPP model, as explained in Sec. II-A. Unless otherwise stated, the simulations parameters are those reported in Table I, summarizing the Urban Micro-cell model (UMi) which is in agreement with IMT ITU-R specifications [14]. The pathloss exponent is chosen based on the empirical evaluations performed in [15].

### A. Stochastic Framework Validation

In Fig. 2, the PSE in Eq. (6) is validated against Monte Carlo simulations. The simulations are obtained by considering several instances for the locations of BSs and MTs, which follow two independent PPPs. For each network realization, the potential throughput of each MT is computed based on its definition in Eq. (2). It is worth mentioning that, to make

TABLE I: Simulation parameters (ITU UMi [14])

System Parameters	Values
Inter-site distance (ISD)	200 m
Base station density ( $\lambda_{BS}$ )	$(\pi \text{ ISD}^2)^{-1}$
Mobile terminal density ( $\lambda_T$ )	$100 \lambda_{BS}$
Decoding threshold ( $\gamma_I$ )	0 dB
Detecting threshold ( $\gamma_A$ )	0 dB
Carrier frequency ( $f_c$ )	$2.1 \cdot 10^9$ Hz
Transmission wavelength ( $\lambda$ )	$\frac{3 \cdot 10^8}{f_c}$ m
Path-loss constant ( $\kappa$ )	$(\frac{4\pi}{\lambda})^2$
Noise power spectral density ( $N_0$ )	$-174$ dBm/Hz
( $\tau_A$ )	$(\kappa \gamma_A N_0)^{-1}$
Path-loss exponent ( $\beta$ ) [15]	3.5
Bandwidth ( $B_{tot}$ )	$20 \cdot 10^6$ Hz
Transmit power ( $P_{tot}$ )	43 dBm
Slice generation process (mean $\mu_{distr}$ )	10
Slice generation process (variance $\sigma_{distr}^2$ )	5
Simulation instances (STORNS)	1000

the validation sound, none of the mathematical equations in Sec. II are used. In particular, the number of users per cell is directly obtained from Monte Carlo simulations. The network spectral efficiency is obtained by summing the potential throughput of all the MTs and normalizing it to the area of the network. Fig. 2 shows a good agreement between analysis and simulations. The small inaccuracies for a large ratio of the density of MTs and BSs is due to the limited number of network realizations that can be simulated in a reasonable amount of time. We note, in particular, that PSE increases as a function of the ratio of densities of MTs and BSs, but saturates as this ratio gets large. This is due to the fact that, as the number of MTs increases, all the BSs are activated and the bandwidth allocated to each MT decreases at the same rate as the number of MTs per unit area. Mathematically speaking, the  $L(\cdot)$  function in Eq. (6) tends to one as the ratio  $\lambda_T/\lambda_{BS}$  increases towards infinity.

### B. Optimality of STORNS

In this section, we validate our algorithm, STORNS, against a benchmark optimal algorithm that is obtained by using a brute-force optimization method that is denoted by *OPT*. In particular, OPT is obtained by means of an exhaustive greedy search algorithm that explores all possible solutions of Problem *MultiTenant-Optimizer*. Due to complexity issues, we are able to employ this method for up to 6 tenants.

In Fig. 3, we show numerical results by setting  $\lambda_T = 100\lambda_{BS}$  and by considering different thresholds  $\gamma_I$  and  $\gamma_A$ . The objective is to compare STORNS and OPT as the number of tenants requesting a RAN slice increases. We assume that each additional tenant asks for a fraction  $\alpha_{T_i}$  of  $\text{PSE}_{\text{NoSlicing}}$  that is drawn from a normal distribution with mean  $\mu_{distr}$  and variance  $\sigma_{distr}$ , as defined in Table I.

The larger the number of slice requests, the higher the PSE. We observe that STORNS exhibits near-optimal performance and the gap with respect to OPT is around 6%, 8% and 12% for  $\gamma_I = \gamma_A = -5\text{dB}, 0\text{dB}$  and  $5\text{dB}$ , respectively.

### C. The RAN slicing benefits

In Fig. 4, we illustrate the performance offered by STORNS as a function of the number of tenants admitted into the cellular network. More precisely, we provide numerical evidence that STORNS is capable of appropriately admitting tenants and allocating their slices (i.e., bandwidth and power) in a way

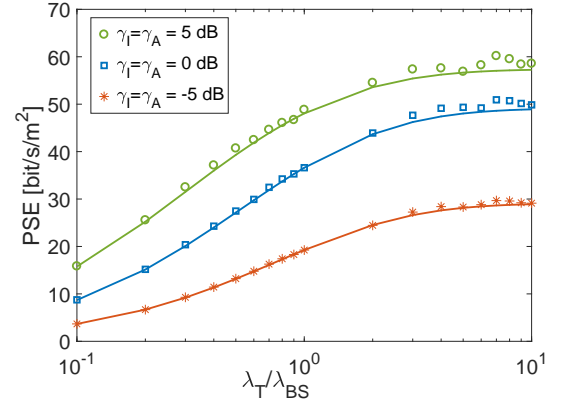


Fig. 2: Framework validation. Solid lines: Eq. (6). Markers: Monte Carlo simulations.

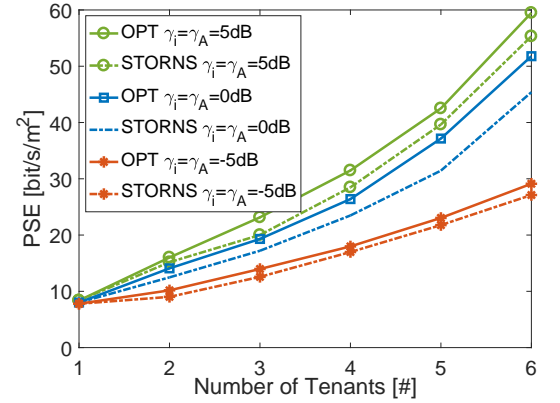


Fig. 3: Optimality study of STORNS

that the PSE of the sliced RAN is higher than its monolithic network counterpart that does not exploit network slicing. We assume a network slices demand up to 32 network slice requests. In addition, we assume that each tenant requests, on average, a network slice that provides a PSE that is 10% of the achievable PSE without using slicing. This implies that the non-sliced network would be able to admit, on average, up to ten tenants (non-shaded region in the figure)<sup>4</sup>. By using STORNS, we can accommodate a larger number of tenants and achieve a sum-PSE that is higher than the non-sliced sum-PSE (shaded region in the figure). This is possible by admitting the “best” network slice requests among the 32 available and by optimally allocating the transmit power and bandwidth to each of them. STORNS allows telecom operators to achieve up to 120% of the throughput of a monolithic non-sliced cellular network. This motivates telecom operators to use network slicing not only as a means for accommodating the specific request of vertical industries, but also as a powerful means for enhancing the overall network performance and, in turn, for increasing their revenues by simply sharing their physical infrastructure among multiple tenants.

In Table II, we evaluate the gain provided by RAN slicing by

<sup>4</sup>It is worth nothing that the overall PSE of 10 admitted network slice requests that corresponds to 100% of  $\text{PSE}_{\text{NoSlicing}}$  is slightly above the dashed line shown in the figure. This is due to the randomness of the network slice generation process. In fact, 10% is only the mean value (see Table I).



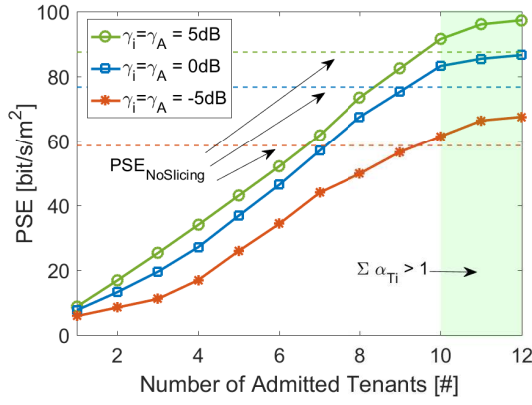


Fig. 4: Potential benefits of network slicing

using STORNS. The gain is defined as  $PSE_{\max}/PSE_{\text{NoSlicing}}$ , where  $PSE_{\max} = \sum_i PSE_{T_i}, \forall i$  admitted. We evaluate different user densities  $\lambda_T$  and threshold parameters  $\gamma_I, \gamma_A$ . When the average number of users increases, we note that network slicing provides additional performance gains (about 19%).

TABLE II: RAN Slicing Gain

	$\frac{\lambda_T}{\lambda_{BS}} = 50$	$\frac{\lambda_T}{\lambda_{BS}} = 200$	$\frac{\lambda_T}{\lambda_{BS}} = 500$
$\gamma_I = \gamma_A = 5\text{dB}$	7.17%	13.2%	18.1%
$\gamma_I = \gamma_A = 0\text{dB}$	6.63%	16.6%	18.8%
$\gamma_I = \gamma_A = -5\text{dB}$	4.52%	14.8%	17.69%

#### D. Algorithm complexity

We study the complexity of our algorithm against that of the exhaustive greedy search. The main parameter for STORNS is the number of rounds to converge and to compute the optimal Lagrange multipliers, as explained in Sec. III-C. In Fig. 5, we show with a solid green line the number of rounds ( $k$ ) that are needed to converge while increasing the number of tenants requesting a RAN slice. The behavior of the curve unveils that the complexity of our algorithm does not exponentially increase with the number of constraints (i.e., the number of tenants) of the optimization problem (Problem MultiTenant-Optimizer) but it converges to a stable number of iterations. On the right y-axis of Fig. 5, we compare the computational time for solving the optimization problem and compare STORNS against OPT. We evince that STORNS is capable of achieving near-optimal performance with a limited complexity compared to greedy approaches.

#### V. CONCLUSIONS

We have analyzed the benefits of applying network slicing to radio access networks. In particular, we have considered network slice requests with diverse service level agreements (SLAs) in terms of required average throughput per tenant. To analytically formulate the problem, we have capitalized on stochastic geometry theory, which allowed us to consider cellular network topologies in a tractable yet sufficiently realistic manner. We have introduced a new mathematical formulation for network slicing throughput and have defined an optimization problem to design an admission control, *STORNS*, that identifies the best tenants to be admitted into the network along with their spectrum and transmit power allocation such that the overall system throughput is maximized.

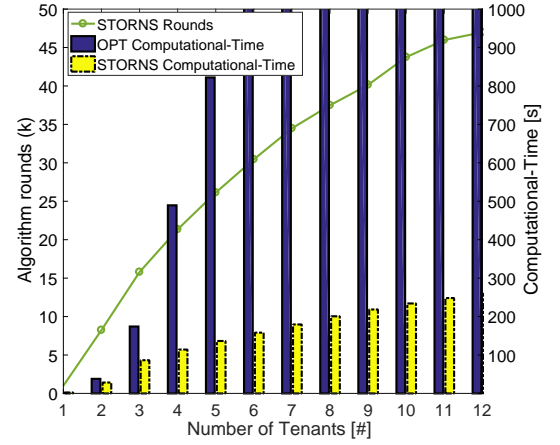


Fig. 5: Computational Analysis

Our results have shown through mathematical proofs, numerical and simulation results that networks where network slicing is applied can achieve a higher network throughput than non-sliced ones. Finally, we have provided quantitative results of reduced computational complexity of STORNS as compared to brute-force optimization methods. Our work puts forth network slicing as a suitable approach for optimizing the radio resource utilization of future sliced cellular networks.

#### REFERENCES

- [1] N. Alliance, "Description of network slicing concept," <http://www.ngmn.org/publications/technical.html>, 2015.
- [2] B. Han et al., "A Utility-driven Multi-Queue Admission Control Solution for Network Slicing," in *Proceedings of IEEE INFOCOM*, Apr. 2019.
- [3] R. Pries et al., "Network as a Service - A Demo on 5G Network Slicing," in *International Teletraffic Congress*, 2016.
- [4] X. Fokas, G. Patounas, A. Elmokashfi, and M. K. Marina, "Network Slicing in 5G: Survey and Challenges," *IEEE Communications Magazine*, vol. 55, pp. 80–87, May 2017.
- [5] V. Sciancalepore et al., "Mobile Traffic Forecasting for Maximizing 5G Network Slicing Resource Utilization," in *Proceedings of IEEE INFOCOM*, May 2017, pp. 4883–4888.
- [6] J. Salvat, L. Zanzi, A. Garcia-Saavedra, V. Sciancalepore, and X. Costa-Perez, "Overbooking network slices through yield-driven end-to-end orchestration," in *ACM CONEXT 2018*, Dec. 2018, pp. 1–12.
- [7] K. Samdanis, X. Costa-Perez, and V. Sciancalepore, "From network sharing to multi-tenancy: The 5g network slice broker," *IEEE Communications Magazine*, vol. 54, no. 7, pp. 32–39, July 2016.
- [8] R. Wen, G. Feng, J. Tang, T. Q. S. Quek, G. Wang, W. Tan, and S. Qin, "On robustness of network slicing for next generation mobile networks," *IEEE Transactions on Communications*, 2018.
- [9] H. ElSawy, A. Sultan-Salem, M. S. Alouini, and M. Z. Win, "Modeling and analysis of cellular networks using stochastic geometry: A tutorial," *IEEE Communications Surveys Tutorials*, vol. 19, pp. 167–203, 2017.
- [10] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.
- [11] M. D. Renzo, W. Lu, and P. Guan, "The intensity matching approach: A tractable stochastic geometry approximation to system-level analysis of cellular networks," *IEEE Transactions on Wireless Communications*, vol. 15, no. 9, pp. 5963–5983, Sep. 2016.
- [12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [13] Z.-J. Shi and J. Shen, "Step-size estimation for unconstrained optimization methods," *Computational and Applied Mathematics*, vol. 24, pp. 399 – 416, Dec. 2005.
- [14] "Guidelines for evaluation of radio interface technologies for IMT-Advanced," Report ITU-R M.2135-1, Dec. 2009.
- [15] S. Sun et al., "Propagation path loss models for 5G urban micro- and macro-cellular scenarios," in *IEEE VTC-Spring 2016*, May.