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# Soliton dynamics in velocity matched ultrafast frequency conversion processes

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**Abstract.** We consider ultrafast second order frequency conversion processes, under group velocity matching conditions. The three wave interaction equations, describing velocity matched ultrafast optical frequency conversion processes, can be reduced to Sine-Gordon equation. In this way, the well known Sine-Gordon solitons can be mapped into three wave soliton solutions. Here, we report the numerical dynamics of three wave solitons, two-solitons and breathers in ultrafast second order frequency conversion processes.

**PACS.** 42.65.Tg Optical solitons – 42.65.Re Ultrafast processes – 42.65.Ky Frequency conversion

## 1 Introduction

Nonlinear frequency conversion in quadratic materials plays a central role in extending the spectral range of light sources [1]. Efficient nonlinear interactions require compensation for the phase mismatch between the interacting waves caused by material dispersion. This compensation is achieved essentially by two means, namely birefringence phase matching and quasi-phase matching. With these methods, efficient nonlinear sum-frequency generation (SFG,  $\omega_1 + \omega_2 \rightarrow \omega_3$ ), difference-frequency generation (DFG,  $\omega_3 - \omega_2 \rightarrow \omega_1$ ), and optical parametric amplification (OPA,  $\omega_3 \rightarrow \omega_1 + \omega_2$ ) have been extensively explored [2–5]. In the ultrafast range domain (pulses as short as tens of femtoseconds), the efficiency of quadratic interactions can be greatly reduced by group velocity mismatch (GVM) of interacting waves, which leads to temporal splitting of pulses at different frequencies. In OPA, as an example, GVM between the pump at  $\omega_3$  and the amplified pulses (signal at  $\omega_1$  and idler at  $\omega_2$ ) limits the interaction length over which parametric amplification takes place, while GVM between the signal and the idler pulses limits phase matching bandwidth [1]. The effectiveness of frequency conversion processes can be increased operating in a frequency range where the group velocity of the high frequency wave is matched to the group velocity of one of the low frequency waves. These extended matching conditions can be easily obtained in birefringent crystals [1] and in periodically poled crystals [6, 7]. By the way, ultrafast quadratic frequency conversion processes are described by three wave interaction (TWI) [8, 9], since with pulses duration in the 100fs range second or higher order dispersion is negligible. TWI is integrable by means of inverse scattering transform (IST) [10, 11]. Integrability gives mathematical tools to investigate several problems such as the evolution of given initial data, the derivation

of (infinitely many) conservation laws and the construction of particular analytic solutions (f.i. solitons). TWI solitons have been extensively studied and applied in the context of nonlinear optics for efficient pulse compression [8], parametric oscillators [12], group velocity control [13, 14], pulse train generation [15]. The TWI model is integrable by means of the Zakharov-Manakov scheme only if the group velocities of the interacting waves are strictly different [10], entailing that the amazing theoretical background and results of TWI soliton literature do not apply under group velocity matching conditions.

In this paper we consider ultrafast second order parametric conversion processes in quadratic media when two of the three interacting waves have the same group velocity. In this framework, we recall that the TWI model is formally equivalent to the Sine-Gordon equation [16, 17]. This equivalence was overlooked in the TWI scenario in the last decades. By virtue of this mapping, we study and report the numerical dynamics of bright-dark-bright solitons, two-solitons and breathers (of Sine-Gordon origin) in velocity matched ultrafast frequency conversion processes.

## 2 Solitons of group-velocity matched TWI

The equations that describe the quadratic mixing of three waves of frequencies  $\omega_1, \omega_2, \omega_3$  ( $\omega_3 = \omega_1 + \omega_2$ ) in a nonlinear optical medium read as [1, 8]

$$\begin{aligned} \frac{\partial A_1}{\partial z} + \frac{1}{v_1} \frac{\partial A_1}{\partial t} &= iA_2^* A_3 \exp(i\Delta kz), \\ \frac{\partial A_2}{\partial z} + \frac{1}{v_2} \frac{\partial A_2}{\partial t} &= iA_1^* A_3 \exp(i\Delta kz), \end{aligned} \quad (1)$$

$$\frac{\partial A_3}{\partial z} + \frac{1}{v_3} \frac{\partial A_3}{\partial t} = iA_1 A_2 \exp(-i\Delta kz),$$

where  $A_j = \pi\chi^{(2)} \sqrt{\frac{n_j\omega_1\omega_2\omega_3}{n_1n_2n_3\omega_j}} E_j$ .  $E_j$  ( $j = 1, 2, 3$ ) is the slowly varying electric field envelope of the wave at frequency  $\omega_j$  with refractive index  $n_j$  and group velocity  $v_j$ ,  $\chi^{(2)}$  is the quadratic nonlinear susceptibility,  $\Delta k = k_3 - k_1 - k_2$  is the wave-vector mismatch,  $z$  and  $t$  are space and time coordinates. In the case of phase-matching ( $\Delta k = 0$ ), group velocity matching at frequency  $\omega_3$  and  $\omega_2$  ( $v_3 = v_2$ ), introducing  $\phi_{1,2} = A_{1,2} \exp(-i\pi/6)$ ,  $\phi_3 = A_3^* \exp(-i\pi/6)$ ,  $\delta_j = 1/v_j$ , assuming real initial conditions  $\phi_j(z = 0, t)$ , we obtain

$$\begin{aligned} \frac{\partial \phi_1}{\partial z} + \delta_1 \frac{\partial \phi_1}{\partial t} &= \phi_2 \phi_3, \\ \frac{\partial \phi_2}{\partial z} + \delta_2 \frac{\partial \phi_2}{\partial t} &= \phi_1 \phi_3, \\ \frac{\partial \phi_3}{\partial z} + \delta_3 \frac{\partial \phi_3}{\partial t} &= -\phi_1 \phi_2, \end{aligned} \quad (2)$$

with  $\delta_2 = \delta_3$ . Note that TWI equations (1) with real conditions  $\phi_j(z = 0, t)$  maintain the reality of fields throughout propagation. The assumption of phase matching  $\Delta k = 0$  is not a loss of generality in the study of TWI solitons, as the results obtained at phase matching can be extended to the case  $\Delta k \neq 0$  by applying a simple phase transform [18]. On the contrary, the reality of input  $\phi_j(z = 0, t)$  represents a loss of generality; indeed, our investigation applies to transform-limited input pulses.

By means of the change of variables  $\zeta = -(t - \delta_1 z)/(\delta_1 - \delta_2)$ ,  $\eta = (t - \delta_2 z)/(\delta_1 - \delta_2)$ , and the substitutions [19]

$$\phi_1 = -\frac{1}{2} \frac{\partial \psi}{\partial \zeta}, \quad \phi_2 = \cos\left(\frac{\psi}{2}\right), \quad \phi_3 = \sin\left(\frac{\psi}{2}\right) \quad (3)$$

we readily obtain from Eqs. (2) the Sine-Gordon (SG) equation for the auxiliary field  $\psi$

$$\frac{\partial^2 \psi}{\partial \eta \partial \zeta} = -\sin \psi \quad (4)$$

that in laboratory frame  $T = \eta + \zeta$ ,  $X = \eta - \zeta$  reads

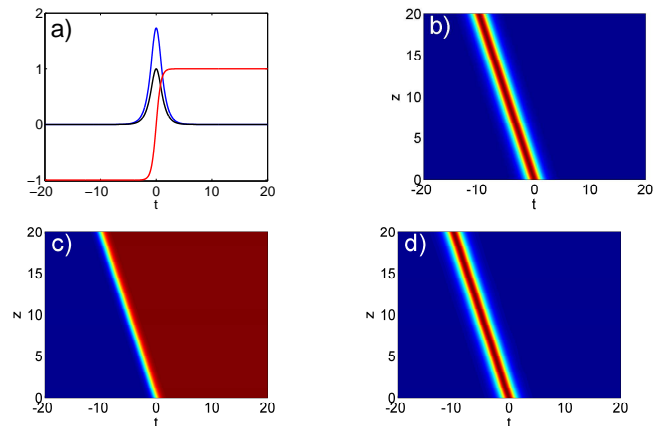
$$\frac{\partial^2 \psi}{\partial T^2} - \frac{\partial^2 \psi}{\partial X^2} = -\sin \psi. \quad (5)$$

SG equation is a well known nonlinear model in theoretical physics, applied in the description of propagation of ultrashort optical pulses in resonant media, dynamics of Josephson junctions and relativistic field theory [16]. SG equation is integrable, its IST was first introduced in the seventies [17], and explicit soliton solutions have been reported.

From SG solitons, through mapping of Eqs. (3), TWI solitons can be derived. SG kink soliton reads:

$$\psi(X, T) = 4 \tan^{-1} \left\{ \exp \left[ \frac{X - uT}{\sqrt{1 - u^2}} \right] \right\} \quad (6)$$

where  $u$  is the propagation speed ( $-1 < u < 1$ ). The velocity limit follows from the Lorentz covariance of the



**Fig. 1.** TWI soliton. a) Initial condition at  $z = 0$ :  $\phi_1$  (blue),  $\phi_2$  (red),  $\phi_3$  (black). b-d) Propagation of  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  soliton components. Values of parameters are  $u = 0.5$ ,  $\delta_1 = -1$ ,  $\delta_2 = 1$ .

SG equation. SG kink soliton maps through Eqs. (3) into a TWI soliton triplet (simulton), that reads:

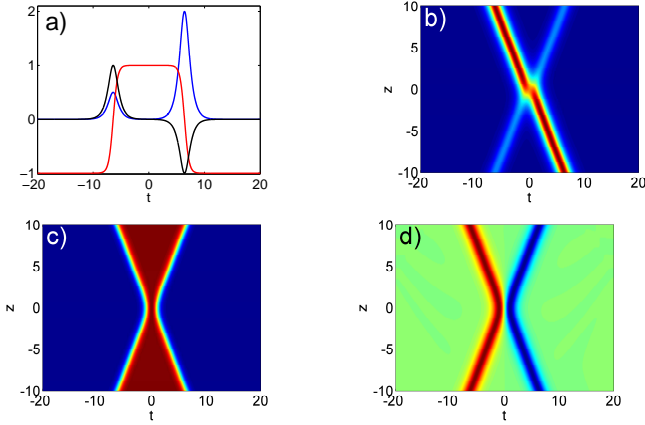
$$\begin{aligned} \phi_1 &= \frac{1+u}{\sqrt{1-u^2}} \operatorname{sech} \left( \frac{X-uT}{\sqrt{1-u^2}} \right) \\ \phi_2 &= -\tanh \left( \frac{X-uT}{\sqrt{1-u^2}} \right) \\ \phi_3 &= \operatorname{sech} \left( \frac{X-uT}{\sqrt{1-u^2}} \right) \end{aligned} \quad (7)$$

In the reference frame  $(z, t)$  the TWI bright-dark-bright soliton triad propagates with the nonlinear group velocity  $v = \delta^{-1}$  with  $\delta = [u(\delta_1 - \delta_2) + \delta_1 + \delta_2]/2$ . Velocity  $v$  can assume values in between the characteristic group velocities  $v_1$  and  $v_2$ . In order to confirm our analysis, we checked expressions (7) by comparing them with numerical simulations of Eqs. (2). Fig. 1 reports the numerical propagation of a TWI soliton triplet. It is worth noting that the numerical propagation exactly coincides with the expression (7).

The SG two-soliton solution, which describe kink-kink collisions, kink-antikink collisions and SG periodic breathers reads:

$$\psi(X, T) = 4 \tan^{-1} \left\{ \left( \frac{a_1 - a_2}{a_1 + a_2} \right) \frac{\sinh \frac{\theta_1 + \theta_2}{2}}{\cosh \frac{\theta_1 - \theta_2}{2}} \right\} \quad (8)$$

where  $\theta_j = 1/2[(a_j - a_j^{-1})T + (a_j + a_j^{-1})X] + \alpha_j$  ( $j = 1, 2$ ). The complex constants  $a_j$  are the discrete eigenvalues that define the soliton content whereas  $\alpha_j$  are translation coefficients and can be set to zero without loss of generality. The discrete eigenvalues may be either real or an anti-Hermitian pair  $a_1 = r + im$ ,  $a_2 = -a_1^* = -r + im$ . In the first case the solution describes the collision between two SG solitons traveling at velocities  $u_j = (1 - a_j^2)/(1 + a_j^2)$  (in the  $(X, T)$  reference frame). In the second case the solution describe a SG oscillating pulse whose barycenter travels at velocity  $u = (1 - a^2)/(1 + a^2)$  ( $a = |a_1| = |a_2|$ ),



**Fig. 2.** TWI solitons collision. a) Initial condition at  $z = -10$ :  $\phi_1$  (blue),  $\phi_2$  (red),  $\phi_3$  (black). b-d) Propagation of  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  soliton components. Here,  $\delta_1 = -1$ ,  $\delta_2 = 1$ ;  $a_1 = 2$ ,  $a_2 = 0.5$ .

and periodic along  $T$  with period  $\Lambda = 4\pi/[r(1 + 1/a^2)]$ . SG two-soliton solutions map through Eqs. (3) into TWI soliton solutions, that reads:

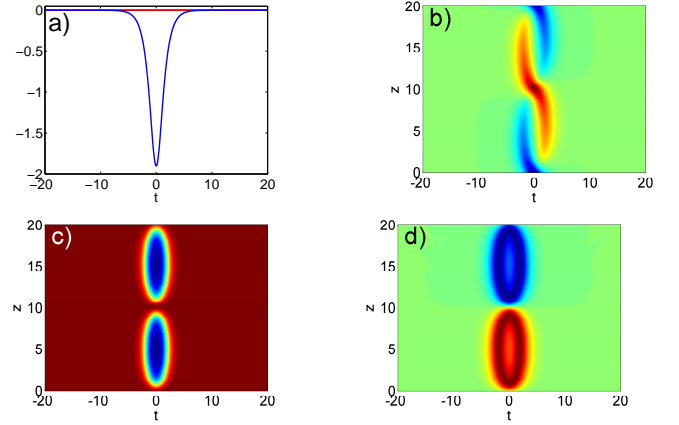
$$\begin{aligned} \phi_1 &= (a_1^2 - a_2^2)(a_1 \cosh(\theta_1) + a_2 \cosh(\theta_2))(a_1 a_2 \Delta)^{-1} \\ \phi_2 &= [2a_1 a_2 \cosh(\theta_1) \cosh(\theta_2) + \\ &\quad (a_1^2 + a_2^2)(1 - \sinh(\theta_1) \sinh(\theta_2))]\Delta^{-1} \\ \phi_3 &= (a_1^2 - a_2^2)(\sinh(\theta_1) + \sinh(\theta_2))\Delta^{-1} \end{aligned} \quad (9)$$

with  $\Delta = (a_1^2 + a_2^2) \cosh(\theta_1) \cosh(\theta_2) + 2a_1 a_2 (1 - \sinh(\theta_1) \sinh(\theta_2))$ . The expression (9) describes TWI solitons collisions, interactions and TWI breathers. Figure 2 shows the propagation of a TWI two-soliton, describing the collision of a soliton with eigenvalue  $a_1 = 2$  traveling with an inverse velocity  $\delta_{sol1} = 0.6$  and a soliton with eigenvalue  $a_2 = 0.5$  traveling with an inverse velocity  $\delta_{sol2} = -0.6$ . Figure 3 shows the propagation of a TWI breather, corresponding to the anti-Hermitian eigenvalue pair  $a_1 = \exp(-i\pi/10)$ ,  $a_2 = -a_1^*$ . This solution describes the interaction of two solitons with equal velocity and placed at a small distance. During propagation the two solitons attract and repel depending on their relative phases, giving rise to a periodic behavior.

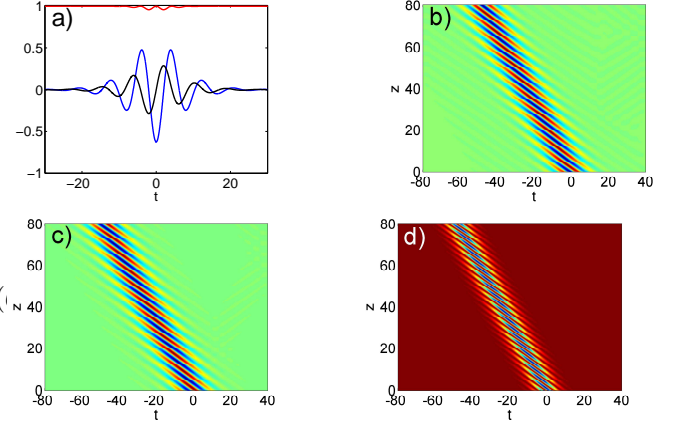
Figure 4 shows the propagation of another kind of TWI breather, corresponding to the anti-Hermitian eigenvalue pair  $a_1 = 0.5 \exp(-i0.45\pi)$ ,  $a_2 = -a_1^*$ . In this case it is not possible to recognize an interaction between two solitons, and the solution describes the propagation of a slowly varying envelope modulated by rapid spatial variation.

These TWI solitons may enable for the enhanced and stable parametric velocity matched frequency conversion of ultrashort pulses, for the intensity dependent control of the speed of light of pulses. TWI breathers also point to the possibility of cavity-less short pulse train generation in ultrafast velocity matched frequency conversion processes.

Let us briefly discuss the experimental conditions for the observation of TWI solitons in nonlinear optics. For instance, when considering a type I oee interaction in a 2



**Fig. 3.** TWI solitons attraction (breather). a) Initial condition at  $z = 0$ :  $\phi_1$  (blue),  $\phi_2$  (red),  $\phi_3$  (black). b-d) Propagation of  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  soliton components. Here,  $\delta_1 = -1$ ,  $\delta_2 = 1$ ;  $a_1 = \exp(-i\pi/10)$ ,  $a_2 = -a_1^*$ .



**Fig. 4.** TWI breather. a) Initial condition at  $z = 0$ :  $\phi_1$  (blue),  $\phi_2$  (red),  $\phi_3$  (black). b-d) Propagation of  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  soliton components. Here,  $\delta_1 = -1$ ,  $\delta_2 = 1$ ;  $a_1 = 0.5 \exp(-i0.45\pi)$ ,  $a_2 = -a_1^*$ .

cm long bulk MgO-doped periodically poled lithium niobate sample with  $20 \mu\text{m}$  periodicity [20], the field envelope carriers  $\lambda_1 = 1560 \text{ nm}$ ,  $\lambda_2 = 1560 \text{ nm}$ ,  $\lambda_3 = 780 \text{ nm}$ , durations of about tens of femtosecond, TWI solitons can be observed with field intensities of a few  $\text{GW}/\text{cm}^2$ .

### 3 Conclusion

In conclusion, we have considered soliton solutions of group velocity matched ultrafast conversion processes. We have carried over TWI solitons of Sine-Gordon origin, which were overlooked in the last decades. We have revealed the numerical dynamics of these TWI solitons solutions representing soliton triplets, soliton interactions and collisions, and three-wave breathers, discussing their applicability in nonlinear optics.

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