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Dispersive shock waves in phase-mismatched second harmonic generation

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We investigate wave-breaking and dispersive shock wave formation driven by a pulse undergoing second-harmonic generation in a quadratic medium. We show that the process is accessible in the regime of high phase-mismatch (cascading limit) and weak dispersion. Insight into the phenomenon is obtained by means of a suitable hydrodynamic reduction of the equations that govern the mixing. © 2012 Optical Society of America

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Parametric nonlinearities characteristic of quadratic media can exactly balance dispersion (and or diffraction) to form solitons [1, 2]. Conversely, in such media, the phenomena occurring when nonlinearities overwhelm dispersion, are left mostly unexplored. Studies performed in this regime for cubic Kerr media show that dispersive (or collisionless) shock waves (also known as undular bore) develop in settings ruled by the defocusing nonlinear Schrödinger equation (NLSE). The distinctive feature of this phenomenon is the onset of oscillations determined by the regularizing action of dispersion over infinite gradients produced by the nonlinear response [3,4]. This process has been observed in fibers [5] and more recently in the spatial domain [6–8] where suitable generalizations of the NLSE account for the specific nature (saturable, nonlocal, etc.) of cubic nonlinearities. On the other hand, a properly generalized NLSE can also describe the propagation of light pulses in second-harmonic generation (SHG) in the so-called cascading limit [1]. In this framework, formation of steep gradients are attributed to self-steepening terms that stem from corrections to the main cascading term (i.e., the effective nonlinear index due to repeated up- and down-conversions), whose effects have been investigated theoretically and experimentally [1, 9–12].

In this letter we show, however, that dispersive shock formation can occur regardless of such corrections, driven by the leading-order index change due to mismatched SHG, when the group-velocity dispersion (GVD) is sufficiently weak. To this end we introduce a hydrodynamic reduction of the standard SHG model that we perform by starting from a suitable renormalization of the latter. We validate the outcome of such an approach by means of numerical analysis of the original model, predicting that this regime is accessible in practice. In this context, the self–steepening term that arises from group-velocity mismatch (GVM) [9–11] appears to play only a perturbative role that modifies the symmetry of the wave-breaking process.

We consider type I mixing of pulse envelopes $w(Z,T)$ and $v(Z,T)$ (electric fields in V/m) at fundamental frequency (FF) and its second harmonic (SH), respectively, which travel along Z in a normally dispersive quadratic medium. In the retarded time frame T moving with the FF group-velocity, the model that governs SHG reads as [1, 2, 11]

$$\begin{aligned} iw_Z - \tfrac{\beta_1''}{2} w_{TT} + \chi v w^* e^{-i\Delta k Z} &= 0, \\ iv_Z + i\delta v_T - \tfrac{\beta_2''}{2} v_{TT} + \chi w^2 e^{i\Delta k Z} &= 0, \end{aligned} \tag{1}$$

where $\beta_{1,2}''$ stands for GVDs, $\delta = \beta_2' - \beta_1'$ is the GVM ($\beta_{1,2}'$ are the inverse group velocities), Δk is the wave–number mismatch, and χ is the effective nonlinearity. We introduce scaled variables suited to investigate the weakly dispersing regime: $z = Z/L$, $t = T/T_0$, $a_1 = w/\sqrt{P}$, $a_2 = v/\sqrt{P}\exp(-i\Delta k Z)$, with $L = \sqrt{L_D L_{NL}}$, $L_{NL} = |\Delta k|/(\chi^2 P)$ and $L_D = T_0^2/\beta_1''$ being the nonlinear and dispersive lengths associated with input pulse duration T_0 and peak FF field $\sqrt{P}$. In terms of such variables, introducing also the small parameter $\varepsilon = \sqrt{L_{NL}/L_D}$ as well as $\delta k = \Delta k L$, $\gamma = \beta_2''/\beta_1''$, and $\eta = L/L_{GVM}$, where $L_{GVM} = T_0/\delta$, we cast Eqs. (1) in dimensionless form

$$\begin{aligned} i\varepsilon a_{1z} - \tfrac{\varepsilon^2}{2} a_{1tt} + \sqrt{|\delta k|\varepsilon}\, a_2 a_1^* &= 0, \\ i\varepsilon a_{2z} + i\varepsilon\eta a_{2t} - \gamma\tfrac{\varepsilon^2}{2} a_{2tt} - \varepsilon\delta k a_2 + \sqrt{|\delta k|\varepsilon}\, a_1^2 &= 0. \end{aligned} \tag{2}$$

We consider the standard case such that only a FF pulse is launched. By using the SH asymptotic expansion $a_2 = \sum_{n=0}^{\infty} a_2^{(n)}/|\delta k|^n$, and the method of repeated substitution [1], retaining leading-order terms, we arrive at a single evolution equation (obtained at $n = 1$ order):

$$i\varepsilon u_z - \frac{\varepsilon^2}{2} u_{tt} + s|u|^2 u + 2i\varepsilon\sigma|u|^2 u_t = 0, \tag{3}$$

where, for sake of clarity we defined $u = a_1$, $s = \mathrm{sign}(\delta k)$ and $\sigma = \eta/(\varepsilon|\delta k|)$. In the following we consider effective self–focusing nonlinearities ($s = 1$), which yields the integrable defocusing NLSE in the limit of null GVM ($\sigma = 0$). When $\sigma \neq 0$, Eq. (3) is also integrable, being equivalent to Chen-Lee-Liu equation (CLLE) [12–14]. We em-

phasize that the validity of Eq. (3) entails weak overall FF to SH conversion, which, assuming small GVM and GVD, requires $|\delta k| \gg 4/\varepsilon$. If we take $|\delta k| = M4/\varepsilon$, $M \gg 1$, then $\sigma = \eta/(4M)$.

For $\varepsilon \ll 1$, the leading order system (geometric optics or hydrodynamic limit), that stems from the WKB transformation $u(z,t) = \sqrt{\rho(z,t)}\exp[iS(z,t)/\varepsilon]$ applied to Eq. (3), takes the form of the following first–order quasi–linear system of equations:

$$\begin{aligned} \rho_z + (v\rho + \sigma\rho^2)_t &= 0 \\ v_z + vv_t + \rho_t + 2\sigma(\rho v)_t &= 0 \end{aligned} \tag{4}$$

where $v = -S_t$ is the equivalent fluid velocity. Equations (4) entail that a smooth input pulse breaks at finite distance z_s [4, 14–16]. At variance with the case of a single hyperbolic equation where such distance can be evaluated a priori, Eqs. (4) allow us to give only a lower and upper bound (i.e. a capture window) for z_s. This can be done by applying a well-known criterion by Lax [14, 16], which requires Eqs. (4) to be cast in diagonal form $r_z^\pm + \lambda^\pm r_t^\pm = 0$ in terms of Riemann invariants $r^\pm = \pm(v + 2\sigma\rho) + \sqrt{\rho(1+2\sigma v)}$, with corresponding eigenvelocities $\lambda^\pm = v + 2\sigma\rho \pm 2\sqrt{\rho(1+2\sigma v)}$ (also expressed through Riemann variables using identical formulas of the NLSE case: $\lambda^- = (-3r^- + r^+)/4$, $\lambda^+ = (-r^- + 3r^+)/4$). We focus henceforth on the initial condition $a_1(0,t) = \sqrt{\rho(0,t)} = \nu + (1-\nu)\exp(-t^2)$, $v(0,t) = 0$ representing an unchirped Gaussian input pulse on a weak pedestal $\nu \ll 1$, for which the results are summarized in Fig. 1. For null background ($\nu = 0$) and $\sigma = 0$ we find $z_s \in [\sqrt{e}/3, \sqrt{2}e/3]$, while the shock distance turns out to decrease with σ [see Fig. 1(a)]. A similar trend is obtained in the presence of pedestal, as displayed in Fig. 1(b) for $\nu = 0.1$, which shows also how the distance z_s computed via direct numerical integration of Eqs. (4) is correctly captured by the bounds from the Lax criterion.

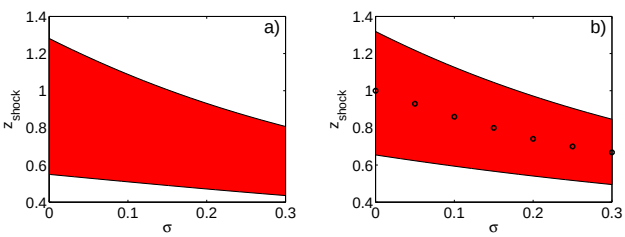

Fig. 1. Shock capture window (shaded area) vs. GVM parameter σ: (a) Gaussian input; (b) Gaussian on pedestal, $\nu = 0.1$. The bullets give the shock distance obtained via direct numerical integration of Eqs. (4).

We then proceeded to verify the shock formation process via numerical integration of the original model (2). The whole physical parameter space can be explored by varying the GVD parameter ε and the GVM parameter η. We report examples for a fixed $\varepsilon = 0.05$ and $\delta k = 200/\varepsilon$ (i.e. $M = 50 \gg 1$) which ensures to be

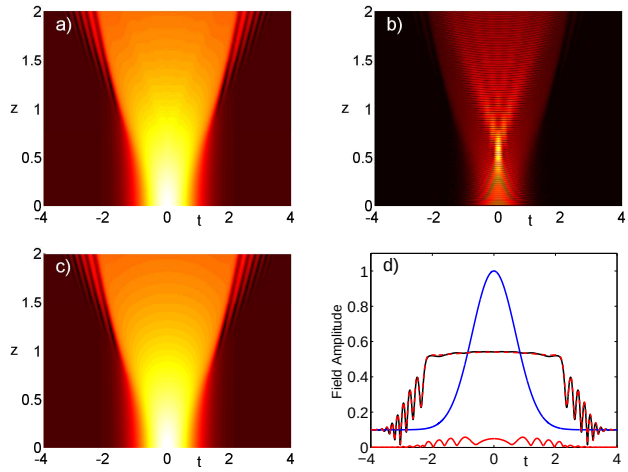

Fig. 2. (a-b) Color level plot of FF field $|a_1|$ (a) and SH field (b), ruled by Eqs. (2); (c) same for FF field $|u|$ from Eq. (3); (d) Snapshots of output field moduli $|a_1|$ (FF, solid black) and $|a_2|$ (SH, solid red) from Eqs. (2), compared with $|u|$ (FF, dashed red) from Eq. (3). The solid blue curve is the input. Here $\eta = \sigma = 0$, $\varepsilon = 0.05$.

in the cascading limit. Without loss of generality we present results for $\nu = 0.1$ (pedestal power equal to 1 % of the peak power) which, though not essential for the physics, enhances the visibility of post-breaking oscillations. First, we consider the case of zero GVM ($\eta = 0$), setting $\gamma = 2$ to account for the (usually) higher GVD at SH. The evolution of the FF field displayed in Fig. 2(a) shows symmetric steepening on both the leading and the trailing edges, until beyond the breaking distance $z_s \sim 1$ (in good agreement with z_s predicted from the hydrodynamic limit), undular bores appear, filling extending portions of time with fast oscillations. The SH component shown in Fig. 2(b) remains always small, though it shows a complicated behavior dominated by periodic back-conversions and a sort of temporal focusing at short distance, which is found to be ruled by the GVD at SH. For comparison Fig. 2(c) reports the corresponding evolution according to Eq. (3), which, as shown, turns out to be in satisfactory agreement with that of Fig. 2(a), as also witnessed by the snapshots displayed in Fig. 2(d).

Next, we consider the case of non-vanishing GVM, taking $\eta = 30$, $\gamma = 2$, for which the evolution of the FF and SH fields are shown in Fig. 3(a,b). In this case, the GVM leads to break the symmetry in time, enhancing the steepening effect on the trailing edge of the pulse which turns out to break at a shorter distance $z \simeq 0.8$ compared with the GVM-free case. This is in agreement with the estimates from the hydrodynamic limit (4) [see Fig. 1(b)], and is satisfactorily reproduced by the reduced CLLE [Eq. 3], whose dynamics is reported in Fig. 3(c) (in this case $\sigma = 0.15$). Furthermore, the leading edge also undergoes a secondary breaking at a longer distance ($z \simeq 1.1$), where the trailing edge already exhibits a well-developed dispersive shock wave. This strong asymmetry

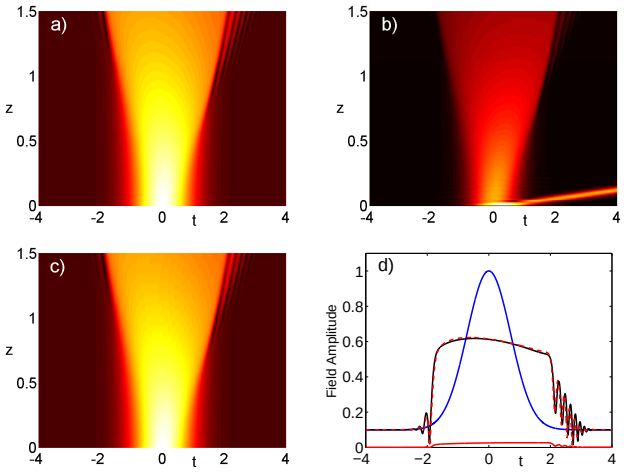

Fig. 3. As in Fig. 2, with non vanishing GVM such that $\eta = 30$ [Eqs. (2)] and $\sigma = 0.15$ [Eq. (3)].

is also clear from the snapshots shown in Fig. 3(d). Note also that the SH evolution exhibits weak initial emission of non-phase-matched component traveling with natural SH group velocity, while the rest of the SH field follows almost adiabatically the FF. An accurate comparison between the output fields obtained from Eqs. (2) and Eq. (3) shows slight discrepancies for higher values of $|\sigma|$. In fact, our numerics suggest that the reduction of Eqs. (2) to Eq. (3) following from truncation at order $n = 1$, holds for small $|\sigma|$ (roughly $|\sigma| < 0.15$ for $\varepsilon = 0.05$), i.e. when GVM is small enough with respect to mismatch. A better accuracy up to the shock point can be obtained even for larger σ by including other terms in Eq. (3) [we find the most significant to be $-2s\varepsilon\sigma^2(uu_t)_t u^*$ arising at order $n = 2$, details will be given elsewhere]. However, beyond z_s, even improved reductions break down for high values of $|\sigma|$. This can be explained as follows: on one hand, shock entails pulse spectral broadening; on the other hand, GVM introduces a frequency dependence of the effective phase-matching ($\omega_{PM} \approx \delta k/\eta$). When the spectrum broadens enough to reach the effective phase-matching frequency ω_{PM}, portions of the FF spectrum are phase-matched and give strong conversion which make the single–field reduction to break down.

Finally, one can address the observability of the process by returning to dimensional quantities (peak power and duration, characteristic length L), as follows:

$$P = \frac{\Delta k^2}{\chi^2|\delta k|\varepsilon}, \quad T_0 = \frac{1}{\chi\varepsilon}\sqrt{\frac{|\Delta k|\beta_1''}{P}}, \quad L = \frac{|\delta k|}{|\Delta k|}. \qquad (5)$$

In order to observe the dynamical interplay between GVM and GVD illustrated in Fig. 3, we can consider for instance a periodically poled stoichiometric lithium tantalate sample [17], which has been successfully exploited to study femtosecond pulse propagation with strong phase- and group-velocity mismatch [11]. A poling period of $\Lambda = 12.66\mu$m at FF wavelength of 1400 nm gives a residual mismatch $\Delta k = 1250$ cm^{-1}, and dispersion parameters $\beta_1'' = 0.1$ ps^2/m, $\delta = 324$ ps/m [17]. Using $\chi = 1.43\cdot 10^{-5}$ m/V (plane-wave limit), the values $\varepsilon = 0.05$ and $\delta k = 4000$ yield $T_0 \approx 260$ fs, $L \approx 3.2$ cm, and an input peak intensity $I \approx 100$ GW/cm^2. The latter compares favorably with the values reported in the literature [10], and will be further reduced with non-negligible contribution from cubic nonlinearities which is of the self-focusing type, or in waveguide geometries. Additionally we get $\eta = 40$ and correspondingly $\sigma = 0.2$.

In conclusion, we have reported a hydrodynamic reduction describing shock formation in highly mismatched SHG. Oscillatory wave-breaking driven by the interplay of cascading and GVD has been demonstrated numerically and predicted to be observable. The perturbative role of GVM-induced self-steepening is expected to cause an asymmetry in the shock process. Future work will address how this scenario changes in the (possibly phase-matched) strong conversion regime.

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