



HAL
open science

Tetraneutron resonance in the presence of a dineutron

A. Deltuva, R. Lazauskas

► **To cite this version:**

A. Deltuva, R. Lazauskas. Tetraneutron resonance in the presence of a dineutron. *Physical Review C*, 2019, 100 (4), pp.044002. <10.1103/PhysRevC.100.044002>. <hal-02394732>

HAL Id: hal-02394732

<https://hal.science/hal-02394732v1>

Submitted on 7 Jan 2025

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Tetraneutron resonance in the presence of a dineutron

A. Deltuva*

Institute of Theoretical Physics and Astronomy, Vilnius University, Saulėtekio al. 3, LT-10257 Vilnius, Lithuania

R. Lazauskas†

IPHC, IN2P3-CNRS/Universite Louis Pasteur BP 28, F-67037 Strasbourg Cedex 2, France

(Received October 29, 2019)

Background: Several previous studies provided contradicting results for the four-neutron system, some claiming the existence of a 0^+ near-threshold resonance, others denying presence of any observable resonant states.

Purpose: Since most of the studies employed enhanced two-neutron interactions to follow the evolution of an artificially bound state into a continuum one, we examine several enhancement schemes that produce a bound dineutron as well.

Methods: We study the four-neutron system by solving exact four-particle equations. By varying the interaction enhancement factor we calculate two-dineutron scattering phase shifts and cross sections.

Results: When the same enhancement factor is used in all partial waves, a bound tetraneutron emerges together with a strongly bound dineutron. Furthermore, such a 0^+ tetraneutron evolves not into a resonance but into a virtual state. Weak enhancement of S waves together with strongly enhanced higher waves is needed for the emergence of the resonant state. Anyhow the resonant behavior disappears well before reaching the physical interaction strength.

Conclusions: The interaction enhancement scheme using the same factor for all waves, employed in several previous works, is misleading for the search of 0^+ resonance as only a virtual state can emerge. Evolution of a bound tetraneutron into a resonance via an intermediate virtual state is possible with strong enhancement of higher two-neutron waves.

I. INTRODUCTION

A possible experimental observation of the tetraneutron resonance [1] triggered a number of theoretical studies [2–6]. Their results are highly contradictory: a resonance with total angular momentum and parity $\mathcal{J}^\pi = 0^+$ just 1 - 2 MeV above threshold was obtained using a harmonic oscillator representation of the continuum [2] and the bound-state quantum Monte Carlo with extrapolation to the continuum [3]. Calculations based on the no-core Gamow shell model [4] and the density matrix renormalization group [4] approaches have not found any narrow resonant states and tolerate only the presence of broad structures with a width of at least 4 MeV, while their positions could not be determined accurately. On the other hand, rigorous solution of four-particle Faddeev-Yakubovsky (FY) equations in the coordinate- [5] and momentum-space [6] representations predict no observable tetraneutron resonance, consistently with the pioneering studies on this subject [7, 8]. The aim of the present work is to shed some light on the origin of these disagreements.

We do not discuss the details of our four-particle bound-state and scattering calculations that can be found in Refs. [5, 6, 8] and in references therein. In Sec. II we present the study of the artificial four-particle system using two different force schemes. The summary and conclusions are contained in Sec. III.

II. RESULTS

Apart from the work [2], none of the aforementioned theoretical studies were able to identify tetraneutron resonances by performing direct calculations considering the physical values of the nuclear interaction. In the studies [4–8] no tetraneutron complex-energy states were observed sufficiently close to the physical energy axis, whereas Monte-Carlo techniques used in [3] are unsuited to determine the positions of unbound states directly. All of the aforementioned studies [9] therefore tried to enhance the nuclear interaction to make the four-neutron ($4n$) system bound, and then follow its evolution to the physical strength of the interaction.

However, important differences were present in the details: Refs. [2–4] enhanced the neutron-neutron (nn) potential in all partial waves by the same factor f , Refs. [5, 8] added an attractive $3n$ or $4n$ force, Ref. [6] enhanced the nn potential in partial waves with orbital momentum $L > 0$ by the same factor f_L but kept the physical strength $f_0 = 1$ for the 1S_0 force. The strategies of Refs. [5, 6, 8] had a particular goal to avoid binding dineutrons. In contrast, when the nn force is enhanced also in the 1S_0 wave as in Refs. [2–4], very soon a bound dineutron is generated. The presence of a bound dineutron with energy $E_d < 0$ relative to the free-particle threshold has several consequences for the $4n$ system. First, the lowest scattering threshold is no longer at the energy $E_{4n} = 0$ but at the two-dineutron threshold $E_{4n} = 2E_d < 0$. Thus, negative energy $4n$ states at $2E_d < E_{4n} < 0$ are not bound but continuum states that may be realized in the scattering of two dineutrons,

* arnoldas.deltuva@tfai.vu.lt

† rimantas.lazauskas@iphc.cnrs.fr

only those with $E_{4n} < 2E_d$ are bound states. Second, the presence of an additional threshold changes the structure of the complex energy plane of $4n$ states, further complicating trajectories of $4n$ states.

To reveal shortcomings of the approach of enhancing the 1S_0 nn force, we study the $4n$ system using rigorous FY equations in the coordinate-space [5] and FY equivalent momentum-space Alt, Grassberger, and Sandhas (AGS) equations for transition operators [6]. We start with the model enhancing all partial waves by the same factor, i.e., $f_0 = f_L = f$. As it is well known, a bound dineutron emerges in the 1S_0 partial wave at $f_0 \approx 1.1$; this critical value of f_0 slightly varies with the nn potential, see Ref. [10] for a number of realistic potentials. The energy of the dineutron E_d relative to the free neutron threshold rapidly decreases with increasing f_0 , i.e., it becomes bound more tightly. To support a bound tetra-neutron, i.e., the one with $E_{4n} < 2E_d$, significantly larger enhancement is needed; it also depends on the potential. A bound tetra-neutron emerges first in the $\mathcal{J}^\Pi = 0^+$ state at $f \approx 2.4$ for several local realistic potentials such as Nijmegen II or Argonne V18, where the dineutron energy is below -20 MeV [11]. In order to make comparison with Refs. [2–4] that used soft nonlocal potentials, our following results will be based on the next-to-leading order (NLO) chiral potential [12] as in Ref. [3]. With this model and the space of Ref. [6] the tetra-neutron becomes bound at $f = 2.665$ where $E_d = -12.01$ MeV and $E_{4n} = -24.02$ MeV. At lower f values the $4n$ system cannot be bound, the lowest-energy $4n$ state is the two-dineutron state. However, Refs. [3, 4] report a bound tetra-neutron at threshold $E_{4n} \rightarrow -0$ with f being as low as 1.3 [3] or 1.6 [4]. Those $4n$ states in Refs. [3, 4] have zero width, which is not compatible with the fact that they are embedded in the two-dineutron continuum. Nevertheless, one might still expect that those are resonant states that evolved from the bound state with decreasing f , and thereby could provide estimation for the real part of the $4n$ resonance energy. To verify this conjecture we study the f -evolution of two-dineutron scattering states using rigorous treatment of the four-particle continuum as provided by AGS equations.

In Fig. 1 we show dineutron-dineutron 0^+ phase shift δ_0 as a function of the kinetic energy E_k in the center-of-mass (c.m.) frame at $f = 2.7, 2.65, 2.6, 2.5, 2.3, 2.0,$ and 1.3 . There is a clear qualitative difference between the $f = 2.7$ and 2.65 phase shift results at low energies: the former monotonically decreases from 180 deg at threshold whereas the latter rapidly increases from 0 deg at threshold exhibiting a bump of 70 deg. This signals the emergence of a bound $4n$ state at f between 2.65 and 2.7 , fully consistent with the result of 2.665 from the direct bound state calculation. However, the most important observation is that at $f = 2.65, 2.6,$ and 2.5 where one could expect the bound state to evolve into a resonance, no resonant phase shift behavior is observed. Note that qualitatively the same behavior is seen in the realistic 1S_0 two-nucleon phase shift, reflecting the presence

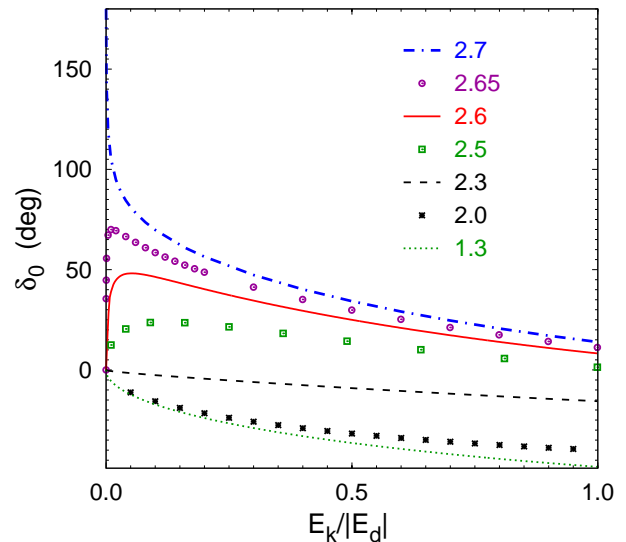


FIG. 1. $J^\Pi = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy E_k . NLO potential enhanced by a factor f as indicated in the plot was used.

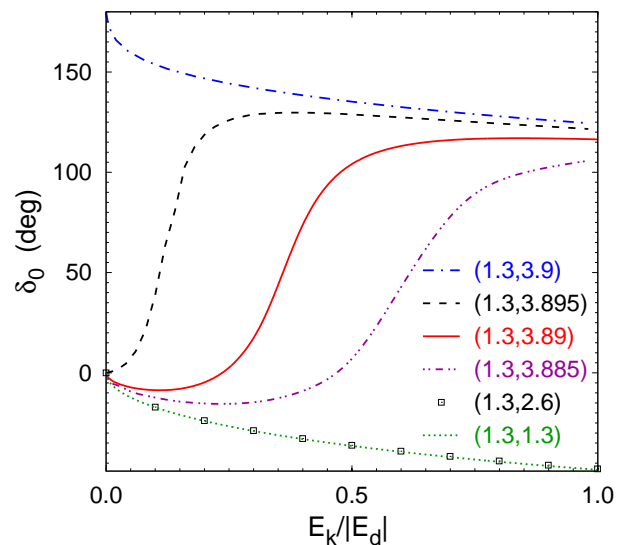


FIG. 2. $J^\Pi = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy E_k . Enhanced NLO potential with S - and higher-wave factors (f_0, f_L) as indicated in the plot was used.

of the well-known virtual state near threshold. Indeed, at $f = 2.65, 2.6,$ and 2.5 the two-dineutron phase shift energy dependence is consistent with the presence of a virtual $4n$ state that with decreasing f moves away from the two-dineutron threshold into the unphysical sheet of the complex energy plane. Reducing f further, the virtual state becomes too far from the threshold to have a visible effect, while the phase shifts approach the universal ones obtained for two fermionic dimers in the unitary

limit [13], signaling an effective repulsion between the two dineutrons. For example, at $f = 1.3$ the deviation from the universal results [13] is below 5%. Thus, in the nn force enhancement scheme $f_0 = f_L = f$ the 0^+ tetra-neutron bound state evolves not into a resonance but into a virtual state, rendering this scheme entirely misleading for the 0^+ resonance study.

The evolution of a bound into a virtual state is typical for S -wave two-body systems [14] unless there is a sufficiently high potential barrier leading to the appearance of a resonance, as shown in the example of Ref. [3] based on a two-Gaussian potential. The angular-momentum barrier is always present in $L > 0$ waves, resulting in the evolution of a bound state into a resonance. In the $4n$ systems both $L = 0$ and $L > 0$ waves are present, but their balance at $f_0 = f_L$ clearly favors a virtual state, not a resonance. The S -wave dominance in the physical $f = 1$ case is demonstrated also in Ref. [6].

In the following we will show a counterexample where the 0^+ tetra-neutron bound state evolves into a resonance seen in the two-dineutron scattering. To achieve this goal one needs to break the S -wave dominance, using larger enhancement factor f_L for $L > 0$ waves as compared to f_0 . In our example we take $f_0 = 1.3$. In that case the dineutron energy is $E_d = -0.316$ MeV. To support a bound tetra-neutron with $E_{4n} < 2E_d$, significantly stronger enhancement $f_L = 3.898$ is needed for higher waves. Results for two-dineutron phase shifts obtained around these values of (f_0, f_L) are presented in Fig. 2. There is clear qualitative difference between $f_L = 3.9$ and 3.895 results, signaling the presence of a bound state in the former case and a resonance in the latter case, consistently with direct bound-state calculations. With decreasing f_L the resonance rapidly moves to higher energy and becomes broader, as is evident from the energy dependence of $f_L = 3.895, 3.89,$ and 3.885 two-dineutron phase shifts shown in Fig. 2. Thus, under these conditions of f_L being considerably larger than f_0 , the 0^+ tetra-neutron bound state evolves into a resonance as there is a strong contribution of higher partial waves creating an effective repulsive barrier via centrifugal terms. However, with decreasing f_L the dominance of S -waves is restored well before reaching the $f_L = f_0$ such that the tetra-neutron pole on the unphysical sheet moves deeply below the two-dineutron threshold, without having a visible effect on the scattering processes, as already known from $f = 1.3$ results in Fig. 1. In fact, already at $f_L = 2f_0 = 2.6$ the phase shift results are very close to those of $f_L = f_0 = 1.3$ and thereby also to the universal ones.

The regime very close to the critical point where the tetra-neutron becomes unbound deserves special consideration. In Fig. 3 we show two-dineutron phase shifts δ_0 obtained with $f_0 = 1.3$ and $f_L = 3.898, 3.8975, 3.897,$ and 3.896 in a narrow energy regime $E_k < |E_d|/10$ very close to the threshold and in Fig. 4 the corresponding 0^+ total cross sections σ_0 ; the latter are normalized by the two-neutron zero-energy cross section σ_{nn}^0 calculated

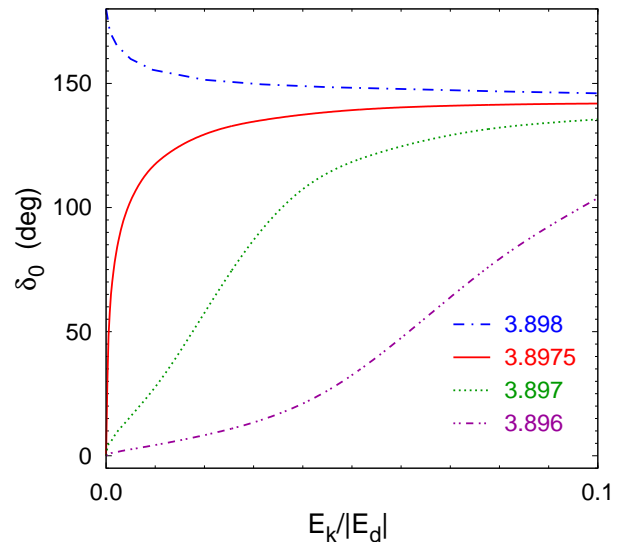


FIG. 3. $J^\Pi = 0^+$ phase shift for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy E_k . Enhanced NLO potential with $f_0 = 1.3$ and different higher-wave factors f_L indicated in the plot was used.

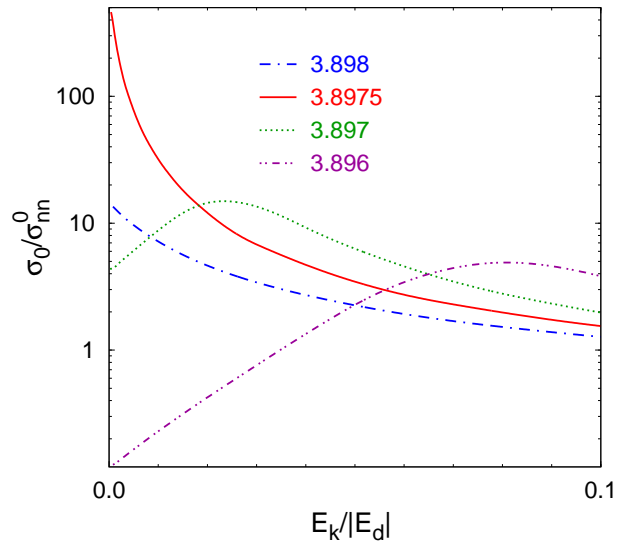


FIG. 4. $J^\Pi = 0^+$ total cross section σ_0 for the scattering of two artificially bound dineutrons as a function of the center-of-mass kinetic energy E_k . Enhanced NLO potential with $f_0 = 1.3$ and different higher-wave factors f_L indicated in the plot was used.

with $f_0 = 1.3$. Clearly, $f_L = 3.898$ corresponds to a bound state, while $f_L = 3.897$ and 3.896 cases appear to be resonant. However, at $f_L = 3.8975$ the energy-dependence of phase shift and cross section is consistent with the presence of a virtual $4n$ state very near to the threshold, not a bound state or resonance. Thus, in the case of the strong $L > 0$ wave enhancement the bound

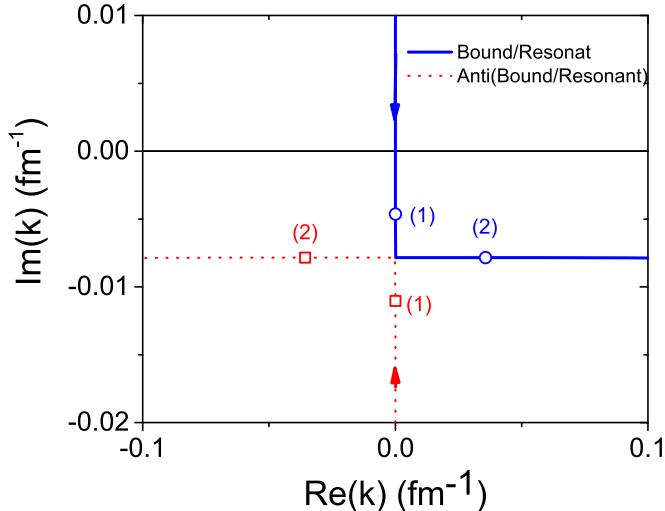


FIG. 5. Two-body S-matrix pole trajectories in the momentum manifold obtained varying the strength f_0 of the two-Gaussian potential in Eq. (2). Arrows indicate the direction of reducing f_0 . Points (1) and (2) are described in text.

state first evolves into a virtual state that, however, further evolves into a resonance.

The threshold behavior of the cross section can be read off from the two-dineutron effective range expansion

$$k \cot \delta_0 = -\frac{1}{a_{dd}} + \frac{1}{2} r_{dd} k^2 + o(k^4) \quad (1)$$

with the on-shell momentum k , the scattering length a_{dd} , and the effective range parameter r_{dd} . Namely, $\sigma_0 \sim 1/|k \cot \delta_0 - ik|^2$ is increasing (decreasing) at $E_k = 0$ if the ratio r_{dd}/a_{dd} is above (below) 1. Since in the regime $f_L < 3.898$ with no bound trineutron a_{dd} is negative, only negative r_{dd} with $|r_{dd}| > |a_{dd}|$ leads to increasing σ_0 as seen for $f_L \leq 3.897$ resonances in Fig. 4. Though $r_{dd} < 0$ in the whole regime $f_L \in [3.885, 3.9]$, large $|a_{dd}|$ at $f_L = 3.8975$ leads to decreasing σ_0 . The values are $a_{dd} \approx -29 a_0$ and $r_{dd} \approx -9 a_0$ at $f_L = 3.8975$, while $a_{dd} \approx -2 a_0$ and $r_{dd} \approx -14 a_0$ at $f_L = 3.897$, expressed in terms of the two-neutron scattering length $a_0 = 12.72$ fm (taken at $f_0 = 1.3$). Thus, in this regime r_{dd} is not only negative, but also of unnaturally large absolute value. As will be discussed below, this feature is essential for the appearance of resonant behavior. Note, that in all studied $f_L = f_0$ cases r_{dd} is positive, as in the universal regime [13].

This kind of bound state evolution into a resonance via the virtual state may also appear in two-body systems for the $L = 0$ (S -wave) state generated by the short range potential with special properties. A general feature of the S-matrix pole trajectory, represented in the momentum (k) manifold, is that it moves down along the imaginary axis when reducing the attraction. In this way

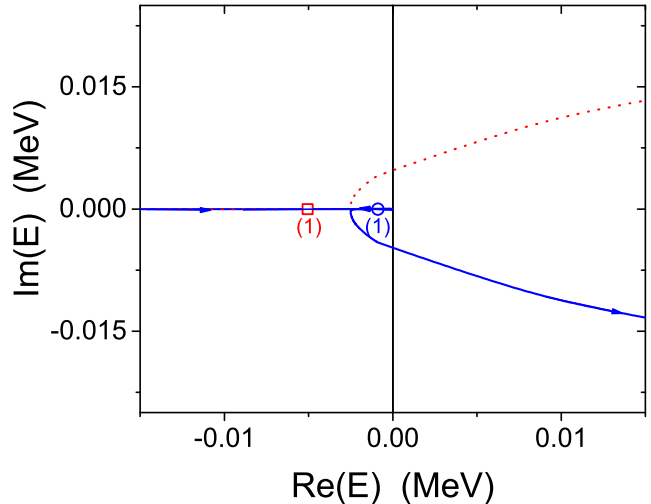


FIG. 6. Two-body S-matrix pole trajectories in the energy manifold close to threshold. Notation is taken from Fig. 5. Point (2) is outside the plot.

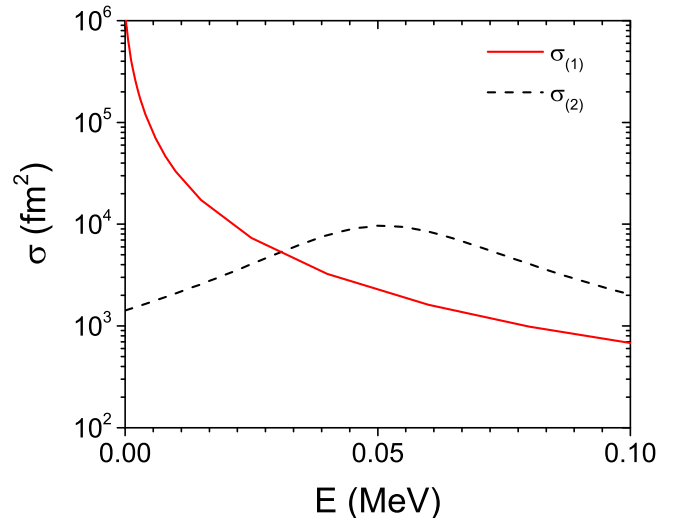


FIG. 7. Total cross sections for two-body scattering calculated using potentials in Eq. (2). Two cases are considered, corresponding to the marked points in Fig. 5: virtual state (1) and near-threshold resonance (2).

the bound state with $\text{Im}(k) > 0$ crosses the threshold becoming a virtual state with $\text{Im}(k) < 0$. Nevertheless, in some particular cases, quite soon this S-matrix pole may move into the fourth quadrant of the complex momentum plane: first, as long as $-\text{Im}(k) > \text{Re}(k)$, appearing as a subthreshold resonance ($\text{Re}(E) < 0$) and then, once $\text{Re}(k) > -\text{Im}(k)$, turning into a resonance above the

threshold ($\text{Re}(E) > 0$). Such a non-standard behavior is determined by the presence of a large negative effective range parameter r_0 in the effective range expansion of the form (1), which can be generated by employing a potential containing a repulsive barrier. To study this case, we adapt the two-Gaussian potential from Ref. [3]:

$$V(r) = f_0 \{-1000e^{-(r/0.4981)^2} + 865e^{-[(r-0.9972)/0.2877]^2}\}. \quad (2)$$

Here the potential $V(r)$ is in units of MeV, whereas the distance between the neutrons r is in fm; neutron mass is fixed by $\hbar^2/m_n = 41.4425 \text{ MeV}\cdot\text{fm}^2$. We study the evolution of the S-matrix pole by varying the potential enhancement factor f_0 . The S-matrix pole trajectory generated in this way is presented in Figs. 5 and 6, along with the trajectory of unphysical anti-bound/antiresonant state. The physical and unphysical poles collide at $k \approx i/r_0$, where r_0 approaches half of the two-body scattering length a_0 . This can be understood from the effective range expansion, which is typically a good approximation near the unitary limit $1/a_0 = 0$ and is suitable to estimate the positions of the near-threshold S-matrix poles given by

$$k = \frac{1}{r_0} (i \pm \sqrt{2r_0/a_0 - 1}). \quad (3)$$

Given a large negative a_0 for a near-threshold virtual state, large negative r_0 ($\sim -125 \text{ fm}$ in the present example) ensures that the two poles collide on the negative imaginary momentum axis in close vicinity of the threshold and then scatter by angle $\pi/2$, see Fig. 5. In Fig. 7 cross sections are plotted at two selected trajectory points: at $f_0^{(1)} = 1.004325$ generating a virtual state at $k = -0.0046i \text{ fm}^{-1}$, and at $f_0^{(2)} = 1.0042$ generating a resonance at $k = 0.0357 - 0.0078i$. The total cross section, dominated by the presence of the virtual state, decreases with energy from $E = 0$. On the other hand, a near-threshold resonance (with $\text{Re}(k) > -\text{Im}(k)$) leads to a cross section which grows with energy at $E = 0$. If the enhancement f_0 is further reduced, the S-matrix pole will recede from the real energy axis at the same time losing impact on the scattering cross section. The S-matrix pole trajectory is even more complicated when presented in the energy manifold, see Fig. 6. At the critical point $E = 0$ the physical pole of the S-matrix is reflected backward while being projected into the next Riemann sheet. At the point where physical and unphysical poles collide, their trajectories turn by $\pi/2$ acquiring imaginary energy parts, whereas the real energy parts once again start to increase. Obviously, such a non-analytic behavior is highly non-linear and cannot be approximated by a simple polynomial as was naively tried in [3].

Returning back to the $4n$ system, we would like to stress again the qualitative difference between the $4n$ $J^\Pi = 0^+$ S-matrix pole trajectories when bound dineutrons are produced from the ones when dineutrons are kept unbound. In the first case the trajectory is dominated by the two-dineutron threshold and thus inherits

features common to a two-body S -state one, i.e., the $4n$ bound state turns into a two-dineutron virtual state. On the contrary, if no bound dineutron states are present, at the critical point the $4n$ bound state evolves directly into a narrow resonance – this feature is determined by the presence of the repulsive $1/r^2$ term in the effective $4n$ hyperradial potential. Notably, such a behavior is also exhibited by three-body Efimov states. On the right side of the unitary limit (positive two-body scattering lengths) bound Efimov states evolve from the virtual atom-dimer states. On the contrary, on the left-hand side of the unitary limit (negative two-body scattering lengths) bound Efimov states appear from three-atom resonant states[15].

III. CONCLUSIONS

Using rigorous treatment of the four-particle continuum as given by FY and AGS equations, we investigated the $4n$ system with artificially enhanced interaction. In contrast to our previous studies [5, 6, 8] with no bound dineutrons, here we enhanced also the two-neutron 1S_0 potential, aiming to elucidate the 0^+ tetra-neutron resonance behavior in the presence of bound dineutrons, naturally appearing in the 1S_0 partial wave once the factor f_0 exceeds roughly 1.1. As even larger f_0 has been used in previous works suggesting the existence of a near-threshold [3] or broad [4] 0^+ tetra-neutron resonance, the present study was aiming to resolve the disagreement between Refs. [3, 4] and the $f_0 = 1$ approach [5, 6, 8] contesting the presence of an observable 0^+ tetra-neutron resonance.

Following the approach of Refs. [3, 4] with $f_0 = f_L = f$ we found that a significant enhancement $f \approx 2.4$ to 2.7 is needed to support a truly bound tetra-neutron below the two-dineutron threshold. However, when f is reduced the $4n$ bound state evolves not into a resonance but into a virtual state. This is convincingly demonstrated by the energy dependence of the two-dineutron phase shifts. Thus, the negative energy $4n$ states, considered in Refs. [3, 4] to be zero-width bound states evolving into resonances at positive energy, are neither bound states nor narrow resonances. From the point of view of rigorous four-particle theory the $E_{4n} < 0$ states above the two-dineutron threshold are either two-dineutron or dineutron plus two-neutron continuum states without resonant character. With decreasing f the scattering observables approach the universal zero-range behavior of the four-fermion system with $E_d = 0$. Note that on the way to the physical potential strength the unitary limit has to be crossed.

We have also shown that enhancing the higher-wave potential considerably stronger as compared to S wave may lead to a resonant $4n$ behavior, as demonstrated by the example of the two-dineutron phase shift and cross section. Anyhow, well before reaching the $f_0 = f_L$ limit, that resonance moves far away from the scattering re-

gion, possibly below the two-dineutron threshold in the unphysical sheet of the complex energy plane, thereby becoming experimentally unobservable. This is consistent with the previous studies [5, 6, 8]. Another remarkable feature of this enhancement scheme is the evolution of the bound state into a resonance via the virtual state appearing in a very narrow transition regime. A simple two-body example exhibiting the same type of evolution was presented. In both cases the essential feature is the presence of a large negative effective range parameter.

ACKNOWLEDGMENTS

The authors acknowledge discussions with J. Carbonell. A.D. acknowledges support by the Alexander von Humboldt Foundation under Grant No. LTU-1185721-HFST-E. We were granted access to the HPC resources of TGCC/IDRIS under the allocation 2018-A0030506006 made by GENCI (Grand Equipement National de Calcul Intensif).

-
- [1] K. Kisamori *et al.*, Phys. Rev. Lett. **116**, 052501 (2016).
 - [2] A. M. Shirokov, G. Papadimitriou, A. I. Mazur, I. A. Mazur, R. Roth, and J. P. Vary, Phys. Rev. Lett. **117**, 182502 (2016).
 - [3] S. Gandolfi, H.-W. Hammer, P. Klos, J. E. Lynn, and A. Schwenk, Phys. Rev. Lett. **118**, 232501 (2017).
 - [4] K. Fossez, J. Rotureau, N. Michel, and M. Płoszajczak, Phys. Rev. Lett. **119**, 032501 (2017).
 - [5] E. Hiyama, R. Lazauskas, J. Carbonell, and M. Kamimura, Phys. Rev. C **93**, 044004 (2016).
 - [6] A. Deltuva, Phys. Lett. B **782**, 238 (2018).
 - [7] S. A. Sofianos, S. A. Rakityansky, and G. P. Vermaak, J. Phys. G **23**, 1619 (1997).
 - [8] R. Lazauskas and J. Carbonell, Phys. Rev. C **72**, 034003 (2005).
 - [9] In Ref. [2] this was an auxiliary method.
 - [10] R. Lazauskas and J. Carbonell, Phys. Rev. C **71**, 044004 (2005).
 - [11] R. Lazauskas, Ph.D. thesis, Université Joseph Fourier, Grenoble, 2003, <http://tel.ccsd.cnrs.fr/documents/archives0/00/00/41/78/>.
 - [12] E. Epelbaum, H. Krebs, and U.-G. Meißner, Phys. Rev. Lett. **115**, 122301 (2015).
 - [13] A. Deltuva, Phys. Rev. A **96**, 022701 (2017).
 - [14] A. M. Badalyan, L. P. Kok, M. I. Polikarpov, and Y. A. Simonov, Phys. Rep. **82**, 31 (1982).
 - [15] P. Naidon and S. Endo, Rep. Prog. Phys. **80**, 056001 (2017).