



**HAL**  
open science

## Dark–antidark solitons in waveguide arrays with alternating positive–negative couplings

Aldo Auditore, Matteo Conforti, Costantino de Angelis, Alejandro B Aceves

► **To cite this version:**

Aldo Auditore, Matteo Conforti, Costantino de Angelis, Alejandro B Aceves. Dark–antidark solitons in waveguide arrays with alternating positive–negative couplings. *Optics Communications*, 2013, 297, pp.125-128. 10.1016/j.optcom.2013.01.068 . hal-02394381

**HAL Id: hal-02394381**

**<https://hal.science/hal-02394381>**

Submitted on 4 Dec 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Dark–antidark solitons in waveguide arrays with alternating positive–negative couplings

Aldo Auditore<sup>1\*</sup>, Matteo Conforti<sup>1</sup>, Costantino De Angelis<sup>1</sup>, Alejandro B. Aceves<sup>2</sup>

<sup>1</sup>*CNISM, Dipartimento di Ingegneria dell'Informazione,  
Università degli Studi di Brescia, Brescia 25123, Italy.*

<sup>2</sup>*Department of Mathematics, Southern Methodist University, Dallas, Texas 75275, USA\**

(Dated: November 14, 2012)

We obtain dark and antidark soliton solutions in binary waveguide arrays with focusing and/or defocusing Kerr nonlinearity and with alternating positive and negative linear couplings between adjacent waveguides. For both stationary and moving solitons, we analyze the properties of these solutions in the presence of uniform and non–uniform nonlinearity along the array.

PACS numbers:

## I. INTRODUCTION

In the last years, discrete optical systems and waveguide arrays have been a very active research area in optics [1]. Binary waveguide arrays in particular have been studied because their intrinsic two bands structure can be very helpful in order to control wave propagation in the linear and nonlinear regimes [2–4]. More recently, the interplay between plasmonic waveguiding and periodicity has been also considered, inasmuch as plasmonic confinement offers an extra degree of freedom to be usefully exploited in all–optical devices [5–7].

In this framework solitons represent an important class of solutions, as their particle like behaviour can be very useful for switching applications and their peculiar features often represent an unvaluable tool to understand the overall dynamics of the system in the nonlinear regime. This certainly explains the huge effort that the scientific community has put in finding soliton solutions in these systems [4, 9–14].

It is well known that in nonlinear discrete systems as those describing light propagation in waveguide arrays, bright localized modes may exist in the form of gap solitons when a gap opens in the linear dispersion relation [13, 15]. On the other hand, without the band gap in the linear dispersion relation, a forbidden frequency region can still exist for a nonuniform nonlinear response and solitons sitting on a pedestal can be found [14, 16, 17]. Among the situations where the nonuniform nonlinearity can be exploited it is certainly worth quoting the case of the linear–nonlinear interlaced waveguide arrays [18].

In previous papers we have used a continuous approximation to exploit bright solitary wave solutions of this system [13] and we have then used a fully discrete model to explore the existence and stability of solitons sitting on a non zero background [14]. In this paper we extend the continuous approximation to the case of a non zero background and we demonstrate that this continuous approximation can capture many features of the discrete system;

remarkably we also show that the continuum approximation gives a reasonable description of discrete states even when they are confined to a very small number of sites.

## II. PHYSICAL SETTINGS AND THEORETICAL ANALYSIS

According to coupled mode theory and taking into account third-order nonlinearities in the form of a pure Kerr effect, the field amplitude propagation in a binary waveguide array can be described by the following two sets of coupled equations with constant coefficients: [10, 13]

$$\begin{aligned} iA_{nz} + \frac{\Delta\beta}{2}A_n + C_1B_n + C_2B_{n+1} + \gamma_1|A_n|^2A_n &= 0 \\ iB_{nz} - \frac{\Delta\beta}{2}B_n + C_2A_{n-1} + C_1A_n + \gamma_2|B_n|^2B_n &= 0 \end{aligned} \quad (1)$$

where  $A_n$ ,  $B_n$  are respectively the mode amplitudes in the even and odd waveguides,  $\Delta\beta$  represents the difference between the propagation constants in even and odd waveguides,  $\gamma_1$  ( $\gamma_2$ ) is the site-dependent nonlinear coefficient for  $n$  even (odd),  $C_1$  and  $C_2$  are the coupling coefficients when  $n$  is odd and even respectively. Finally, without loss of generality we can set  $C_2 = 1$ .

In close proximity of the band edge (i.e. around  $k_x = 0$  for  $C_1 < 0$  and for  $k_x$  around  $\pi$  for  $C_1 > 0$ ) a very useful equivalent continuous model can be derived by performing a Taylor expansion to obtain (as a first order approximation):

$$\begin{aligned} iu_z + \frac{\Delta\beta}{2}u + w_x + \epsilon w + \gamma_1|u|^2u &= 0 \\ iw_z - \frac{\Delta\beta}{2}w - u_x + \epsilon u + \gamma_2|w|^2w &= 0 \end{aligned} \quad (2)$$

where we have also defined  $C_1 = \pm 1 + \epsilon$ , with the  $+$  ( $-$ ) sign that has to be used for  $k_x = 0$  ( $k_x = \pi$ ). To look for both stationary and walking self confined solutions of the system defined by Eqs. (2), we use the following trial

---

\*Electronic address: aldo.auditore@ing.unibs.it

functions [19]:

$$\begin{aligned}
u(x, z) &= \frac{1}{2} (K_1 g_1(\xi) + i K_2 g_2(\xi)) \exp(i \cos(Q) \psi) \\
w(x, z) &= \frac{1}{2i} (K_1 g_1(\xi) - i K_2 g_2(\xi)) \exp(i \cos(Q) \psi) \\
\xi &= \frac{x + vz}{\sqrt{1-v^2}} \quad , \quad \psi = \frac{vx + z}{\sqrt{1-v^2}} \\
K_1 &= \left(\frac{1+v}{1-v}\right)^{1/4} \quad , \quad K_2 = \left(\frac{1-v}{1+v}\right)^{1/4} \quad (3)
\end{aligned}$$

with  $g_{1,2}$  two arbitrary complex functions,  $-1 < v < 1$ . Although not necessary, for the sake of clarity, from now on we set  $\Delta\beta = 0$ .

Substituting the ansatz (3) into Eqs. (2) and following the procedure as in [13], it is straightforward to obtain an Hamiltonian form for the equations, observing that  $P = |g_1|^2 - |g_2|^2$  is a constant of motion for the dynamical system. Indeed, setting  $g_{1,2}(\xi) = f_{1,2}(\xi) \exp[i\theta_{1,2}(\xi)]$ ,  $\eta = f_2^2$  and  $\mu = \theta_1 - \theta_2$ ,  $\eta$  and  $\mu$  obey the following Hamiltonian system:

$$\begin{aligned}
\dot{\eta} &= -\frac{\partial H}{\partial \mu} \\
\dot{\mu} &= \frac{\partial H}{\partial \eta} \\
H &= 2\eta \cos Q + 2\sqrt{\eta(\eta+P)}\epsilon \cos \mu \\
&\quad - \frac{s}{8}\eta \left( \eta \left( \frac{K_1^4}{2} + \frac{K_2^4}{2} + 2 - \cos(2\mu) \right) \right. \\
&\quad \left. + P \left( K_1^4 + 2 - \cos(2\mu) \right) \right) \\
&\quad - \frac{d}{4}\sqrt{\eta(\eta+P)} \left( \eta(K_1^2 + K_2^2) + PK_1^2 \right) \sin \mu. \quad (4)
\end{aligned}$$

where we also set  $s = \gamma_1 + \gamma_2$  and  $d = \gamma_1 - \gamma_2$ .

It is straightforward to show that this Hamiltonian has the following symmetry: mapping  $P$  and  $v$  into  $-P$  and  $-v$  induces only a non nonessential shift by a constant into the Hamiltonian's value. For this reason, from now on we set  $P \geq 0$ . Note also that the Hamiltonian system described by Eqs.(4) reduces obviously to the one considered in [13] for  $P = 0$ . Moreover in the situation considered in [13] the condition  $\epsilon \neq 0$  was necessary in the quest for bright soliton solutions; here, on the contrary, we are interested in discussing solitons with a non zero background and their existence is not related to the presence of a band gap in the linear spectrum, i.e. they exist even in the case  $\epsilon = 0$  as we have discussed in [14]. As a matter of fact, the key properties of these solutions sitting on a non zero background do not depend on the presence of a bandgap in the linear spectrum; for these reasons from now on we consider the case  $\epsilon = 0$ , corresponding to the presence of a Dirac point at zero transverse momentum in the linear spectrum. Soliton solutions of Eqs. 2 correspond to separatrix trajectories emanating from and sinking into unstable fixed points of the dynamical system described by Eqs. 4.

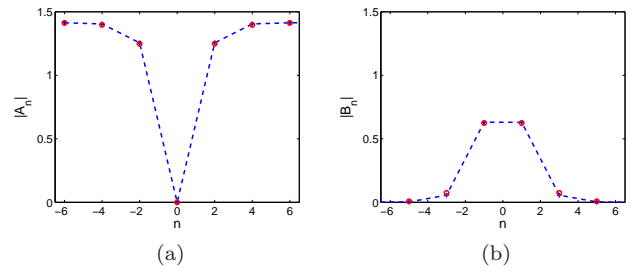


FIG. 1: Comparison between approximated soliton states obtained from the continuum limit (blue crosses and dashed line) and exact numerical results obtained using the Newton conjugate–gradient method (red open circles). With reference to Eqs. 1 here we set  $\gamma_1 = 1, \gamma_2 = 0, C_1 = -1, C_2 = 1, \Delta\beta = 0$  and  $\lim_{n \rightarrow \pm\infty} |A_n| = \sqrt{2}$ . a): even sites; b): odd sites.

Obviously the validity of the continuum approximation becomes more questionable as the degree of localization of the solitons increases; in order to get a feeling of how far one can push the use of the continuum model while still having a reasonable description of the discrete system, we performed a thorough comparison between the approximated soliton solutions obtained in the continuum limit and exact soliton states obtained numerically using the Newton conjugate–gradient method [20].

To summarize our findings we report in figure 1 the comparison between the modulus of the approximated soliton states obtained from the continuum limit (blue crosses and dashed line) and exact numerical results obtained using the Newton conjugate–gradient method (red open circles). Note that despite the very strong degree of localization (the soliton structure is basically confined to 5 sites only), the continuum approximation still shows an excellent agreement with the exact solution and this holds true both for the modulus (reported in figure 1) and the phase (not shown here). If we further increase the degree of confinement we gradually lose the validity of the continuum approximation and for soliton states confined to 3 sites only the continuum approximation is not able to capture closely the features of the discrete solutions. For such a high level of confinement one has to go back to the discrete model and find there solutions as was recently done by using asymptotic expansions in [14]. It is remarkable to note that by using these two different approximations (asymptotic expansion and continuum model) one can describe in a simple and accurate fashion the entire spectrum of the dark–antidark soliton states of this system.

The derived hamiltonian system thus represents a valuable tool in describing and in understanding the features of the solutions of the problem; the goal of the rest of the paper is to prove that thanks to this hamiltonian system we are able to introduce new soliton solutions for the problem at hand. Moreover, we will always test the validity of our results, obtained in the framework of a continuum approximation, in the original (truly discrete)

system. The above goals will be pursued using some representative examples; we will thus focus our attention on three different binary arrays: the first case we will discuss is the case of uniform nonlinearity in the array; the second case we will face is that of the linear–nonlinear interlaced binary array and the third case will be the case of nonlinearities with different signs along the array, i.e. the focusing–defocusing interlaced binary waveguide array.

### III. EXAMPLES

In this section we present some results derived from the analysis of the Hamiltonian system (Eqs. 4). We thus first look for unstable fixed points of the dynamical system and then obtain the separatrices corresponding to solitary wave solutions. To assess the validity of our approach we then propagate the obtained waveforms in the truly discrete system described by Eqs.1. If we were looking for bright solitons, as we did in [13], we would pick  $P = 0$  and this in turn would simplify considerably the algebra required in the analysis of the dynamical system described by Eqs.4; here, on the contrary, we can not limit ourselves to the simplest case  $P = 0$  and thus we focus on the more general case  $P \neq 0$ .

As a first example, we consider an array with uniform nonlinearity (i.e.  $d = 0$ ). In figure 2a we report the bifurcation diagram of the amplitude  $\eta$  of the fixed points as a function of  $Q$ ; unstable fixed points and thus dark solitons do exist only for  $Q < \arccos(sP(3 + k_1^2)/16)$ ; note that the unstable fixed points here correspond to two different branches with different generalized phase ( $\mu = \pm\pi/2$ ). In figure 2b we report the phase plane analysis of the system and we see there two saddle points at  $\mu = \pm\pi/2$  and  $\eta = (16 \cos(Q) - sP(K_1^4 + 3))/(s(K_1^4 + K_2^4 + 6))$ ; the separatrices emanating from and sinking into the saddles turn around the center located at  $\mu = 0$  and  $\eta = (16 \cos Q - sP(K_1^4 + 1))/(s(K_1^4 + K_2^4 + 2))$ . The existence of these saddle points is possible if the following constraints on  $Q$  and  $P$  are satisfied:  $0 < Q < \arccos(sP(K_1^4 + 3)/16)$  and  $0 < P < 16/(s(K_1^4 + 3))$ . To test the validity of these results we use the obtained solutions as the initial condition (i.e. at  $z=0$ ) in the numerical integration of the equations describing propagation in the binary waveguide array (i.e. Eqs. 1); the results are reported in figure 3a where the initial condition corresponds to the trajectory enlightened with a thicker line in the inset of the figure. Note that propagation of this solution into the array reveals only a small amount of radiation (almost non visible) thus proving the validity of the continuum approximation; moreover these solutions are strong enough to survive for non zero transverse velocities; in figure 3b we report one example of non zero transverse velocity and we observe once again that the continuum system is able to capture the nature of the discrete one. Obviously the solution cannot move without radiating some energy; however the amount of

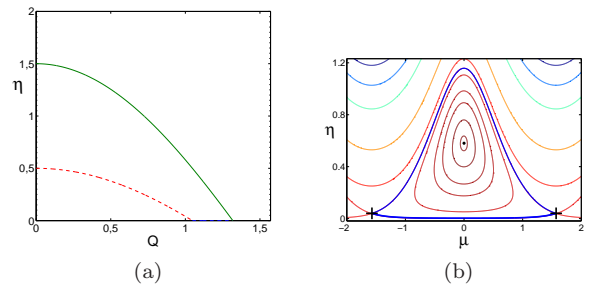


FIG. 2:  $s = 2$ ,  $d = 0$ ,  $P = 1$ ,  $v = 0$ . a) Bifurcation diagram of the Hamiltonian system: continuous lines for stable centers, dashed lines for unstable saddle points. b) Phase plane analysis ( $Q = 1$ ): crosses correspond to the unstable points; the dot shows the stable center. The thicker lines are the heteroclinic separatrices connecting the two unstable saddles and correspond to solitons on a non zero background.

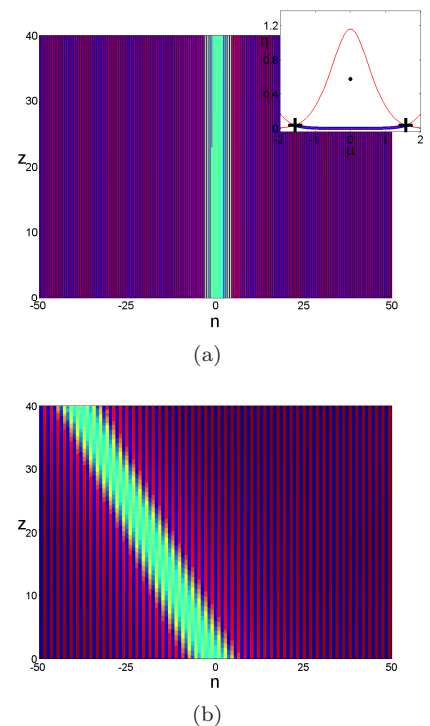


FIG. 3: Field evolution along the array for  $s = 2$ ,  $d = 0$ ,  $P = 1$ : the initial condition is the dark soliton solution corresponding to the thicker line in the inset where the phase plane of system 4 is reported. a):  $Q = 1$ ,  $v = 0$ ; b):  $Q = 0.7$ ,  $v = 0.5$

radiation is almost negligible and the soliton can propagate with high velocity along the array. As a second example we consider the case of an array of alternating linear-nonlinear waveguides, i.e.  $s = d$ . The situation is quite different with respect to the  $d = 0$  case; two fixed points (one center and one saddle) exist for  $Q < \pi/2$ ; only one stable center exists for  $Q > \pi/2$  (see figure 4a). In figure 5 we report the beam propagation along the array

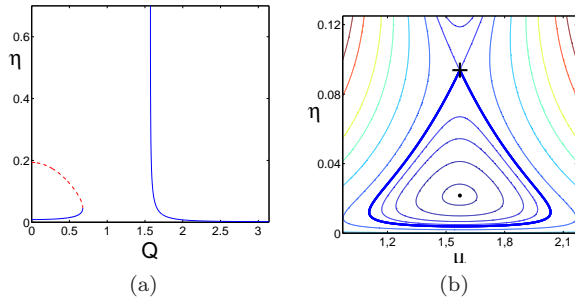


FIG. 4:  $s = 2$ ,  $d = 2$ ,  $P = 0.6$ ,  $v = 0$ . a) Bifurcation diagram of the Hamiltonian system: continuous lines for stable centers, dashed lines for unstable saddle points. b) Phase plane analysis ( $Q = 0.6$ ): the cross corresponds to the unstable point; the dot shows the stable center. The thicker line is the homoclinic separatrix corresponding to solitons on a non zero background.

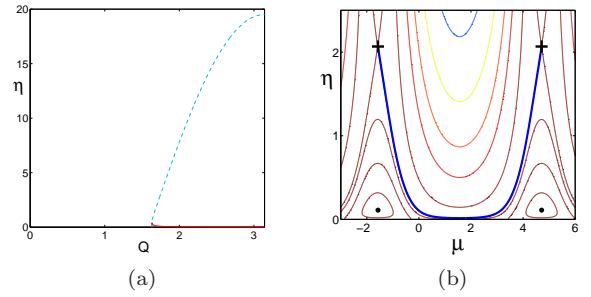


FIG. 6:  $s = 2$ ,  $d = 2.1$ ,  $P = 1.0$ ,  $v = 0$ . a) Bifurcation diagram of the Hamiltonian system: continuous lines for stable centers, dashed lines for unstable saddle points. b) Phase plane analysis ( $Q = 1.7$ ): the crosses correspond to the unstable points; the dots show the stable centers. The thicker line is the heteroclinic separatrix corresponding to solitons on a non zero background.

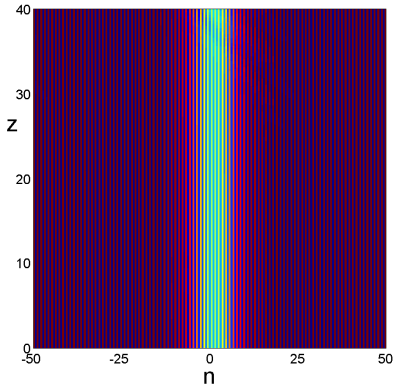


FIG. 5: Field evolution along the array: the initial condition corresponds to the separatrix enlightened in figure 4b.

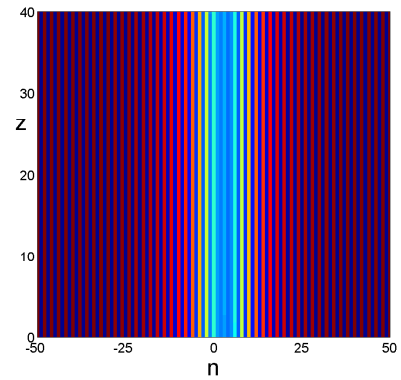


FIG. 7: Field evolution along the array for  $s = 2$ ,  $d = 2.1$ ,  $Q = 1.7$ ,  $P = 1$  and  $v = 0$ . The initial condition corresponds to the separatrix enlightened in figure 6b.

using as initial condition the field profiles obtained from Eqs. 3 after having solved for the trajectory along the separatrix described as a thick line in figure 4b. The third example we consider corresponds to an array of alternating focusing-defocusing nonlinearities, i.e.  $d > s$  ( $s = 2$  and  $d = 2.1$  in what follows). As we can see in figure 6a, for  $Q > \pi/2$  we find one center and one saddle. In the corresponding phase plane in figure 6b we report the heteroclinic trajectory emanating from and sinking into the saddle points at  $\mu = -\pi/2$  and  $\mu = 2\pi - \pi/2$ . In figure 7 we show the field evolution along the array using as initial condition the field profiles obtained from Eqs. 3 and corresponding to the separatrix enlightened in figure 6b. Once again we can observe that propagation of this solution into the array reveals only an almost invisible small amount of radiation thus proving the validity of our analytical approach.

## CONCLUSIONS

In this work we have obtained dark and antidark soliton solutions in a binary waveguide array with alternating positive and negative linear couplings between adjacent waveguides and in the presence of focusing and/or defocusing Kerr nonlinearity. These solutions do exist also in a linear–nonlinear interlaced array and they even survive in focusing–defocusing interlaced arrays. We have also numerically verified the soundness of our approach by a detailed comparison with exact results obtained by numerically solving the discrete system; remarkably our results, obtained in the framework of a continuum approximation, retain their validity also for very strong degrees of localization.

## ACKNOWLEDGMENTS

The authors wish to thank Professor T. R. Akylas for very helpful discussions. AA, CDA and MC acknowl-

edge financial support from CARIPLO Foundation under grant no. 2010-0595 and US ARMY under grant no. W911NF-12-1-0202.

- 
- [1] F. Lederer, G. I. Stegeman, D. N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, “Discrete solitons in optics”, *Phys. Rep.* 463, 1 (2008).
- [2] S. Longhi, “Multiband diffraction and refraction control in binary arrays of periodically curved waveguides”, *Opt. Lett.* 31, 1857-1859 (2006).
- [3] M. Guasoni, A. Locatelli, and C. De Angelis, “Peculiar properties of photonic crystal binary waveguide array”, *Journal of the Optical Society of America B* 25, 1515 (2008).
- [4] N. K. Efremidis, P. Zhang, Z. Chen, D. N. Christodoulides, C. E. Rüter, and Detlef Kip, “Wave propagation in waveguide arrays with alternating positive and negative couplings”, *Phys. Rev. A* 81, 053817 (2010).
- [5] M. Conforti, M. Guasoni, and C. De Angelis, “Sub-wavelength diffraction management”, *Opt. Lett.* 33, 2662 (2008).
- [6] M. Guasoni, M. Conforti, and C. De Angelis, “Light propagation in nonuniform plasmonic subwavelength waveguide arrays”, *Opt. Comm.* 283, 1161 (2010).
- [7] S. H. Nam, E. Ulin-Avila, G. Bartal, and X. Zhang, “Deep subwavelength surface modes in metal-dielectric metamaterials”, *Opt. Lett.* 35, 1847 (2010).
- [8] C. M. de Sterke, L. C. Botten, A. A. Asatryan, T. P. White, and R. C. McPhedran, “Modes of coupled photonic crystal waveguides”, *Opt. Lett.* 29, 1384 (2004).
- [9] Y. S. Kivshar, and A. A. Sukhorukov, “Optical solitons: From Fibers to Photonic Crystals”, (Academic Press, San Diego, 2003).
- [10] A. A. Sukhorukov, and Y. S. Kivshar, “Discrete gap solitons in modulated waveguide arrays”, *Opt. Lett.* 27, 2112-2114 (2002).
- [11] R. Morandotti, D. Mandelik, Y. Silberberg, J. S. Aitchison, M. Sorel, D. N. Christodoulides, A. A. Sukhorukov, and Y. S. Kivshar, “Observation of discrete gap solitons in binary waveguide arrays,” *Opt. Lett.* 29, 2890-2892 (2004).
- [12] A. A. Sukhorukov, and Y. S. Kivshar, “Soliton control and Bloch-wave filtering in periodic photonic lattices”, *Opt. Lett.* 30, 1849-1851 (2005).
- [13] M. Conforti, C. De Angelis, and T.R. Akylas, “Energy localization and transport in binary waveguide arrays,” *Phys. Rev. A* 83, 043822 (2011).
- [14] M. Conforti, C. De Angelis, T. R. Akylas, A. B. Aceves, “Modulational stability and gap solitons of gapless systems: Continuous versus discrete limits”, *Physical Review A* 85, 063836(1-4) (2012).
- [15] A. Marini, A. V. Gorbach, and D. V. Skryabin, “Coupled-mode approach to surface plasmon polaritons in nonlinear periodic structures”, *Opt. Lett.* 35, 3532 (2010).
- [16] Y. S. Kivshar, “Self-induced gap solitons,” *Phys. Rev. Lett.* 70, 3055 (1993).
- [17] N. Flytzanis and B. A. Malomed, “Gap solitons in gapless coupled chains,” *Phys. Lett. A* 227, 335 (1997).
- [18] K. Hizanidis, Y. Kominis, N. K. Efremidis,
- [19] A. B. Aceves, and S. Wabnitz, “Self-induced transparency solitons in nonlinear refractive periodic media”, *Physics Letters A* 141, 37 (1989).
- [20] J. Yang, “Newton-conjugate-gradient methods for solitary wave computations”, *Journal of Computational Physics* 228, 7007 (2009).