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# Resonance line polarization and the Hanle effect in optically thick media

## I. Formulation for the two-level atom

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**Abstract.** A general formalism is presented to describe resonance line polarization for a two-level atom in an optically thick, three-dimensional medium embedded in an arbitrary varying magnetic field and irradiated by an arbitrary radiation field. The magnetic field is supposed sufficiently small to induce a Zeeman splitting much smaller than the typical line width. By neglecting atomic polarization in the lower level and stimulated emission, an integral equation is derived for the multipole moments of the density matrix of the upper level. This equation shows how the multipole moments at any assigned point of the medium are coupled to the multipole moments relative at a different point as a consequence of the propagation of polarized radiation between the two points. The equation also accounts for the effect of the magnetic field, described by a kernel locally connecting multipole moments of the same rank, and for the role of inelastic and elastic (or depolarizing) collisions.

After having given its formal derivation for the general case, the integral equation is particularized to the one-dimensional and two-dimensional cases. For the one-dimensional case of a plane parallel atmosphere, neglecting both the magnetic field and depolarizing collisions, the equation here derived reduces to a previous one given by Rees (1978).

**Key words:** radiation transfer – polarization – magnetic field – solar and stellar atmosphere

### 1. Introduction

Resonance scattering is one of the fundamental physical processes that are responsible for the appearance of linear polarization—called in this case resonance polarization—in spectral lines. Resonance polarization is heavily modified by the presence of a magnetic field when the Zeeman splitting induced by this field is comparable to or larger than the natural width of the upper level involved in the transition (Hanle effect). Resonance

polarization and the Hanle effect have proven in recent years to be a powerful diagnostic tool for the analysis of various physical properties of the solar atmosphere; in particular, the Hanle effect is, in most cases, the unique physical process that makes possible the measurement of the magnetic field vector through a convenient analysis of spectropolarimetric observations.

Resonance polarization has been widely observed in solar spectral lines, either on the disk close to the limb or in coronal (or prominence) plasma. Important contributions in this field are those of Stenflo et al. (1983a,b) and Wiehr (1975, 1978, 1981) for disk observations and those of Leroy et al. (1977), Querfeld (1977), Arnaud (1977), Leroy (1981) for the coronal plasma.

The modification of resonance polarization due to the presence of a magnetic field (Hanle effect) has also been observed in solar spectral lines; most of the observations were obtained in prominences where the Hanle effect generally manifests itself through a depolarization of resonance radiation and the rotation of the plane of linear polarization. Extensive series of measurements on the He I D<sub>3</sub> line and various Balmer lines from prominences have been reported by Leroy et al. (1977, 1984), Leroy (1981) and Athay et al. (1983). A single observation in the Ca I  $\lambda$ 4227 line by Stenflo (1982) has however shown that the Hanle effect can be observed also on the solar disk.

Although the physical processes that are at the basis of resonance polarization and the Hanle effect are nowadays well understood, little effort has been devoted to the development of a suitable theoretical scheme for the description of resonance polarization in an optically thick medium embedded in a magnetic field. Early contributions in this field are those of Dumont et al. (1973, 1977), Stenflo and Stenholm (1976), and Rees (1978), where resonance polarization is derived for non-magnetic, plane-parallel atmospheres (or for slabs of finite optical thickness) in the approximation of complete (or partial) redistribution in frequency. In successive works, more general redistribution functions are considered but the calculations are still restricted to non-magnetic plane parallel atmospheres or slabs (Rees and Saliba, 1982; Saliba, 1985, 1986; Faurobert, 1987, 1988).

Resonance scattering in magnetic media with geometries different from the plane-parallel case have been considered by Landi Degl'Innocenti et al. (1987) and by Bommier et al. (1989) in a series of papers that was aimed at the interpretation of

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linear polarization observed in Balmer lines from optically thick prominences (or filaments). In the last paper the prominence plasma was schematized as an infinite cylinder having an elliptical cross-section and standing horizontally over the solar surface. Resonance polarization, as modified by the presence of a magnetic field, was computed by means of a perturbative approach starting from a zero-order prominence model developed by Heasley and Milkey (1978). Although the results presented in the papers by Landi Degl'Innocenti et al. (1987) and Bommier et al. (1989) can be safely applied to the interpretation of the observations, the approach followed in these papers for the description of resonance polarization is rather empirical and needs to be improved from the point of view of internal consistency.

The aim of the present paper is to derive a convenient formalism able to describe the physics of resonance polarization in an optically thick, magnetic medium. The approach that we will follow is to describe the atomic system through its density-matrix operator. The value of the density-matrix at any assigned point of the medium is coupled to the polarized radiation field, and, through the effect of collisions, to the local thermodynamical properties like temperature and density. This coupling is suitably described by the statistical equilibrium equations which also contain the effect of the local magnetic field. On the other hand, the polarized radiation field propagating, at any assigned point, along a given direction is coupled, through the equations of radiative transfer, to the atomic density-matrix at the points belonging to the associated ray-path. From this point of view, the problem that we are going to attack is nothing but a generalization of the well-known non-LTE problem with the only difference that the atom is now described in terms of its density-matrix (rather than just its level populations) while the radiation field is described through the four Stokes parameters (and not only through its intensity). For this reason, we will refer in the following to this problem as a *non-LTE problem of the 2nd kind*.

The approach followed in the present paper is quite different from the one of the papers quoted above, where the introduction of the atomic density-matrix is avoided and the physics of resonance polarization is just described in terms of the Rayleigh scattering phase-matrix. This makes the generalization of this last approach to more complicated cases either more difficult or impossible. Indeed, in the presence of a magnetic field the scattering phase-matrix becomes very involved (see Landi Degl'Innocenti and Landi Degl'Innocenti (1988) for its analytical expression). This fact has practically precluded any generalization to the magnetic case of previous works on resonance polarization. On the other hand, in those cases where resonance scattering also induces atomic polarization in the lower level (depopulation pumping), the Stokes parameters of the scattered radiation cannot be expressed as a linear function of the Stokes parameters of the incident radiation, so that the concept itself of scattering phase-matrix loses any meaning. On the contrary, the approach of the present paper does not suffer from this kind of limitation, and the influence of the magnetic field on resonance polarization can be taken into account, as we will see in the following, just by the addition of one term in the relevant equations.

The formalism that will be used in the present paper is the one developed by Landi Degl'Innocenti (1983a,b, 1984, 1985; hereafter referred to as Papers I, II, III and IV); in particular, the atomic density matrix will be described through its irreducible spherical components (or multipole moments) which have the advantage of making the underlying physics more transparent.

Obviously, the results that will be obtained in the following suffer from the same limitations contained in Papers I to IV; namely, the scattering process is described in the approximation of complete redistribution in frequency. Dumont et al. (1977) have shown that this is not a severe limitation for weak and medium lines and even for the core of the strongest lines, so that the theory developed here can be safely applied to a large number of astrophysical problems.

In this paper, the non-LTE problem of the 2nd kind is attacked for the simplest atomic model, namely for a two-level atom having an unpolarized ground level. Moreover, stimulated emission is neglected and the magnetic field is supposed to be sufficiently weak that its Zeeman splitting is negligible with respect to the Doppler broadening of the line (Hanle effect regime according to the classification scheme of Paper II). This simple model deserves particular attention and can serve as a standard reference for more complicated models that will be the subject of further investigations. In this respect it is sufficient to remind the reader about the importance of the analogous two-level atom problem in the usual non-LTE theory.

In Sect. 2, the basic theory for the two-level atom will be established in the general three-dimensional case for a medium of arbitrary geometrical shape. The theory will then be particularized to simpler geometrical situations where all the physical quantities depend only on one coordinate (Sect. 3) or only on two coordinates (Sect. 4). The basic equations for resonance scattering in a magnetized, plane-parallel atmosphere are thus obtained as a particular case of a more general formulation.

This paper is only aimed at establishing the basic theoretical formulation of the problem. The numerical solutions of the equations derived here will be the subject of further papers of this series.

## 2. Formulation

We consider a two-level atom having a lower level with angular momentum  $J$  and an upper level with angular momentum  $J'$  and Landé factor  $g_{J'}$  and we suppose that the lower level has no atomic polarization, so that its only non-vanishing density-matrix element on the basis of the statistical tensors is  $\rho_0^0(J)$ . The definition used here for the statistical tensors is the standard one given in Eq. (28) of Paper III, namely:

$$\rho_Q^K(J) = \sum_{MM'} (-1)^{J-M} (2K+1)^{1/2} \begin{pmatrix} J & K & J \\ -M & Q & M' \end{pmatrix} \rho_J(M, M'), \quad (1)$$

and we normalize the density matrix elements to the overall population of the ground level so that we have:

$$\rho_0^0(J) = (2J+1)^{-1/2} \sum_M \rho_J(M, M) = (2J+1)^{-1/2}. \quad (2)$$

Neglecting stimulated emission we now write the statistical equilibrium equations for the statistical tensors of the upper level in a reference system having its  $z$ -axis directed along the magnetic field. Considering, for the time being, only the contribution from radiative processes, we have, from Eq. (12) of Paper IV:

$$\begin{aligned} 0 &= \frac{d}{dt} \rho_Q^K \\ &= -2\pi\nu_L g_{J'} Q \rho_Q^K - A \rho_Q^K + B(2J'+1)^{-1/2} w_J^{(K)} (-1)^Q \bar{J}_Q^K \end{aligned} \quad (3)$$

where  $\rho_Q^K$  is the density matrix of the upper level that will be written, from now on, without its argument ( $J'$ ) to shorten notations;  $\nu_L$  is the Larmor frequency that is connected to the local value  $B_0$  of the magnetic field:  $\nu_L = e_0 B_0 / (4\pi mc)$ ;  $A$  and  $B$  are the Einstein coefficients for spontaneous emission and for absorption respectively;  $w_{J'J}^{(K)}$  is the symbol introduced in Eq. (38) of Paper III. Finally  $\bar{J}_Q^K$  is the irreducible tensor of the polarized radiation field that is defined in terms of the Stokes parameters  $S_i(\nu, \Omega)$  propagating at frequency  $\nu$  along the direction  $\Omega$ , and in terms of the absorption profile  $\phi(\nu_0 - \nu)$  centered at the frequency  $\nu_0$  of the atomic transition:

$$\bar{J}_Q^K = \int_0^\infty d\nu \phi(\nu_0 - \nu) \oint \frac{d\Omega}{4\pi} \sum_{i=0}^3 \mathcal{T}_Q^K(i, \Omega) S_i(\nu, \Omega) \quad (4)$$

where  $\mathcal{T}_Q^K(i, \Omega)$  is a spherical tensor defined in the Appendix of Paper III.

Equation (1) holds in a reference system having its z-axis directed along the magnetic field. To obtain a more general equation we consider a new reference system  $xyz$  (see Fig. 1). In this new system the magnetic field direction is specified by the angles  $\theta_B$  and  $\chi_B$ , while the rotation  $R_B$  bringing the old reference system into the new one is specified by the Euler angles:

$$R_B \equiv (\alpha, -\theta_B, -\chi_B) \quad (5)$$

where  $\alpha$  is an angle that can be arbitrarily set equal to zero and whose actual value is related to the orientation of the  $x$  and  $y$  axes of the old system.

Under a rotation of the coordinate system the statistical tensors transform according to the equation [see Eq. (31) of Paper III]:

$$[\rho_Q^K]_{\text{new}} = \sum_{Q'} [\rho_Q^K]_{\text{old}} \mathcal{D}_{Q'Q}^{K*}(R_B) \quad (6)$$

where  $\mathcal{D}$  are the ordinary rotation matrices. On the other hand, the radiation tensor  $\bar{J}_Q^K$  transforms according to [see Eq. (A18) of Paper III]:

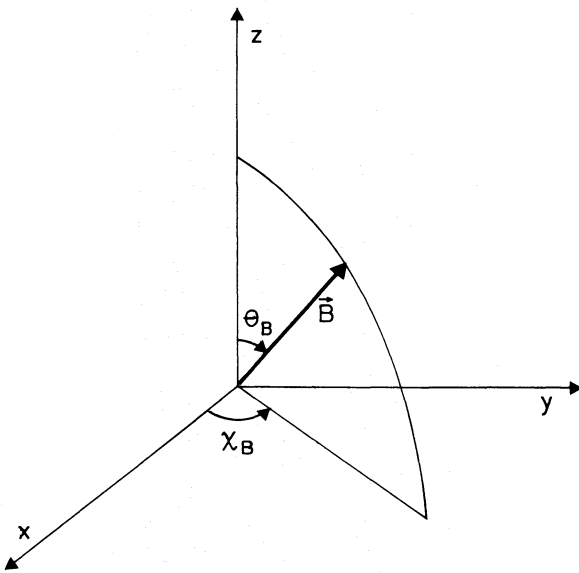


Fig. 1. The direction of the magnetic field vector is specified through the angles  $\theta_B$  and  $\chi_B$

$$[J_Q^K]_{\text{new}} = \sum_{Q'} [J_Q^K]_{\text{old}} \mathcal{D}_{Q'Q}^K(R_B). \quad (7)$$

Multiplying Eq. (3) by  $\mathcal{D}_{Q'Q}^{K*}(R_B)$ , summing over  $Q$ , and dividing the same equation by  $A$ , we obtain after some algebra the following equation for the tensors defined in the new reference system:

$$0 = -i\Gamma \sum_{Q'} \mathcal{X}_{Q'Q}^K \rho_Q^K - \rho_Q^K + \frac{B}{A} (2J' + 1)^{-1/2} w_{J'J}^{(K)} (-1)^Q \bar{J}_Q^K \quad (8)$$

where  $\Gamma = (e_0 B_0 g_J) / (2mcA)$  is the value of the magnetic field in relative units and where  $\mathcal{X}_{Q'Q}^K$ , the magnetic kernel, is defined by:

$$\mathcal{X}_{Q'Q}^K = \sum_{Q''} Q'' \mathcal{D}_{Q''Q}^{K*}(R_B) \mathcal{D}_{Q''Q'}^K(R_B). \quad (9)$$

Through some Racah algebra, whose details are given in Appendix A, the magnetic kernel can be expressed in the form:

$$\mathcal{X}_{Q'Q}^K = [K(K+1)(2K+1)]^{1/2} (-1)^{K+Q} \times \begin{pmatrix} K & K & 1 \\ Q & -Q & Q' - Q \end{pmatrix} \mathcal{D}_{0, Q' - Q}^1(R_B). \quad (10)$$

Substituting for  $R_B$  its expression given in Eq. (5), we obtain for the magnetic kernel the analytical values contained in Table 1.

We now turn to the equations of radiative transfer to find a convenient expression for the radiation tensor appearing in Eq. (8). Neglecting stimulated emission we have, for the Stokes vector propagating at frequency  $\nu$  along the direction  $\Omega$  (see Eq. (65) of Paper I):

$$\frac{d}{ds} S_i(\nu, \Omega) = -\sum_j K_{ij}(\nu, \Omega) S_j(\nu, \Omega) + \varepsilon_i(\nu, \Omega) \quad (11)$$

where  $K_{ij}$  is the absorption matrix written here in condensed form,  $\varepsilon_i(\nu, \Omega)$  is the emission vector and  $s$  is the coordinate measured along the ray path.

As we have supposed the ground level to be unpolarized and as the magnetic field is supposed to be sufficiently weak to induce a Zeeman splitting much smaller than the typical width of the line profile, the absorption matrix  $K_{ij}$  is proportional to the identity matrix so that we can write:

$$K_{ij} = \eta_\nu \delta_{ij} \quad (12)$$

where the absorption coefficient  $\eta_\nu$  is given, in terms of the local density  $N_J$  of atoms in the lower level by:

$$\eta_\nu = \frac{h\nu_0}{4\pi} B N_J \phi(\nu_0 - \nu) = \eta_0 \phi(\nu_0 - \nu). \quad (13)$$

The emission coefficient can be expressed in terms of the statistical tensors of the upper level. Referring to Eq. (33) of Paper III we have<sup>1</sup>:

$$\begin{aligned} \varepsilon_i(\nu, \Omega) &= \frac{h\nu_0}{4\pi} A (2J' + 1)^{1/2} N_J \sum_{KQ} w_{J'J}^{(K)} \rho_Q^K \mathcal{T}_Q^K(i, \Omega) \phi(\nu_0 - \nu) \\ &= \eta_\nu \frac{A (2J' + 1)^{1/2}}{B} \sum_{KQ} w_{J'J}^{(K)} \rho_Q^K \mathcal{T}_Q^K(i, \Omega) \\ &= \eta_\nu s_i(\Omega) \end{aligned} \quad (14)$$

<sup>1</sup> Note that the factor  $\mathcal{N}$  contained in Eq. (33) of Paper III is substituted here with  $N_J$ . This is due to a different choice of the normalization factor for the density matrix.

**Table 1.** Analytical expressions for the magnetic kernel  $\mathcal{H}_{\mathcal{Q}\mathcal{Q}'}^K$ 

$$\begin{aligned}
\mathcal{H}_{00}^0 &= 0 \\
\mathcal{H}_{-1-1}^1 &= -\mathcal{H}_{11}^1 = -\cos\theta_B \\
\mathcal{H}_{-10}^1 &= \mathcal{H}_{01}^1 = \frac{1}{\sqrt{2}} \sin\theta_B e^{i\chi_B} \\
\mathcal{H}_{0-1}^1 &= \mathcal{H}_{10}^1 = \frac{1}{\sqrt{2}} \sin\theta_B e^{-i\chi_B} \\
\mathcal{H}_{-11}^1 &= \mathcal{H}_{00}^1 = \mathcal{H}_{1-1}^1 = 0 \\
\mathcal{H}_{-2-2}^2 &= -\mathcal{H}_{22}^2 = -2\cos\theta_B \\
\mathcal{H}_{-2-1}^2 &= -\mathcal{H}_{11}^2 = -\cos\theta_B \\
\mathcal{H}_{-2-1}^2 &= \mathcal{H}_{12}^2 = \sin\theta_B e^{i\chi_B} \\
\mathcal{H}_{-2-1}^2 &= \mathcal{H}_{21}^2 = \sin\theta_B e^{-i\chi_B} \\
\mathcal{H}_{-2-1}^2 &= \mathcal{H}_{01}^2 = \frac{\sqrt{3}}{\sqrt{2}} \sin\theta_B e^{i\chi_B} \\
\mathcal{H}_{0-1}^2 &= \mathcal{H}_{10}^2 = \frac{\sqrt{3}}{\sqrt{2}} \sin\theta_B e^{-i\chi_B} \\
\mathcal{H}_{-20}^2 &= \mathcal{H}_{-21}^2 = \mathcal{H}_{-22}^2 = \mathcal{H}_{-11}^2 = \mathcal{H}_{-12}^2 = \mathcal{H}_{0-2}^2 = \mathcal{H}_{00}^2 \\
&= \mathcal{H}_{02}^2 = \mathcal{H}_{1-2}^2 = \mathcal{H}_{1-1}^2 = \mathcal{H}_{2-2}^2 = \mathcal{H}_{2-1}^2 = \mathcal{H}_{20}^2 = 0
\end{aligned}$$

where we have introduced the (angle-dependent) source function  $S_i(\mathbf{\Omega})$  for the various Stokes parameters.

For the equation of radiative transfer a formal solution can easily be given. For any point  $P$  inside the medium, having coordinate vector  $\mathbf{x}$ , the Stokes vector can be written in the form (see Fig. 2):

$$[S_i(v, \mathbf{\Omega})]_{\mathbf{x}} = \int_0^{l(\mathbf{x}, \mathbf{\Omega})} [S_i(\mathbf{\Omega})]_{\mathbf{x}'} e^{-\tau_v(\mathbf{x}, \mathbf{x}')} \eta_v(\mathbf{x}') ds + S_i^{(0)}(v, \mathbf{\Omega}) e^{-\tau_v(\mathbf{x}, x_0)} \quad (15)$$

where  $s$  is the coordinate measured from the boundary along the ray path having direction  $\mathbf{\Omega}$  and passing through  $P$ ;  $l(\mathbf{x}, \mathbf{\Omega})$  is the distance of point  $P$  from the point  $P_0$  where the same ray-path crosses the boundary;  $\mathbf{x}'$  is the coordinate of a given point  $P'$  along the ray-path;  $\tau_v(\mathbf{x}, \mathbf{x}')$  is the optical depth between points  $P$  and  $P'$ ; finally,  $S_i^{(0)}(v, \mathbf{\Omega})$  is the Stokes vector entering the medium at point  $P_0$  along the same ray-path.

We can now evaluate the radiative rates entering Eq. (8) remembering the definition of  $\bar{J}_{\mathcal{Q}}^K$  given in Eq. (4) and substituting the formal solution for the Stokes parameters now obtained.

Introducing the symbol:

$$R_{-Q}^K = \frac{B}{A} (2J' + 1)^{-1/2} w_{J'J}^{(K)} (-1)^Q \bar{J}_{-Q}^K \quad (16)$$

and observing that the expression for  $S_i$  [Eq. (15)] contains two terms (an “internal” term due to the emission inside the medium, and an “external” one due to the boundary conditions) we can write:

$$R_{-Q}^K = (R_{-Q}^K)^{(I)} + (R_{-Q}^K)^{(E)}. \quad (17)$$

For the “internal” part of the radiative rate, evaluated at point  $P$ , we have:

$$\begin{aligned}
(R_{-Q}^K)^{(I)} &= (-1)^Q w_{J'J}^{(K)} \int_0^{\infty} dv \phi(v_0 - v) \oint \frac{d\mathbf{\Omega}}{4\pi} \int_0^{l(\mathbf{x}, \mathbf{\Omega})} \left\{ \eta_v(\mathbf{x}') e^{-\tau_v(\mathbf{x}, \mathbf{x}')} \right. \\
&\quad \times \sum_{K'Q'} w_{J'J}^{(K')} \rho_{Q'}^{K'}(\mathbf{x}') \left( \sum_{i=0}^3 \mathcal{T}_{Q'}^{K'}(i, \mathbf{\Omega}) \mathcal{T}_{-Q}^K(i, \mathbf{\Omega}) \right) \left. \right\} ds \quad (18)
\end{aligned}$$

where we have introduced the argument  $\mathbf{x}'$  in the expression for  $\rho_{Q'}^{K'}$  to point out explicitly its dependence on the point  $P'$  in the medium. Moreover, for the “external” part we obtain:

$$\begin{aligned}
(R_{-Q}^K)^{(E)} &= \frac{B}{A} (2J' + 1)^{-1/2} w_{J'J}^{(K)} (-1)^Q \int_0^{\infty} dv \phi(v_0 - v) \\
&\quad \times \oint \frac{d\mathbf{\Omega}}{4\pi} e^{-\tau_v(\mathbf{x}, x_0)} \sum_{i=0}^3 S_i^{(0)}(v, \mathbf{\Omega}) \mathcal{T}_{-Q}^K(i, \mathbf{\Omega}). \quad (19)
\end{aligned}$$

We now introduce the following notation:

$$G_{KQ, K'Q'}(\mathbf{\Omega}) = w_{J'J}^{(K)} w_{J'J}^{(K')} (-1)^Q \sum_{i=0}^3 \mathcal{T}_{Q'}^{K'}(i, \mathbf{\Omega}) \mathcal{T}_{-Q}^K(i, \mathbf{\Omega}). \quad (20)$$

From the definition given in Eq. (A6) of Paper III, we have:

$$G_{KQ, K'Q'}(\mathbf{\Omega}) = w_{J'J}^{(K)} w_{J'J}^{(K')} (-1)^Q \sum_{i=0}^3 \sum_{PP'} t_{P'}^{K'}(i) \mathcal{D}_{P'Q'}^{K'}(R) t_P^K(i) \mathcal{D}_{P-Q}^K(R) \quad (21)$$

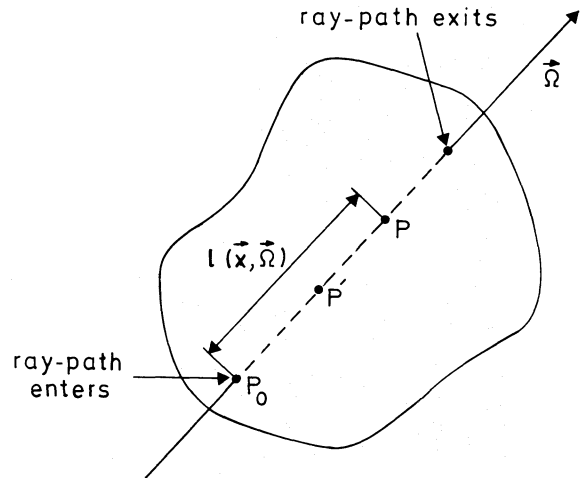
where, referring to Fig. 3,  $R$  is the rotation bringing the coordinate system  $(\mathbf{e}_a(\mathbf{\Omega}), \mathbf{e}_b(\mathbf{\Omega}), \mathbf{\Omega})$  into the system  $xyz$ :

$$R \equiv (-\gamma, -\theta, -\chi). \quad (22)$$

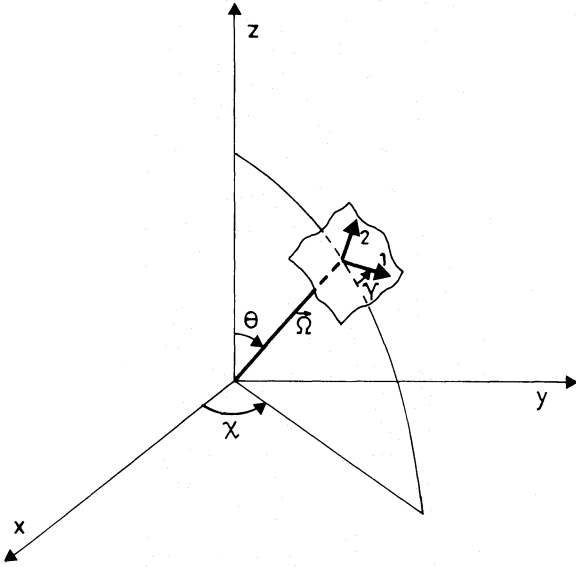
The calculations for the quantities  $G_{KQ, K'Q'}$  are rather involved and their details are given in Appendix 2. Here it is more important to concentrate over their physical meaning. To this purpose we rewrite Eq. (8) in the form:

$$\begin{aligned}
\rho_{Q'}^K(\mathbf{x}) &= -i\Gamma \sum_{\mathcal{Q}\mathcal{Q}'} \mathcal{H}_{\mathcal{Q}\mathcal{Q}'}^K \rho_{\mathcal{Q}}^K(\mathbf{x}) + \int_0^{\infty} dv \phi(v_0 - v) \oint \frac{d\mathbf{\Omega}}{4\pi} \\
&\quad \times \int_0^{l(\mathbf{x}, \mathbf{\Omega})} \left\{ \eta_v(\mathbf{x}') e^{-\tau_v(\mathbf{x}, \mathbf{x}')} \sum_{K'Q'} G_{KQ, K'Q'}(\mathbf{\Omega}) \rho_{Q'}^{K'}(\mathbf{x}') \right\} ds \\
&\quad + (R_{-Q}^K)^{(E)} \quad (23)
\end{aligned}$$

and we observe that the integrals over the solid angle and the ray-path can be transformed into a volume integral; we then



**Fig. 2.** Explanation of the symbols introduced in Eq. (15)



**Fig. 3.** For any given direction  $\Omega$ , the Stokes parameters are defined with respects to a couple of unit vectors  $e_1(\Omega)$  and  $e_2(\Omega)$ , labeled with 1 and 2 in the figure, both perpendicular to the direction  $\Omega$ . Note that the angle  $\gamma$  is arbitrary

obtain:

$$\begin{aligned} \rho_Q^K(x) = & -i\Gamma \sum_{Q'Q''} \mathcal{K}_{Q'Q''}^K \rho_{Q''}^K(x) + \int_0^\infty dv \phi(v_0 - v) \\ & \times \frac{1}{4\pi} \int d^3x' \eta_v(x') \frac{e^{-\tau_v(x, x')}}{(x - x')^2} \\ & \times \sum_{K'Q'} G_{KQ, K'Q'}(\Omega) \rho_{Q'}^{K'}(x') + (R_{-Q}^K)_x^{(E)}. \end{aligned} \quad (24)$$

The physical meaning of the various quantities is now clear. The statistical tensor at point  $P$ ,  $\rho_Q^K(x)$ , is determined by three different contributions. The first reflects the influence of the local magnetic field through its amplitude ( $\Gamma$ ) and its orientation (contained in the magnetic kernel  $\mathcal{K}_{Q'Q''}^K$ ); this is the term responsible for the Hanle effect. Through the second term,  $\rho_Q^K(x)$  is connected to the different statistical tensors present at the various points of the medium. The term  $G_{KQ, K'Q'}$  represents the contribution to  $\rho_Q^K(x)$  arising from  $\rho_{Q'}^{K'}(x')$  as a consequence of radiative coupling. Obviously this contribution is weighted through the usual attenuation operator  $e^{-\tau_v(x, x')}/(x - x')^2$  and through the local density of the medium proportional to  $\eta_v(x')$ . In the following we will refer to  $G_{KQ, K'Q'}$  as the multipole coupling coefficients. Finally, the third term appearing in the former equation describes the contribution to  $\rho_Q^K(x)$  arising from boundary conditions.

From the expression given in Appendix B, the multipole coupling coefficients can be evaluated in analytical form as a function of the angles  $\theta$  and  $\chi$  defined in Fig. 3. Obviously, these expressions do not depend on the angle  $\gamma$  which is not related to any physical direction but just reflects an arbitrary choice to define the linear polarization Stokes parameters for any  $\Omega$  direction. The analytical expressions for the non-vanishing multipole coupling coefficients are given in Table 2.

An important result concerning the multipole coupling coefficients is the fact that the  $K = 1$  multipole moments of the statistical tensors (describing atomic orientation) are coupled only

among each other; on the other hand there is a direct coupling of the  $K = 0$  multipole moment (the level population) with the  $K = 2$  multipole moments (atomic alignment).

If we return now to the “external” part of the radiative rate [Eq. (19)], and suppose that the scattering medium is irradiated only by the unpolarized radiation field  $I^{(0)}(v, \Omega)$ , we can formally write, for an expression there contained:

$$\begin{aligned} w_{J'J}^{(K)}(-1)^Q \sum_{i=0}^3 S_i^{(0)}(v, \Omega) \mathcal{F}_{-Q}^K(i, \Omega) = I^{(0)}(v, \Omega) \\ \times w_{J'J}^{(0)} w_{J'J}^{(K)}(-1)^Q \sum_{i=0}^3 \mathcal{F}_0^0(i, \Omega) \mathcal{F}_{-Q}^K(i, \Omega) = I^{(0)}(v, \Omega) G_{KQ, 00}(\Omega), \end{aligned} \quad (25)$$

so that the “external” rate can be written in the form:

$$\begin{aligned} (R_{-Q}^K)_x^{(E)} = \frac{B}{A} (2J' + 1)^{-1/2} \int_0^\infty dv \phi(v_0 - v) \oint \frac{d\Omega}{4\pi} \\ \times e^{-\tau_v(x, x_0)} I^{(0)}(v, \Omega) G_{KQ, 00}(\Omega). \end{aligned} \quad (26)$$

In more complicated physical situations this expression does not hold and one has to return to Eq. (19), which can be written in a more compact form introducing the “attenuated radiation field tensor”  $\bar{J}_Q^K$  defined by [see Eq. (4) for comparison]:

$$\bar{J}_Q^K(x) = \int_0^\infty dv \phi(v_0 - v) \oint \frac{d\Omega}{4\pi} e^{-\tau_v(x, x_0)} \sum_{i=0}^3 \mathcal{F}_Q^K(i, \Omega) S_i^{(0)}(v, \Omega). \quad (27)$$

Thus one can write:

$$(R_{-Q}^K)_x^{(E)} = \frac{B}{A} (2J' + 1)^{-1/2} w_{J'J}^{(K)}(-1)^Q \bar{J}_{-Q}^K(x). \quad (28)$$

In Paper III, an analytical expression is given for  $\bar{J}_Q^K$ ; this expression holds also for  $\bar{J}_Q^K$  provided that the local values for the Stokes parameters propagating along the  $\Omega$  direction are substituted by the corresponding boundary values attenuated by the factor  $e^{-\tau_v(x, x_0)}$ . From this expression one can note that, if the scattering medium is irradiated by a radiation with no circular polarization component ( $S_3^{(0)}(v, \Omega) = 0$ ), then the “external rates” for the  $K = 1$  multipole moments are identically zero. As these multipole moments are not coupled with the others, and as they have no source term in the statistical equilibrium equations, one obtains the important conclusion that for such a scattering medium the atoms do not show any atomic orientation irrespectively of the presence of a (weak) magnetic field.

Summarizing the results so far obtained, we write down, in their definitive form, the coupled statistical equilibrium equations for the statistical tensors in a scattering medium [see Eqs. (24) and (28)]:

$$\begin{aligned} \rho_Q^K(x) = & -i\Gamma \sum_{Q'} \mathcal{K}_{QQ'}^K \rho_{Q'}^K(x) + \frac{1}{4\pi} \int_0^\infty dv \phi(v_0 - v) \\ & \times \int d^3x' \eta_v(x') \frac{e^{-\tau_v(x, x')}}{(x - x')^2} \sum_{K'Q'} G_{KQ, K'Q'}(\Omega) \rho_{Q'}^{K'}(x') \\ & + \frac{B}{A} (2J' + 1)^{-1/2} w_{J'J}^{(K)}(-1)^Q \bar{J}_{-Q}^K(x). \end{aligned} \quad (29)$$

This equation contains only the contribution of the radiative rates; we want now to add the contribution due to collisions. We will restrict ourselves to the case where the velocity distribution of the colliders is isotropic and Maxwellian, as the analysis of impact polarization is beyond the scope of this paper. We will

**Table 2.** Analytical expressions for the non-vanishing symbols  $G_{KQ,K'Q'}(\Omega)$  as a function of  $\theta$  and  $\chi$ 

$G_{00,00} = 1$	$G_{20,20} = a_3 \frac{1}{4} (5 - 12C^2 + 9C^4)$
$G_{10,10} = a_1 \frac{3}{2} C^2$	$G_{22,21} = -G_{2-1,2-2} = a_3 \frac{3}{4} SC(1 + C^2)e^{-ix}$
$G_{11,11} = G_{1-1,1-1} = a_1 \frac{3}{4} S^2$	$G_{21,20} = -G_{20,2-1} = a_3 \frac{\sqrt{3}}{2\sqrt{2}} SC(2 - 3C^2)e^{-ix}$
$G_{11,10} = -G_{10,1-1} = -a_1 \frac{3}{2\sqrt{2}} SCe^{-ix}$	$G_{21,22} = -G_{2-2,2-1} = a_3 \frac{3}{4} SC(1 + C^2)e^{ix}$
$G_{10,11} = -G_{1-1,10} = -a_1 \frac{3}{2\sqrt{2}} SCe^{ix}$	$G_{20,21} = -G_{2-1,20} = a_3 \frac{\sqrt{3}}{2\sqrt{2}} SC(2 - 3C^2)e^{ix}$
$G_{11,1-1} = -a_1 \frac{3}{4} S^2 e^{-2ix}$	$G_{22,20} = G_{20,2-2} = a_3 \frac{\sqrt{3}}{4\sqrt{2}} S^2(1 + 3C^2)e^{-2ix}$
$G_{1-1,11} = -a_1 \frac{3}{4} S^2 e^{2ix}$	$G_{21,2-1} = a_3 \frac{3}{4} S^2(1 - 2C^2)e^{-2ix}$
$G_{22,00} = G_{00,2-2} = a_2 \frac{\sqrt{3}}{4} S^2 e^{-2ix}$	$G_{20,22} = G_{2-2,20} = a_3 \frac{\sqrt{3}}{4\sqrt{2}} S^2(1 + 3C^2)e^{2ix}$
$G_{21,00} = -G_{00,2-1} = -a_2 \frac{\sqrt{3}}{2} SCe^{-ix}$	$G_{2-1,21} = a_3 \frac{3}{4} S^2(1 - 2C^2)e^{2ix}$
$G_{20,00} = G_{00,20} = a_2 \frac{1}{2\sqrt{2}} (3C^2 - 1)$	$G_{22,2-1} = -G_{21,2-2} = a_3 \frac{3}{4} S^3 Ce^{-3ix}$
$G_{2-1,00} = -G_{00,21} = a_2 \frac{\sqrt{3}}{2} SCe^{ix}$	$G_{2-1,22} = -G_{2-2,21} = a_3 \frac{3}{4} S^3 Ce^{3ix}$
$G_{2-2,00} = G_{00,22} = a_2 \frac{\sqrt{3}}{4} S^2 e^{2ix}$	$G_{22,2-2} = a_3 \frac{3}{8} S^4 e^{-4ix}$
$G_{22,22} = G_{2-2,2-2} = a_3 \frac{3}{8} (1 + C^2)^2$	$G_{2-2,22} = a_3 \frac{3}{8} S^4 e^{4ix}$
$G_{21,21} = G_{2-1,2-1} = a_3 \frac{3}{4} S^2(1 + 2C^2)$	

Note:  $C = \cos \theta$ ,  $S = \sin \theta$ , the quantities  $a_1$ ,  $a_2$ ,  $a_3$  are defined in Appendix B

also make the further assumption that the collision time is much smaller than the time interval between successive collisions and also much smaller than the lifetime of the excited level (impact approximation). Under these assumptions the atom-photon and atom-perturber interactions are decoupled and the collisional rates can be simply added to the radiation rates in the statistical equilibrium equations.

Two different kinds of collisions are effective in altering the statistical tensors of the upper level: inelastic and elastic (or depolarizing) collisions. Inelastic collisions correspond to the excitation or deexcitation of the upper level with an energy loss or gain of the perturber (typically an electron). As the velocity distribution of the electrons is isotropic, and as the lower level is supposed to be unpolarized, the effect of inelastic collisional excitation is described in the statistical equilibrium equations for the statistical tensor of the upper level by a term of the form:

$$\frac{d}{dt} \rho_Q^K = (2J' + 1)^{-1/2} C_{JJ'} \delta_{K,0} \delta_{Q,0} \quad (30)$$

where  $C_{JJ'}$  is proportional to the density of the perturbers and depends on their temperature  $T$  and on the relevant atomic cross-section (see for instance Mihalas, 1970) and where the factor  $(2J' + 1)^{-1/2}$  has been introduced to make our notations compatible with those used in the standard non-LTE theory.

The inverse process (inelastic collisional de-excitation) induces a relaxation of the various multipoles of the upper level. Again, as the velocity distribution of the perturbers is isotropic, all the multipoles relax at the same rate, so that we have, adding the two contributions:

$$\frac{d}{dt} \rho_Q^K = (2J' + 1)^{-1/2} C_{JJ'} \delta_{K,0} \delta_{Q,0} - C_{JJ'} \rho_Q^K \quad (31)$$

The relation between  $C_{JJ'}$  and  $C_{J'J}$  can be found through the principle of detailed balance applied to the  $K = 0$  multipole moments. Indeed, if only collisions are present we have to find for  $\rho_Q^0$  its thermodynamical equilibrium value so that, remembering our normalization for the statistical tensors, we obtain:

$$\rho_Q^0 = \frac{(2J' + 1)^{1/2}}{(2J + 1)} e^{-h\nu/kT} = \frac{(2J' + 1)^{-1/2} C_{JJ'}}{C_{J'J}} \quad (32)$$

which gives:

$$C_{JJ'} = \frac{2J' + 1}{2J + 1} e^{-h\nu/kT} C_{J'J} \quad (33)$$

Finally, for elastic collisions, it can be shown on theoretical grounds (Ben Reuven, 1966; Omont, 1977) that, in an isotropic environment, each multipole moment of the statistical tensor

relaxes independently. The relaxation can be described by a rate  $D^{(K)}$  that will be proportional to the density of the perturbers. Obviously the rate  $D^{(0)}$  is identically zero.

Adding together all the terms due to the collisions we have:

$$\frac{d}{dt} \rho_Q^K = C_{JJ} \left[ \frac{(2J'+1)^{1/2}}{(2J+1)} e^{-h\nu/kT} \delta_{K,0} \delta_{Q,0} - \rho_Q^K \right] - D^{(K)} \rho_Q^K \quad (34)$$

and introducing the following notations:

$$\begin{aligned} \varepsilon &= \frac{C_{JJ}}{A} \\ \beta_0 &= \frac{(2J'+1)^{1/2}}{2J+1} e^{-h\nu/kT} \\ \delta^{(K)} &= \frac{D^{(K)}}{A} \end{aligned} \quad (35)$$

we generalize Eq. (29) to account for collisions:

$$\begin{aligned} (1 + \varepsilon + \delta^{(K)}) \rho_Q^K(x) &= -i\Gamma \sum_{Q'} \mathcal{K}_{QQ'}^K \rho_{Q'}^K(x) \\ &+ \frac{1}{4\pi} \int_0^\infty dv \phi(v_0 - v) \int_V d^3x' \eta_\nu(x') \frac{e^{-\tau_\nu(x,x')}}{(x-x')^2} \sum_{K'Q'} G_{KQ,K'Q'}(\Omega) \rho_{Q'}^{K'}(x') \\ &+ \frac{B}{A} (2J'+1)^{-1/2} w_{JJ'}^{(K)} (-1)^Q \tilde{J}_{-Q}^K(x) + \varepsilon \beta_0 \delta_{K,0} \delta_{Q,0}. \end{aligned} \quad (36)$$

This is an integral equation that can in principle be solved once the boundary conditions and the properties of the medium are specified. When the statistical tensors are known, the Stokes parameters for any line of sight can be computed. Referring to Fig. 4 we have, for the Stokes parameters at frequency  $\nu$  emerging from the medium along the direction  $\Omega_0$  [see Eq. (15)]:

$$S_i(\nu, \Omega_0) = \int_0^{L(\vec{\Omega}_0)} [s_i(\Omega_0)]_x e^{-\tau_\nu(x_1, x)} \eta_\nu(x) ds + S_i^{(0)}(\nu, \Omega_0) e^{-\tau_\nu(x_1, x_0)} \quad (37)$$

where  $x_0$  and  $x_1$  are, respectively, the coordinates of the points  $P_0$  and  $P_1$  marking the intersections of the line of sight with the

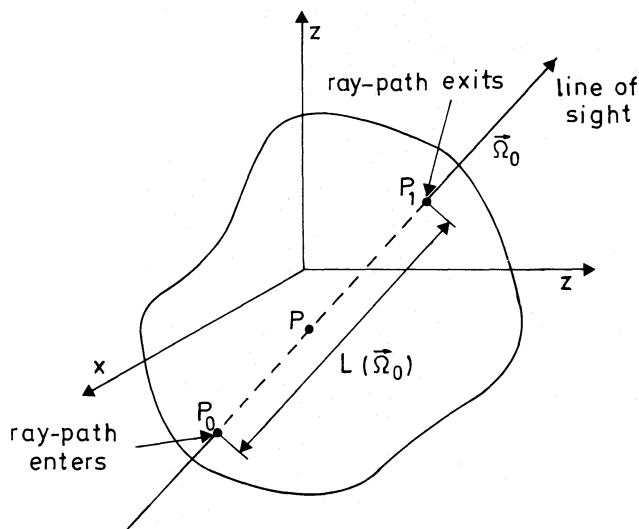


Fig. 4. Explanation of the symbols employed in Eq. (37)

boundaries of the medium,  $s$  is the spatial coordinate measured along the ray-path,  $x$  is the coordinate of a given point  $P$  along the ray-path, and all the other symbols have the same meaning as in Eq. (15). In particular we have:

$$[s_i(\Omega_0)]_x = \frac{A(2J'+1)^{1/2}}{B} \sum_{KQ} \rho_Q^K(x) \mathcal{F}_Q^K(i, \Omega_0) \quad (38)$$

where

$$\mathcal{F}_Q^K(i, \Omega_0) = \sum_P t_P^K(i) \mathcal{D}_{PQ}^K(R_0), \quad (39)$$

$R_0$  being the rotation that brings the system  $(e_a(\Omega_0), e_b(\Omega_0), \Omega_0)$  into the system  $(x, y, z)$ :

$$R_0 \equiv (-\gamma_0, -\theta_0, -\chi_0) \quad (40)$$

where  $\gamma_0$ ,  $\theta_0$ , and  $\chi_0$  have, for the line of sight, the same meaning as the corresponding angles defined in Fig. 3.

The formalism presented in this paper is very general and can in principle be applied to any given three-dimensional structure. In the following sections we will particularize the formalism to the special cases where the properties of the scattering medium (and the boundary conditions) depend only on one or two coordinates.

### 3. Unidimensional case

We suppose that all the parameters entering Eq. (36) depend only on the spatial coordinate  $z$  and not on  $x$  and  $y$ . In other words we suppose that the scattering medium is composed of a plane parallel slab (that can be eventually extended to infinity on each side) invariant with respect to any translation in the  $x$ - $y$  plane. The radiation field that is illuminating the slab has obviously to fulfil the same requirement.

Introducing cylindrical coordinates and supposing that the slab extends between  $z_a$  and  $z_b$ , the volume integral appearing in Eq. (36)—for which we introduce the symbol  $I_\nu$ —can be expressed in the form (see Fig. 5):

$$\begin{aligned} I_\nu &= \int_{z_a}^{z_b} dz' \eta_\nu(z') \sum_{K'Q'} \rho_{Q'}^{K'}(z') \int_0^\infty r dr \\ &\times \int_0^{2\pi} d\varphi \frac{e^{-\tau_\nu(x,x')}}{(z-z')^2 + r^2} G_{KQ,K'Q'}(\Omega). \end{aligned} \quad (41)$$

We now introduce the optical depth  $t_\nu(z)$  measured from the top of the slab along the  $z$  direction

$$t_\nu(z) = \int_z^{z_b} \eta_\nu(z') dz' \quad (42)$$

and observe that the quantity  $\tau_\nu(x, x')$  can be written in the form<sup>2</sup>

$$\tau_\nu(x, x') = (t_\nu(z') - t_\nu(z)) / \cos \psi \quad (43)$$

where  $\psi$  is the angle defined in Fig. 5.

<sup>2</sup> Indeed, Eq. (43) holds only if there is no macroscopic motion in the medium. This requirement is not necessary for the equations derived in the former section.



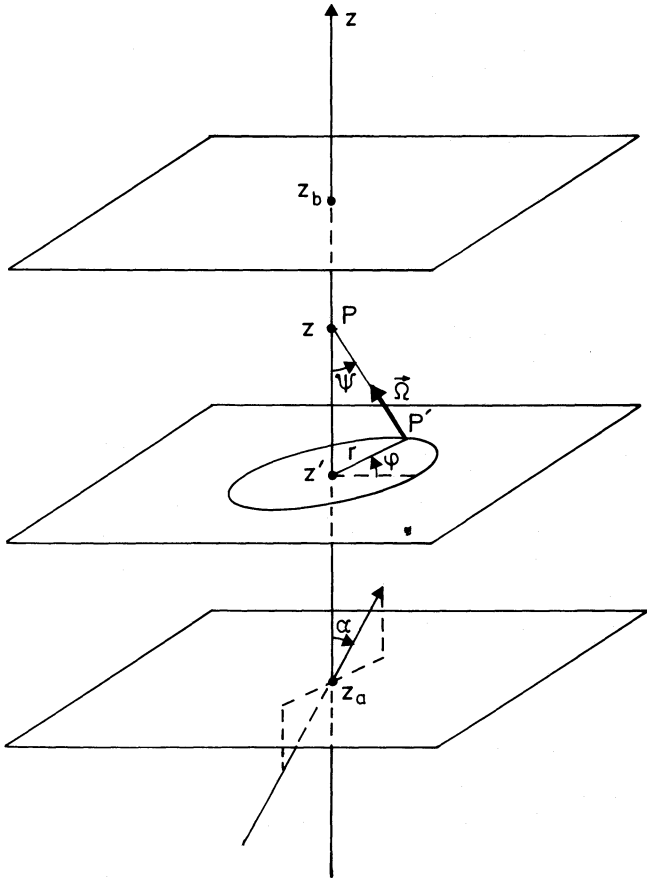


Fig. 5. Geometry for the plane-parallel case

We next observe, comparing the geometries of Figs. 3 and 5, that the angles  $\theta$  and  $\chi$ , entering the expressions for the multipole coupling coefficients in Table 2, are the same as  $\psi$  and  $\varphi$  defined in Fig. 5. Performing the integral over  $\varphi$ , we observe that this integration leads to a contribution equal to  $2\pi$  for those multipole coupling coefficients not depending on  $\varphi$ , and to a vanishing result for those having a dependence of the form  $e^{-im\varphi}$  with  $m$  integer. Next we observe that the multipole coupling coefficients which lead to a non-vanishing contribution after the  $\varphi$  integration can be expressed as a linear combination of  $\cos^{2n}\psi$ , with  $n$  integer. Thus we are left with the evaluation of integrals of the form

$$\int_0^\infty dr r \frac{e^{-(t_\nu(z') - t_\nu(z))/\cos \Psi}}{(z - z')^2 + r^2} \cos^{2n}\psi. \quad (44)$$

By changing to the new variable  $\psi$  with the substitution

$$r = (z - z') \operatorname{tg} \psi \quad (45)$$

this integral can be transformed to the following one (for  $z > z'$ ):

$$\int_0^{\pi/2} e^{-(t_\nu(z') - t_\nu(z))/\cos \psi} \operatorname{tg} \psi \cos^{2n}\psi \, d\psi, \quad (46)$$

and introducing the new variable  $y = 1/\cos \psi$ , we finally obtain:

$$\int_1^\infty e^{-(t_\nu(z') - t_\nu(z))y} y^{-2n-1} dy = E_{2n+1}(t_\nu(z') - t_\nu(z)) \quad (47)$$

where  $E_n(x)$  is the usual exponential-integral function.

By considering also the case where  $z < z'$  one can easily prove that the result of the integral can be expressed in the general form:

$$E_{2n+1}(|t_\nu(z') - t_\nu(z)|). \quad (48)$$

Summarizing the results now obtained, one has for  $I_V$ :

$$I_V = 2\pi \int_{z_a}^{z_b} dz' \eta_\nu(z') \sum_{K'Q'} \rho_{Q'}^{K'}(z') \mathcal{G}_{K'Q'}^{(1)} \quad (49)$$

where the non-vanishing components of  $\mathcal{G}^{(1)}$  are given in Table 3.

As far as the "external rate" is concerned, no obvious simplification of Eq. (27) can be obtained in the one-dimensional case as the boundary-value radiation field  $S_1^{(0)}(\nu, \Omega)$  may have an arbitrary dependence on the  $\Omega$  direction. However, if we restrict to the particular case where the radiation field is unpolarized and we suppose that it can be described by an expression of the form:

$$I^{(0)}(\nu, \Omega) = \sum_n I_n(\nu) \cos^n \alpha \quad (50)$$

where  $\alpha$  is the angle defined as in Fig. 5, we find from Eq. (26) that the only non-vanishing components of the external rates are:

$$\begin{aligned} (R_{0z}^{(E)}) &= \frac{1}{2} \frac{B}{A} (2J' + 1)^{-1/2} \\ &\times \int_0^\infty d\nu \phi(\nu_0 - \nu) \sum_n I_n(\nu) E_{n+2}(t_\nu(z_a) - t_\nu(z)) \\ (R_{2z}^{(E)}) &= \frac{1}{4\sqrt{2}} a_2 \frac{B}{A} (2J' + 1)^{-1/2} \int_0^\infty d\nu \phi(\nu_0 - \nu) \sum_n I_n(\nu) \\ &\times [3 E_{n+4}(t_\nu(z_a) - t_\nu(z)) - E_{n+2}(t_\nu(z_a) - t_\nu(z))]. \end{aligned} \quad (51)$$

The results now obtained can be easily applied to the description of a semi-infinite atmosphere ( $z_b = 0, z_a \rightarrow -\infty$ ).

In the absence of magnetic fields ( $\Gamma = 0$ ) and neglecting the influence of depolarizing collisions ( $\delta^{(K)} = 0$ ), the resulting equations for the only non-vanishing multipole moments assume the

Table 3. Expressions for the non-vanishing symbols  $\mathcal{G}_{K'Q'}^{(1)}$ . The argument of the various exponential-integrals is  $|t_\nu(z') - t_\nu(z)|$

$\mathcal{G}_{00,00}^{(1)} = E_1$	$\mathcal{G}_{10,10}^{(1)} = a_1 \frac{3}{2} E_3$
$\mathcal{G}_{11,11}^{(1)} = \mathcal{G}_{1-1,1-1}^{(1)} = a_1 \frac{3}{4} (E_1 - E_3)$	$\mathcal{G}_{20,00}^{(1)} = \mathcal{G}_{00,20}^{(1)} = a_2 \frac{1}{2\sqrt{2}} (3E_3 - E_1)$
$\mathcal{G}_{22,22}^{(1)} = \mathcal{G}_{2-2,2-2}^{(1)} = a_3 \frac{3}{8} (E_1 + 2E_3 + E_5)$	$\mathcal{G}_{21,21}^{(1)} = \mathcal{G}_{2-1,2-1}^{(1)} = a_3 \frac{3}{4} (E_1 + E_3 - 2E_5)$
$\mathcal{G}_{20,20}^{(1)} = a_3 \frac{1}{4} (5E_1 - 12E_3 + 9E_5)$	

form:

$$\begin{aligned}\rho_0^0(\tau) &= (1 - \varepsilon') \int_0^\infty K_{11}(|\tau - \tau'|) \rho_0^0(\tau') d\tau' \\ &\quad + (1 - \varepsilon') \int_0^\infty K_{12}(|\tau - \tau'|) \rho_0^2(\tau') d\tau' + \varepsilon' \beta_0, \\ \rho_0^2(\tau) &= (1 - \varepsilon') \int_0^\infty K_{21}(|\tau - \tau'|) \rho_0^0(\tau') d\tau' \\ &\quad + (1 - \varepsilon') \int_0^\infty K_{22}(|\tau - \tau'|) \rho_0^2(\tau') d\tau'\end{aligned}$$

where

$$\varepsilon' = \frac{\varepsilon}{1 + \varepsilon} = \frac{C_{JJ}}{A + C_{JJ}} \quad (53)$$

and where we have introduced the optical depth  $\tau$  in strict analogy to Eq. (42):

$$\tau(z) = \int_z^{z_b=0} \eta_0(z') dz'. \quad (54)$$

For an atmosphere where the line profile does not vary with  $z$ , the quantities  $t_v$  and  $\tau$  are simply related through the relation:

$$t_v(z) = \phi(v_0 - v) \tau = \phi(x) \tau \quad (55)$$

and the various kernels are given by:

$$\begin{aligned}K_{11}(\tau) &= \frac{1}{2} \int_{-\infty}^{+\infty} dx \phi^2(x) E_1[\tau \phi(x)], \\ K_{12}(\tau) &= \frac{a_2}{4\sqrt{2}} \int_{-\infty}^{+\infty} dx \phi^2(x) \{3E_3[\tau \phi(x)] - E_1[\tau \phi(x)]\}, \\ K_{21}(\tau) &= K_{12}(\tau), \\ K_{22}(\tau) &= \frac{a_3}{8} \int_{-\infty}^{+\infty} dx \phi^2(x) \\ &\quad \times \{5E_1[\tau \phi(x)] - 12E_3[\tau \phi(x)] + 9E_5[\tau \phi(x)]\}.\end{aligned} \quad (56)$$

These equations are very similar to those derived by previous authors (Rees, 1978; Faurobert-Scholl and Frisch, 1989). In their formalism, the reduced source functions  $S(\tau)$  and  $\bar{P}(\tau)$  obey coupled integral equations of the form of Eq. (52). Their equations can be obtained from our ones by equating to one the quantities  $a_2$  and  $a_3$  and performing the formal substitutions:

$$\begin{cases} \frac{2J+1}{(2J'+1)^{1/2}} \rho_0^0 \rightarrow S, \\ \frac{2J+1}{(2J'+1)^{1/2}} \rho_0^2 \rightarrow -\frac{2\sqrt{2}}{3} \bar{P}, \\ \frac{2J+1}{(2J'+1)^{1/2}} \beta_0 \rightarrow B. \end{cases} \quad (57)$$

In the more general case of a magnetic, semi-infinite atmosphere with depolarizing collisions the equations assume the following form:

$$\begin{aligned}\rho_0^0(\tau) &= (1 - \varepsilon') \int_0^\infty K_{11}(|\tau - \tau'|) \rho_0^0(\tau') d\tau' + (1 - \varepsilon') \\ &\quad \times \int_0^\infty K_{12}(|\tau - \tau'|) \rho_0^2(\tau') d\tau' + \varepsilon' \beta_0,\end{aligned}$$

$$\begin{aligned}\rho_0^2(\tau) &= -i\Gamma' \sum_Q \mathcal{X}_{0Q}^2 \rho_0^2(\tau) + (1 - \varepsilon'') \int_0^\infty K_{21}(|\tau - \tau'|) \rho_0^0(\tau') d\tau' \\ &\quad + (1 - \varepsilon'') \int_0^\infty K_{22}(|\tau - \tau'|) \rho_0^2(\tau') d\tau', \\ \rho_{\pm 1}^2(\tau) &= -i\Gamma' \sum_Q \mathcal{X}_{\pm 1Q}^2 \rho_0^2(\tau) + (1 - \varepsilon'') \\ &\quad \times \int_0^\infty K_{33}(|\tau - \tau'|) \rho_{\pm 1}^2(\tau') d\tau', \\ \rho_{\pm 2}^2(\tau) &= -i\Gamma' \sum_Q \mathcal{X}_{\pm 2Q}^2 \rho_0^2(\tau) + (1 - \varepsilon'') \\ &\quad \times \int_0^\infty K_{44}(|\tau - \tau'|) \rho_{\pm 2}^2(\tau') d\tau'\end{aligned} \quad (52)$$

where, besides the symbols previously introduced, we have:

$$\Gamma' = \frac{\Gamma}{1 + \varepsilon + \delta^{(2)}} = \frac{e_0 B}{2mc(A + C_{JJ} + D^{(2)})}, \quad (59)$$

$$\varepsilon'' = \frac{\varepsilon + \delta^{(2)}}{1 + \varepsilon + \delta^{(2)}} = \frac{C_{JJ} + D^{(2)}}{A + C_{JJ} + D^{(2)}}, \quad (60)$$

$$\begin{aligned}K_{33}(\tau) &= \frac{3}{8} a_3 \int_{-\infty}^{+\infty} dx \phi^2(x) \{E_1[\tau \phi(x)] + E_3[\tau \phi(x)] \\ &\quad - 2E_5[\tau \phi(x)]\}, \\ K_{44}(\tau) &= \frac{3}{16} a_3 \int_{-\infty}^{+\infty} dx \phi^2(x) \{E_1[\tau \phi(x)] + 2E_3[\tau \phi(x)] \\ &\quad + E_5[\tau \phi(x)]\}.\end{aligned} \quad (61)$$

Note that in these equations all the parameters introduced, namely  $\varepsilon'$ ,  $\varepsilon''$ ,  $\Gamma'$ ,  $B$ , plus the magnetic field direction contained in the magnetic kernel are arbitrary functions of  $\tau$ .

#### 4. Two-dimensional case

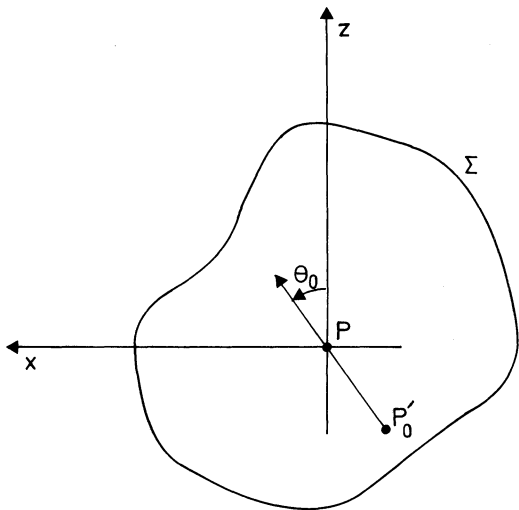
In strict analogy to what has been done in the previous section, we now suppose that the parameters entering Eq. (36) depend on two spatial coordinates,  $z$  and  $x$ , but do not depend on  $y$ . In other words, we now suppose that the scattering medium is composed of an infinite cylinder of arbitrary cross-section invariant with respect to any translation along the  $y$ -direction. Obviously, the radiation field that is illuminating the slab has to fulfil the same requirements.

Again, we turn our attention to the volume integral appearing in Eq. (36) that we express in the form:

$$\begin{aligned}I_V &= \int_{\Sigma} d^2 \xi' \eta_V(\xi') \sum_{K'Q'} \rho_{Q'}^{K'}(\xi') \\ &\quad \times \int_{-\infty}^{+\infty} dy' \frac{e^{-\tau_V(x, x')}}{(\xi - \xi')^2 + (y - y')^2} G_{KQ, K'Q'}(\Omega)\end{aligned} \quad (62)$$

where  $\Sigma$  is the cross-section of the cylinder and where we have introduced the reduced coordinate  $\xi \equiv (x, z)$ .

The integral in  $dy'$  represents the contribution to the volume integral arising from all the points  $P'$  lying on a straight line parallel to the  $y$ -axis. Each of these straight lines intersects the cylinder cross-section containing  $P$  in a point  $P'_0$  which can be characterized, in polar coordinates, by the quantities  $\sigma = |\xi - \xi'|$  and  $\theta_0$ , defined as in Fig. 6. Any point  $P'$  belonging to a given straight line can be specified through the angle  $\psi$  defined in Fig.



**Fig. 6.** The cylinder cross-section  $\Sigma$  passing through  $P$  is intersected, by any straight line parallel to  $y$ , in the point  $P_0$ . The  $y$ -axis is perpendicular to the plane of the figure and is directed towards the reader

7, and the polar angles  $\theta$  and  $\chi$ , relative to the direction  $\Omega$  from  $P'$  to  $P$ , can be evaluated through the spherical triangle of Fig. 8, from which we have:

$$\begin{aligned} \cos \theta &= \cos \theta_0 \cos \psi, \\ \sin \theta \sin \chi &= \sin \psi, \\ \sin \theta \cos \chi &= \sin \theta_0 \cos \psi. \end{aligned} \tag{63}$$

The integral in  $dy'$  can now be performed by introducing the new variable  $\psi$  through the substitution:

$$y' = -\sigma \tan \psi$$

We obtain:

$$\int_{-\infty}^{+\infty} dy' \dots = \frac{1}{\sigma} \int_{-\pi/2}^{+\pi/2} d\psi e^{-\tau_v(\xi, \xi')/\cos \psi} G_{KQ, K'Q'}(\Omega) \tag{64}$$

where the trigonometric functions contained in  $G_{KQ, K'Q'}$  have to be expressed through  $\psi$  by means of Eq. (63), and where we have introduced the expression  $\tau_v(\xi, \xi')$  to denote the optical depth between two points belonging to the cylinder cross-section  $\Sigma$ .

With the help of Eqs. (63), the non-vanishing integrals contained in Eq. (64) can all be transformed to integrals of the following form:

$$\begin{aligned} \int_{-\pi/2}^{\pi/2} e^{-\tau_v(\xi, \xi')/\cos \psi} \cos^n \psi d\psi &= 2 \int_0^{\pi/2} e^{-\tau_v(\xi, \xi')/\cos \psi} \cos^n \psi d\psi \\ &= 2 \mathcal{J}_n(\tau_v(\xi, \xi')) \end{aligned} \tag{65}$$

where  $\mathcal{J}_n(x)$  is a well-known mathematical function that can be related to the repeated integrals of the modified Bessel functions of the second kind,  $K_0(x)$ . [In the notation of Abramowitz and Stegun (1964) the same function is denoted by the symbol  $Ki_{n+1}(x)$ .]

Taking into account Eq. (65), we can finally write:

$$I_V = 2 \int_{\Sigma} d^2 \xi' \eta_v(\xi') \sum_{K'Q'} \rho_{Q'}^{K'}(\xi') \frac{1}{|\xi - \xi'|} \mathcal{G}_{KQ, K'Q'}^{(2)} \tag{66}$$

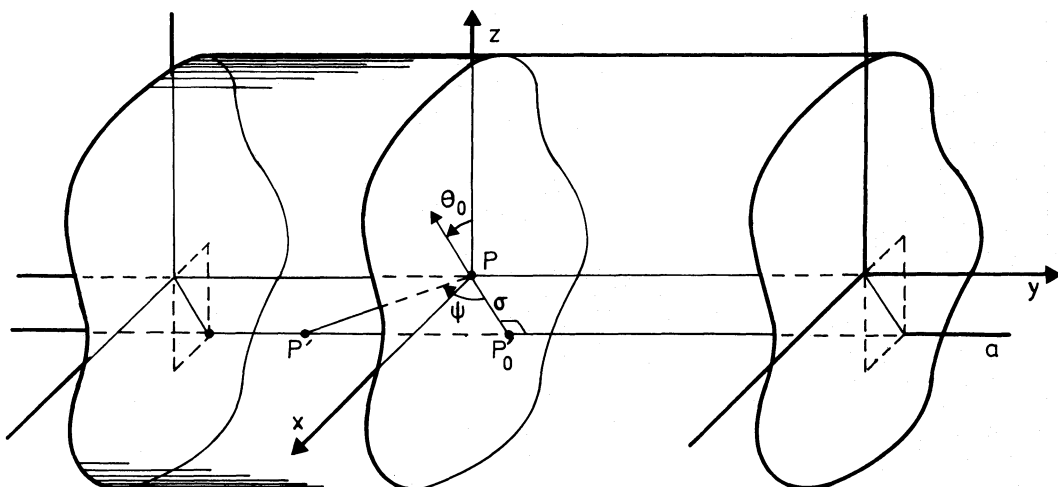
where the analytical expressions of the symbols  $\mathcal{G}_{KQ, K'Q'}^{(2)}$  are given in Table 4.

For the two-dimensional case, Eq. (36) can then be written in the form:

$$\begin{aligned} (1 + \varepsilon + \delta^{(K)}) \rho_Q^K(\xi) &= -i\Gamma \sum_{Q'} \mathcal{X}_{Q'}^K \rho_{Q'}^K(\xi) + \frac{1}{2\pi} \int_0^{\infty} dv \phi(v_0 - v) \\ &\times \int_{\Sigma} d^2 \xi' \eta_v(\xi') \sum_{K'Q'} \rho_{Q'}^{K'}(\xi') \frac{1}{|\xi - \xi'|} \mathcal{G}_{KQ, K'Q'}^2 \\ &+ \frac{B}{A} (2J' + 1)^{-1/2} w_{J', J}^{(K)}(-1) \rho_{Q'}^K(\xi) \\ &+ \varepsilon \beta_0 \delta_{K,0} \delta_{Q,0}. \end{aligned} \tag{67}$$

**5. Conclusion**

The density-matrix formalism in the representation of the irreducible spherical tensors has been employed to give a self-consistent description of resonance scattering in an optically-thick, three-dimensional medium of arbitrary geometrical shape. The formalism, that has been developed for a two-level atom having an unpolarized ground level, allows for the presence of an arbitrary magnetic field, supposed sufficiently weak to introduce a



**Fig. 7.** Definition of the angle  $\psi$  introduced to reckon the point  $P'$  along the straight line  $a$

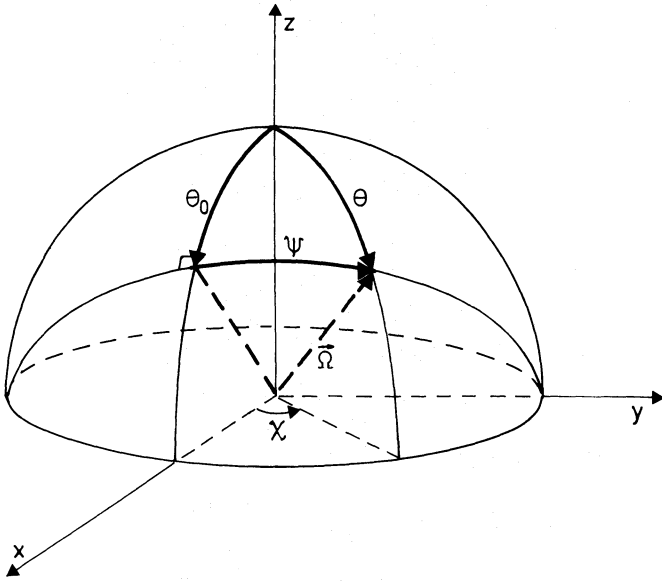


Fig. 8. The sphere is centered on the point  $P$ . The direction  $\Omega$  is the one from  $P'$  to  $P$

Zeeman splitting much smaller than the typical width of the line profile. These two hypotheses (unpolarized ground level and weak magnetic field) lead to the decoupling of the transfer equations for the Stokes parameters propagating between different points inside the medium. The further hypothesis of neglecting stimulating emission produces a system of coupled linear, integral equations for the multipole moments of the density-matrix.

These equations account for the presence of the magnetic field and describe the coupling, due to the propagation of polarized radiation, between multipole moments relative to different points in the medium. Both elastic and inelastic collisions can be easily introduced in the formalism through suitable rates.

Once the geometry of the scattering medium, its physical properties and the relevant boundary conditions are specified, the coupled linear equations for the multipole moments of the density-matrix can, in principle, be solved. From the solution, the radiation field propagating along any line of sight can be finally recovered.

The formalism presented in this paper significantly differs from the one presented in previous works on resonance scattering in the sense that the unknowns of the problem are now the multipole moments of the atomic density-matrix instead of the radiation field propagating inside the medium.

The general equations for the multipole moments can be easily particularized to the one-dimensional or bi-dimensional cases. For the one-dimensional case of the plane parallel atmosphere, neglecting the magnetic field and depolarizing collisions, these equations reduce to similar equations previously derived by Rees (1978).

#### Appendix A: calculations for the magnetic kernel

In Eq. (9) we substitute for  $Q''$  by taking into account the identity:

$$Q'' = [K(K+1)(2K+1)]^{1/2} \begin{pmatrix} K & K & 1 \\ Q'' & -Q'' & 0 \end{pmatrix} (-1)^{K-Q''} \quad (\text{A1})$$

and we express the product of the two rotation matrices through their conjugation and contraction properties (Brink and Satchler,

1968):

$$\begin{aligned} & \mathcal{D}_{Q''}^{K*}(R) \mathcal{D}_{Q''}^{K'}(R) \\ &= (-1)^{Q''-Q} \mathcal{D}_{-Q''-Q}^K(R) \mathcal{D}_{Q''}^{K'}(R) \\ &= (-1)^{Q''-Q} \sum_{K'} (2K'+1) \\ & \times \begin{pmatrix} K & K & K' \\ -Q'' & Q'' & 0 \end{pmatrix} \begin{pmatrix} K & K & K' \\ -Q & Q & Q''' \end{pmatrix} \mathcal{D}_{0Q''}^{K'*}(R) \end{aligned} \quad (\text{A2})$$

Table 4. Expressions for the non-vanishing symbols  $\mathcal{G}_{KQ,K'Q'}^{(2)}$ . The argument of the functions  $\mathcal{F}_n$  is  $\tau_n(\xi, \xi')$

$$\begin{aligned} \mathcal{G}_{00,00}^{(2)} &= \mathcal{F}_0 \\ \mathcal{G}_{10,10}^{(2)} &= a_1 \frac{3}{2} C_0^2 \mathcal{F}_2 \\ \mathcal{G}_{11,11}^{(2)} &= \mathcal{G}_{1-1,1-1}^{(2)} = a_1 \frac{3}{4} (\mathcal{F}_0 - C_0^2 \mathcal{F}_2) \\ \mathcal{G}_{11,10}^{(2)} &= \mathcal{G}_{10,11}^{(2)} = -\mathcal{G}_{10,1-1}^{(2)} = -\mathcal{G}_{1-1,10}^{(2)} = -a_1 \frac{3}{2\sqrt{2}} S_0 C_0 \mathcal{F}_2 \\ \mathcal{G}_{11,1-1}^{(2)} &= \mathcal{G}_{1-1,11}^{(2)} = a_1 \frac{3}{4} (\mathcal{F}_0 - (2 - C_0^2) \mathcal{F}_2) \\ \mathcal{G}_{22,00}^{(2)} &= \mathcal{G}_{00,22}^{(2)} = \mathcal{G}_{00,2-2}^{(2)} = \mathcal{G}_{2-2,00}^{(2)} \\ &= a_2 \frac{\sqrt{3}}{4} (-\mathcal{F}_0 + (2 - C_0^2) \mathcal{F}_2) \\ \mathcal{G}_{21,00}^{(2)} &= \mathcal{G}_{00,21}^{(2)} = -\mathcal{G}_{00,2-1}^{(2)} = -\mathcal{G}_{2-1,00}^{(2)} \\ &= -a_2 \frac{\sqrt{3}}{2} S_0 C_0 \mathcal{F}_2 \\ \mathcal{G}_{20,00}^{(2)} &= \mathcal{G}_{00,20}^{(2)} = a_2 \frac{1}{2\sqrt{2}} (-\mathcal{F}_0 + 3C_0^2 \mathcal{F}_2) \\ \mathcal{G}_{22,22}^{(2)} &= \mathcal{G}_{2-2,2-2}^{(2)} = a_3 \frac{3}{8} (\mathcal{F}_0 + 2C_0^2 \mathcal{F}_2 + C_0^4 \mathcal{F}_4) \\ \mathcal{G}_{21,21}^{(2)} &= \mathcal{G}_{2-1,2-1}^{(2)} = a_3 \frac{3}{4} (\mathcal{F}_0 + C_0^2 \mathcal{F}_2 - 2C_0^4 \mathcal{F}_4) \\ \mathcal{G}_{20,20}^{(2)} &= a_3 \frac{1}{4} (5\mathcal{F}_0 - 12C_0^2 \mathcal{F}_2 + 9C_0^4 \mathcal{F}_4) \\ \mathcal{G}_{22,21}^{(2)} &= \mathcal{G}_{21,22}^{(2)} = -\mathcal{G}_{2-1,2-2}^{(2)} = -\mathcal{G}_{2-2,2-1}^{(2)} \\ &= a_3 \frac{3}{4} S_0 C_0 (\mathcal{F}_2 + C_0^2 \mathcal{F}_4) \\ \mathcal{G}_{21,20}^{(2)} &= \mathcal{G}_{20,21}^{(2)} = -\mathcal{G}_{20,2-1}^{(2)} = -\mathcal{G}_{2-1,20}^{(2)} \\ &= a_3 \frac{\sqrt{3}}{2\sqrt{2}} S_0 C_0 (2\mathcal{F}_2 - 3C_0^2 \mathcal{F}_4) \\ \mathcal{G}_{22,20}^{(2)} &= \mathcal{G}_{20,22}^{(2)} = \mathcal{G}_{20,2-2}^{(2)} = \mathcal{G}_{2-2,20}^{(2)} \\ &= a_3 \frac{\sqrt{3}}{4\sqrt{2}} (-\mathcal{F}_0 + 2(1 - 2C_0^2) \mathcal{F}_2 + 3C_0^2(2 - C_0^2) \mathcal{F}_4) \\ \mathcal{G}_{21,2-1}^{(2)} &= \mathcal{G}_{2-1,21}^{(2)} \\ &= a_3 \frac{3}{4} (-\mathcal{F}_0 + (2 + C_0^2) \mathcal{F}_2 - 2C_0^2(2 - C_0^2) \mathcal{F}_4) \\ \mathcal{G}_{22,2-1}^{(2)} &= \mathcal{G}_{2-1,22}^{(2)} = -\mathcal{G}_{21,2-2}^{(2)} = -\mathcal{G}_{2-2,2-1}^{(2)} \\ &= a_3 \frac{3}{4} S_0 C_0 (-3\mathcal{F}_2 + (4 - C_0^2) \mathcal{F}_4) \\ \mathcal{G}_{22,2-2}^{(2)} &= \mathcal{G}_{2-2,22}^{(2)} \\ &= a_3 \frac{3}{8} (\mathcal{F}_0 - 2(4 - 3C_0^2) \mathcal{F}_2 + (8 - 8C_0^2 + C_0^4) \mathcal{F}_4) \end{aligned}$$

Note:  $C_0 = \cos \theta_0$ ,  $S_0 = \sin \theta_0$

We thus obtain:

$$\begin{aligned} \mathcal{K}_{Q'Q''}^K &= [K(K+1)(2K+1)]^{1/2} (-1)^{K-Q} \\ &\times \sum_{Q''} \begin{pmatrix} K & K & 1 \\ Q'' & -Q'' & 0 \end{pmatrix} \sum_{K'} (2K'+1) \\ &\times \begin{pmatrix} K & K & K' \\ -Q'' & Q'' & 0 \end{pmatrix} \begin{pmatrix} K & K & K' \\ -Q & Q & Q'' \end{pmatrix} \mathcal{D}_{0Q''}^{K'K^*}(R) \end{aligned} \quad (\text{A3})$$

We then perform the sum over  $Q''$ :

$$\sum_{Q''} \begin{pmatrix} K & K & 1 \\ Q'' & -Q'' & 0 \end{pmatrix} \begin{pmatrix} K & K & K' \\ -Q'' & Q'' & 0 \end{pmatrix} = -\frac{1}{3} \delta_{K',1} \quad (\text{A4})$$

and substituting in (A3) we finally obtain, after some transformations, Eq. (10) of the text.

From Eq. (10) one can note that the magnetic kernel only connects  $Q$ -values such that  $Q - Q' = 0, \pm 1$ . Moreover, the following symmetry properties can be easily proven:

$$\begin{aligned} (\mathcal{K}_{QQ'}^K)^* &= \mathcal{K}_{Q'Q}^K, \\ \mathcal{K}_{QQ'}^K &= (-1)^{1+Q'-Q} \mathcal{K}_{-Q'-Q}^K. \end{aligned} \quad (\text{A5})$$

## Appendix B: calculations for the multipole coupling coefficients

For the evaluation of Eq. (21) we start by observing that, due to Eqs. (A7) of Paper III, we can write:

$$\begin{aligned} \sum_{i=0}^3 t_P^K(i) t_P^{K'}(i) &= \frac{3}{2} [(2K+1)(2K'+1)]^{1/2} \\ &\times \left\{ \begin{pmatrix} 1 & 1 & K \\ 1 & -1 & -P \end{pmatrix} \begin{pmatrix} 1 & 1 & K' \\ 1 & -1 & -P' \end{pmatrix} \right. \\ &+ \begin{pmatrix} 1 & 1 & K \\ -1 & 1 & -P \end{pmatrix} \begin{pmatrix} 1 & 1 & K' \\ -1 & 1 & -P' \end{pmatrix} \\ &+ \begin{pmatrix} 1 & 1 & K \\ 1 & 1 & -P \end{pmatrix} \begin{pmatrix} 1 & 1 & K' \\ -1 & -1 & -P' \end{pmatrix} \\ &\left. + \begin{pmatrix} 1 & 1 & K \\ -1 & -1 & -P \end{pmatrix} \begin{pmatrix} 1 & 1 & K' \\ 1 & 1 & -P' \end{pmatrix} \right\}. \end{aligned} \quad (\text{B1})$$

Evaluating this sum we obtain, for the only non-vanishing terms, the following results:

$$\begin{aligned} \sum_i t_0^0(i) t_0^0(i) &= 1, \\ \sum_i t_0^1(i) t_0^1(i) &= 3/2, \\ \sum_i t_0^2(i) t_0^0(i) &= 1/\sqrt{2}, \\ \sum_i t_0^2(i) t_0^2(i) &= 1/2, \\ \sum_i t_2^2(i) t_{-2}^2(i) &= 3/2 \end{aligned} \quad (\text{B2})$$

and substituting into Eq. (21), we obtain for the only non-vanishing coefficients:

$$\begin{aligned} G_{00,00}(\Omega) &= 1, \\ G_{10,10}(\Omega) &= \frac{3}{2} a_1 (-1)^Q \mathcal{D}_{0Q}^1(R) \mathcal{D}_{0-Q}^1(R), \\ G_{20,00}(\Omega) &= \frac{1}{\sqrt{2}} a_2 (-1)^Q \mathcal{D}_{0-Q}^2(R), \end{aligned}$$

$$G_{00,20}(\Omega) = \frac{1}{\sqrt{2}} a_2 \mathcal{D}_{0Q}^2(R),$$

$$\begin{aligned} G_{20,20}(\Omega) &= \frac{1}{2} a_3 (-1)^Q [\mathcal{D}_{0Q}^2(R) \mathcal{D}_{0-Q}^2(R) + 3(\mathcal{D}_{-2Q}^2(R) \mathcal{D}_{2-Q}^2(R) \\ &+ \mathcal{D}_{2Q}^2(R) \mathcal{D}_{-2-Q}^2(R))] \end{aligned} \quad (\text{B3})$$

where we have introduced the notations:

$$\begin{aligned} a_1 &= (w_{J'J}^1)^2 = W_1(J, J'), \\ a_2 &= w_{J'J}^{(2)}, \\ a_3 &= (w_{J'J}^2)^2 = W_2(J, J'); \end{aligned} \quad (\text{B4})$$

a table of these symbols for the various atomic transitions can be found in Paper III; for the simplest case of a transition  $J = 0$ ,  $J' = 1$  we just have:  $a_1 = a_2 = a_3 = 1$ .

The multipole coupling coefficients obey some symmetry relations that can be easily deduced from their definition:

$$\begin{aligned} (G_{KQ,K'Q'})^* &= G_{K'Q',KQ}, \\ G_{K-Q,K'-Q'} &= (-1)^{Q+Q'} G_{K'Q',KQ}. \end{aligned} \quad (\text{B5})$$

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